Acoustic band gaps of immersed porous structures consisting of alternating plates and linear arrays of cylinders.

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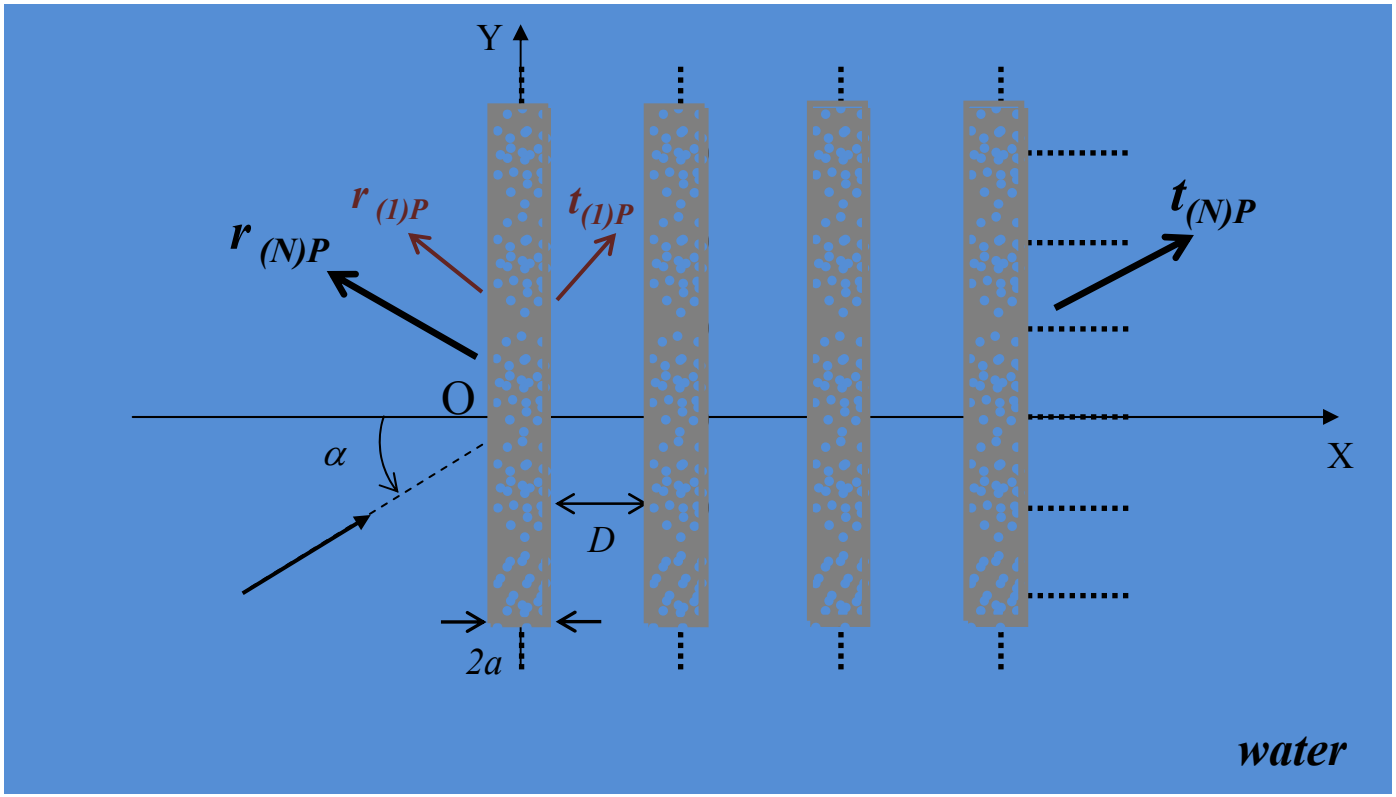
OUTLINE

- **I - Periodic systems of porous plates**
- **II - Periodic systems of linear arrays of porous cylinders**
- **III - Periodic systems of alternating plates and linear arrays of cylinders**
- **IV - Conclusion**

QF20 and saturating water parameters (All data from Johnson et al. (1994) except pore radius estimated by us).

Bulk modulus of grains	K_r (Pa)	36.6×10^9
Dried frame bulk modulus	K_b (Pa)	9.47×10^9
Dried frame shear modulus	μ (Pa)	7.63×10^9
Solid density	ρ_s (kg m^{-3})	2760
Bulk modulus of water	K_0 (Pa)	2.22×10^9
Density of water	ρ_0 (kg m^{-3})	1000
Viscosity of saturating water	η (kg m^{-1} s^{-1})	1.14×10^{-3}
Porosity	ϕ	0.402
Permeability	k (m^2)	1.68×10^{-11}
Pores radius	a_p (m)	3.26×10^{-5}
Tortuosity	α	1.89

I - Periodic systems of porous plates



Structure consists of N identical water saturated porous plates separated by water

I - Periodic systems of porous plates

- *Reflection and transmission coefficients by the system of N plates*

$$r_{(N)P} = r_{(N-1)P} + \frac{r_{(1)P} t^2_{(N-1)P} e^{-i\varphi_r}}{1 - r_{(1)P} r_{(N-1)P} e^{-i\varphi_r}}$$

$$t_{(N)P} = \frac{t_{(1)p}\left(t_{(N-1)P}\right) e^{-i\frac{\varphi_r}{2}}}{1 - r_{(1)P} r_{(N-1)P} e^{-i\varphi_r}} \quad with\ \varphi_r = 2k_0 D\cos\alpha$$

- *The application of the Bloch–Floquet's theorem yields the dispersion equation :*

$$\cos(\Gamma D) = \frac{1}{2t_{(1)P}}\left[\left(t^2_{(1)P} - r^2_{(1)P} + 1\right)\cos\chi + i\left(t^2_{(1)P} - r^2_{(1)P} - 1\right)\sin\chi\right]$$

$$with\ \Gamma = \Gamma' + i\Gamma''\ and\ \chi = \varphi_r/2$$

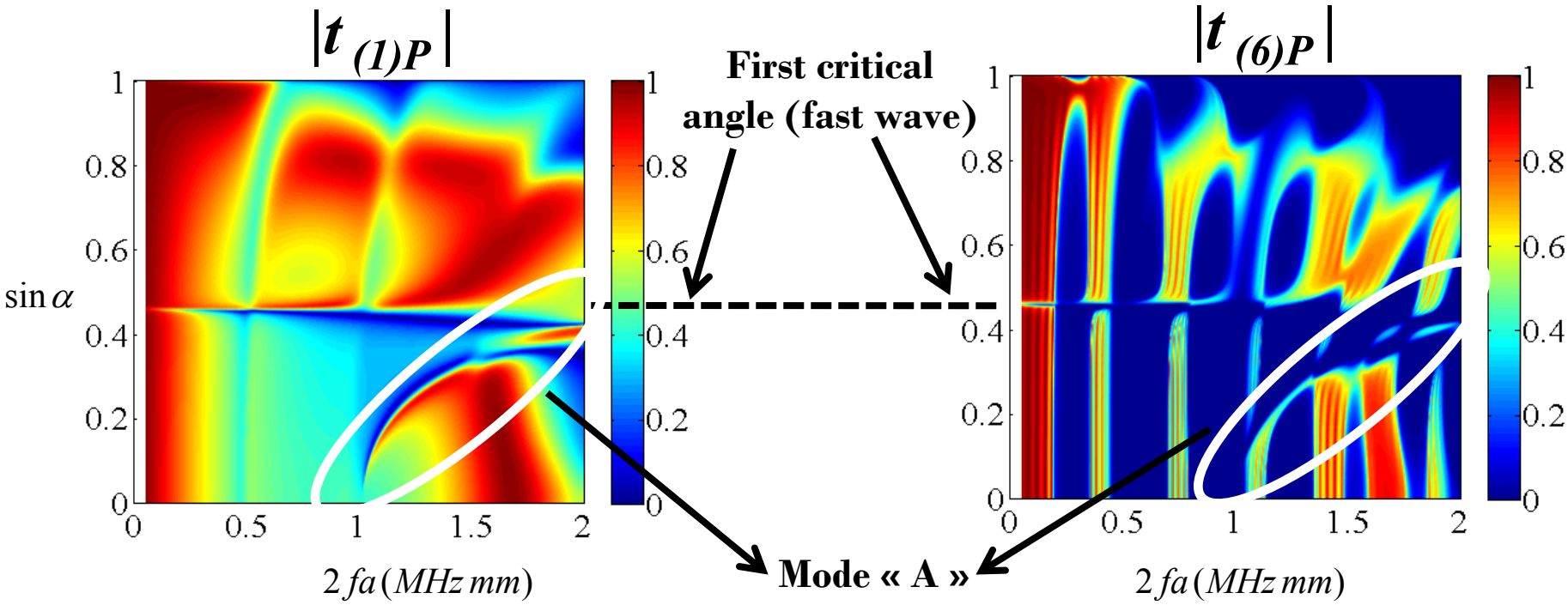
Trabelsi et al. (2014)

I - Periodic systems of porous plates

Sealed pore condition

System composed of **single** water-saturated porous plate (thickness 2a= 5*mm*)

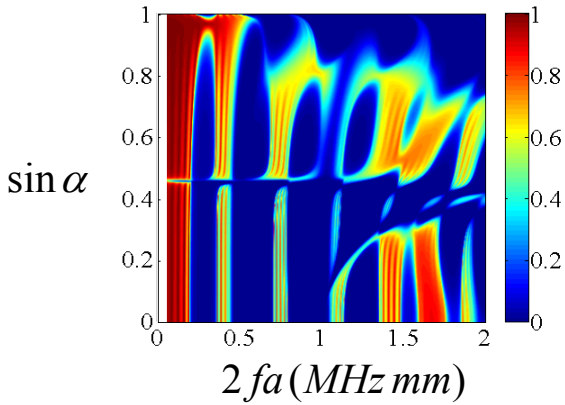
Finit periodic system composed of **6** water-saturated porous plates(thickness 2a= 5*mm*) separated by a layer of water (D= 10*mm*)



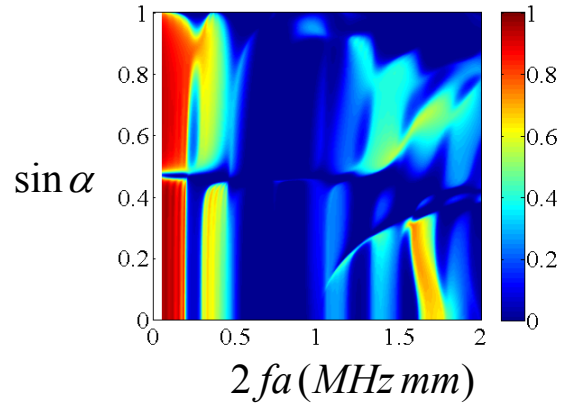
I - Periodic systems of porous plates

Sealed pores | **Open pores**

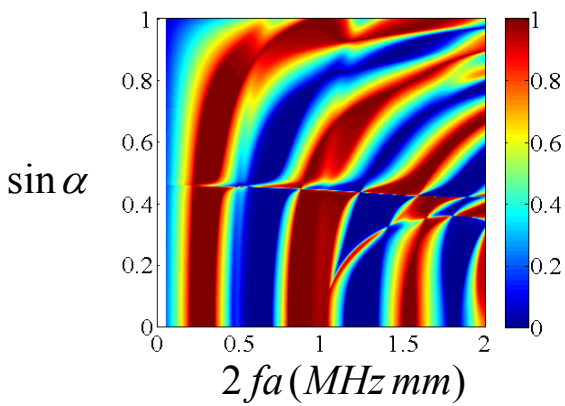
Finite system : 6 water-saturated porous plates(thickness 2a= 5*mm*) separated by a layer of water (D= 10*mm*)



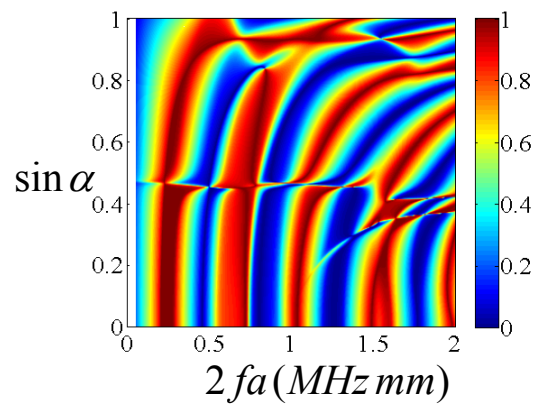
$|t_{(6)P}|$



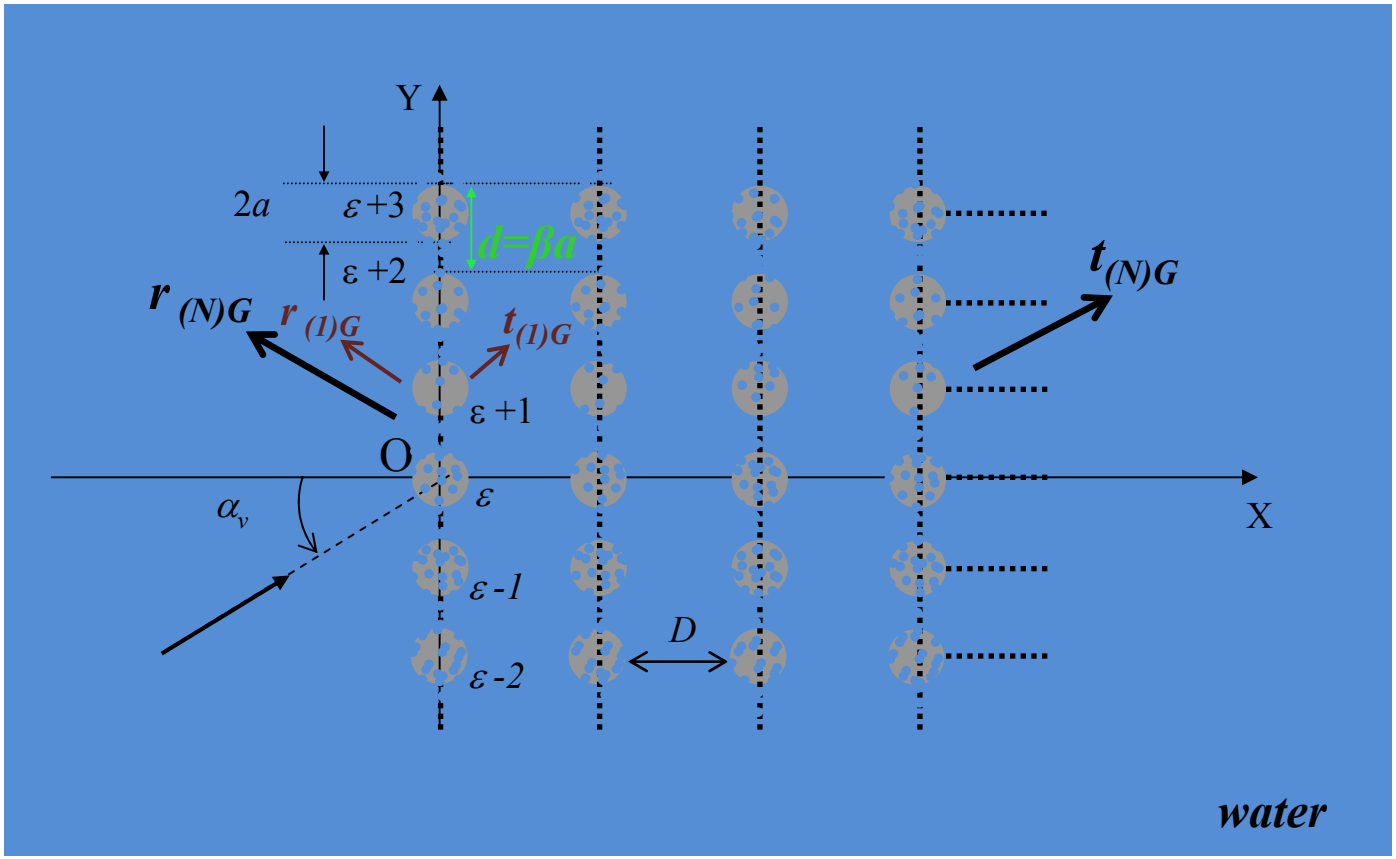
Infinite system : water-saturated porous plates(thickness 2a= 5*mm*) separated by a layer of water (D= 10*mm*)



$$\frac{\Gamma' D}{\pi}$$



II - Periodic systems of linear arrays of porous cylinders



2-d grating consists of N infinite identical linear arrays separated by D

II - Periodic systems of linear arrays of porous cylinders

The linear system for computing the coefficients of multiple scattering C_m^ν between cylinders

Kronecker symbol

Schlömilch series (V. Twersky (1961))

$$\sum_{m=-\infty}^{+\infty}\left[\delta_{mn}-T_n\,\sigma\left(m-n,\alpha_\nu\right)\right]C_m^\nu=T_nA_n^\nu$$

Scattering coefficients by single cylinder

The amplitude reflection coefficient :

$$R^{\nu,\varepsilon}=A_R^{\nu,\varepsilon}\big/A_{inc}^\nu \quad \text{with} \quad A_R^{\nu,\varepsilon}=\frac{2}{k_\varepsilon^\nu d}\sum_{n=-\infty}^{+\infty}C_n^\nu\left(\frac{\ell_\varepsilon^\nu+ik_\varepsilon^\nu}{k}\right)^n; \quad \ell_\varepsilon^\nu=k\sin\alpha_\nu+2\pi\varepsilon/d$$

$$k_\varepsilon^\nu=+i\sqrt{\left(\ell_\varepsilon^\nu\right)^2-k^2}$$

The amplitude transmission coefficient :

$$T^{\nu,\varepsilon}=A_T^{\nu,\varepsilon}\big/A_{inc}^\nu \quad \text{with} \quad A_T^{\nu,\varepsilon}=A_{inc}^\nu\delta_{\varepsilon\nu}+\frac{2}{k_\varepsilon^\nu d}\sum_{n=-\infty}^{+\infty}C_n^\nu\left(\frac{\ell_\varepsilon^\nu-ik_\varepsilon^\nu}{k}\right)^n$$

Only the zeroth diffraction order $\varepsilon=0$ propagates at frequencies such as : $f\le f_c=\dfrac{c}{d(1+\sin\alpha_\nu)}$

Mulholland et al. (1994) & Robert et al. (2006)

II - Periodic systems of linear arrays of porous cylinders

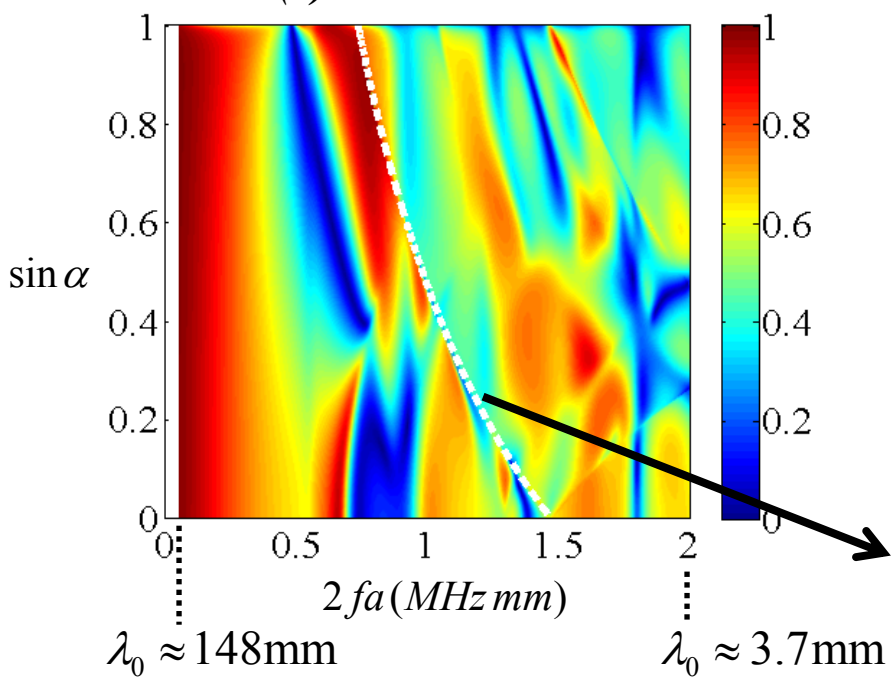
II.1. Finite periodic structures

Sealed pore condition

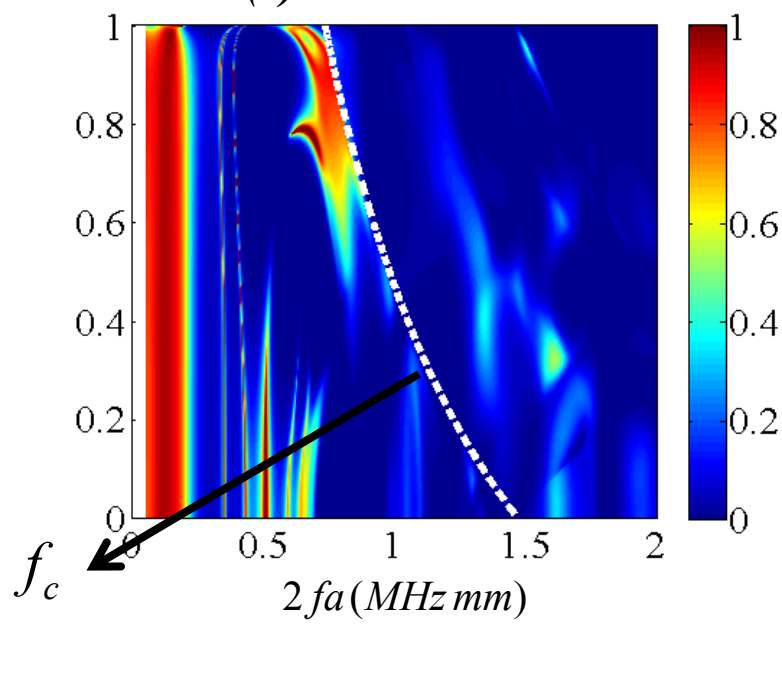
System composed of a **single** grating (diameter of cylinders 2a= 5*mm*)

Finite periodic system composed of **6** gratings (diameter of cylinders 2a= 5*mm*) separated by a layer of water (D= 10*mm*)

$|t_{(1)G}|$ *for* $\beta = 2.01$



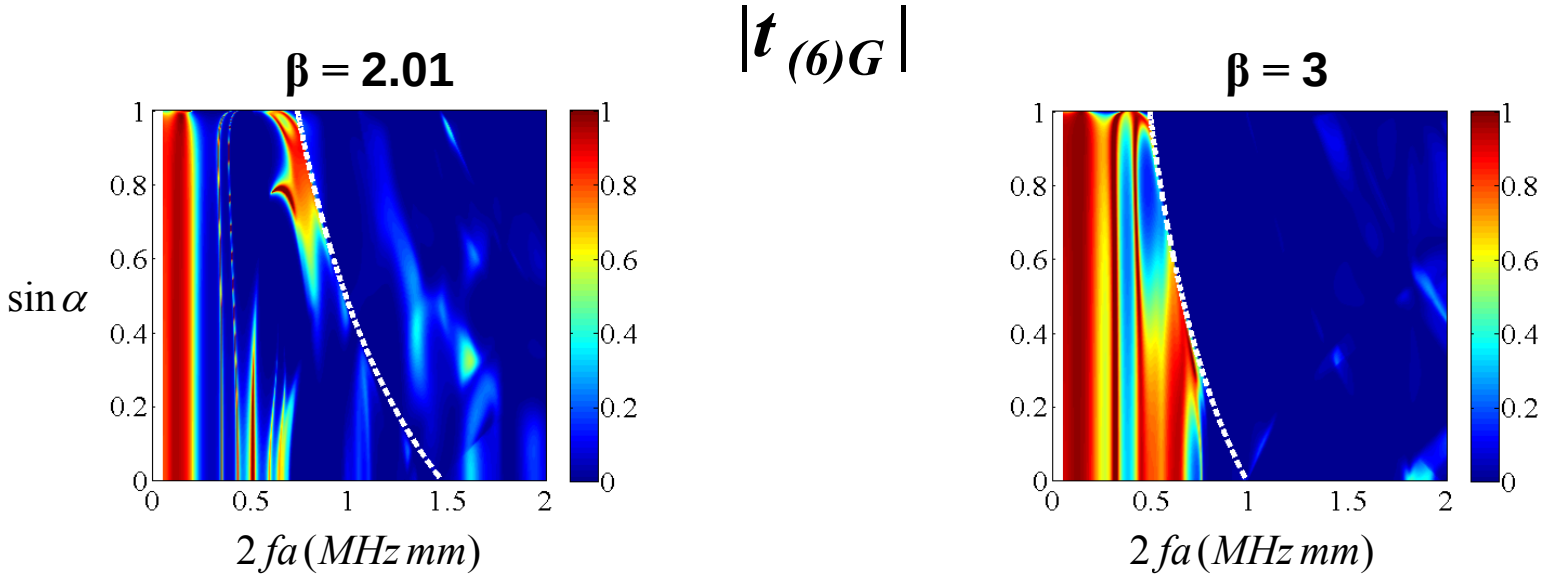
$|t_{(6)G}|$ *for* $\beta = 2.01$



II - Periodic systems of linear arrays of porous cylinders

II.1. Finite periodic structures

Sealed pore condition

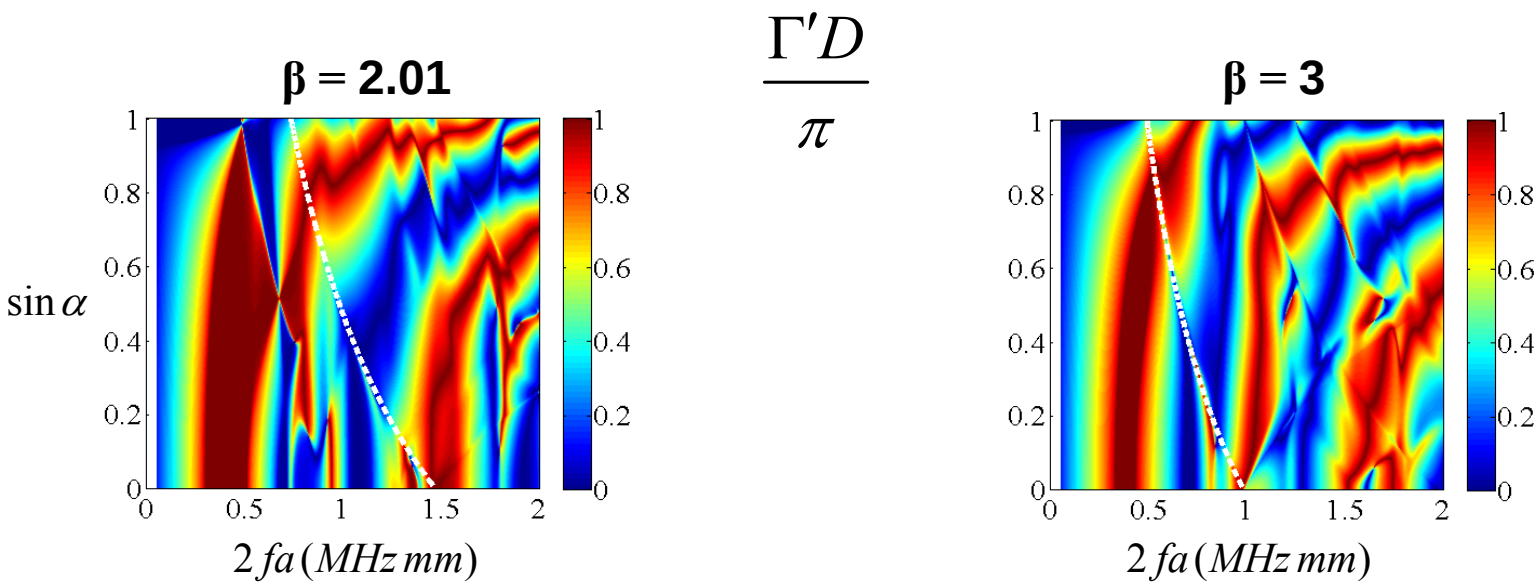


Effect of β on $t_{(6)G}$

II - Periodic systems of linear arrays of porous cylinders

II.2. Infinite periodic structures

Sealed pore condition

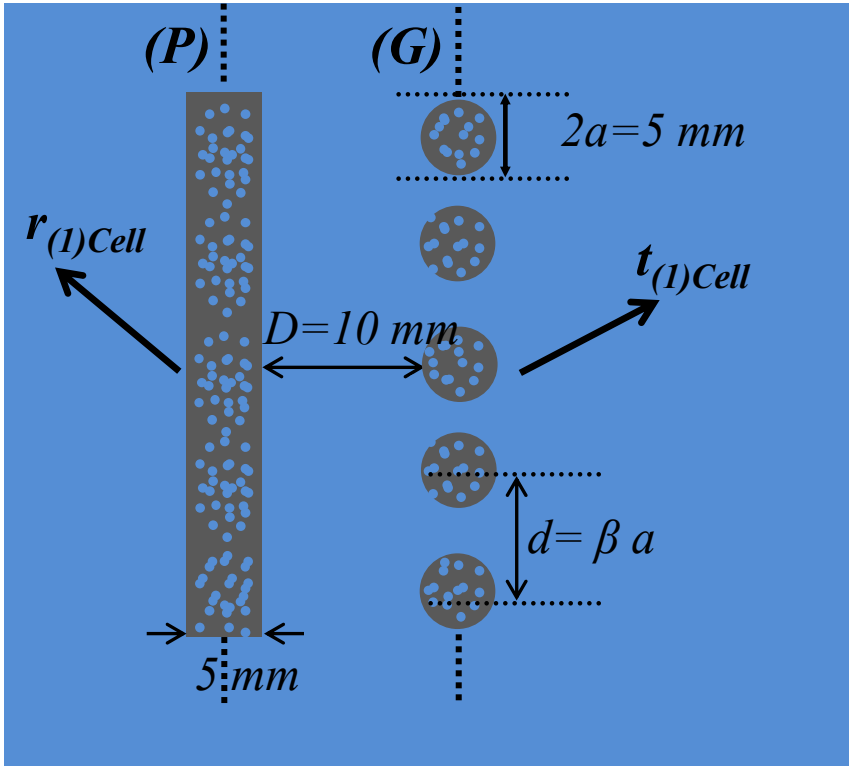


Effect of β on $\frac{\Gamma' D}{\pi}$

III - Periodic systems of alternating plates and linear arrays of cylinders

III.1. Finite periodic structures

III.1.1. Unit cell



$$\begin{cases} r_{(1)cell} = r_{(1)i} + \dfrac{r_{(1)j} t_{(1)i}^2 e^{-i\varphi_r}}{1 - r_{(1)i} r_{(1)j} e^{-i\varphi_r}} \\ t_{(1)cell} = \dfrac{t_{(1)i} \left(t_{(1)j} \right) e^{-i\frac{\varphi_r}{2}}}{1 - r_{(1)i} r_{(1)j} e^{-i\varphi_r}} \end{cases};$$

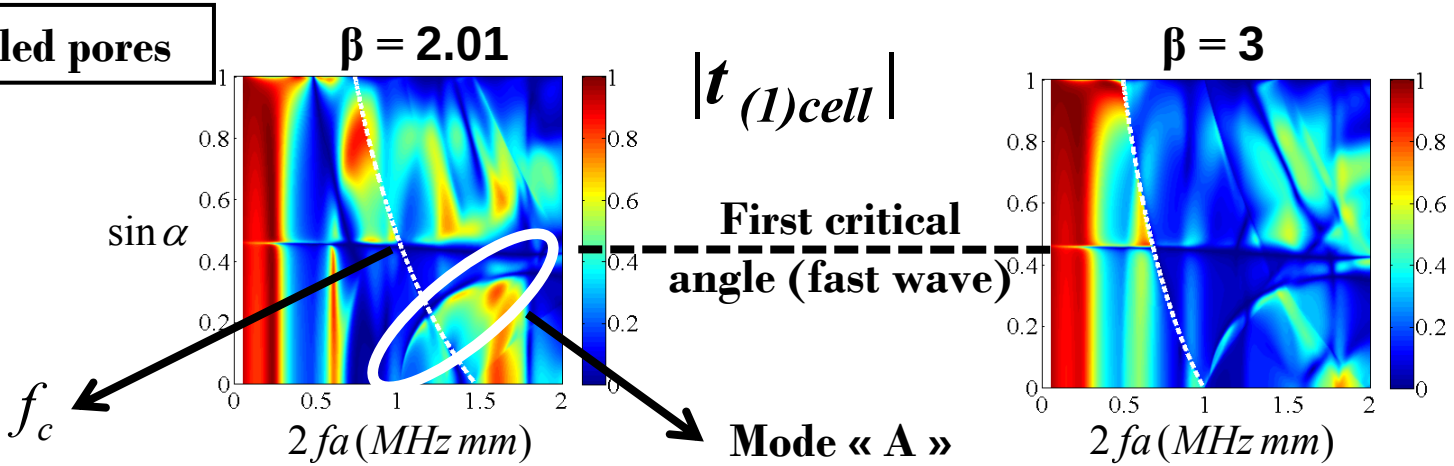
$$i \neq j = P\,(for\ plate)\ or\ G\,(for\ grating)$$

III - Periodic systems of alternating plates and linear arrays of cylinders

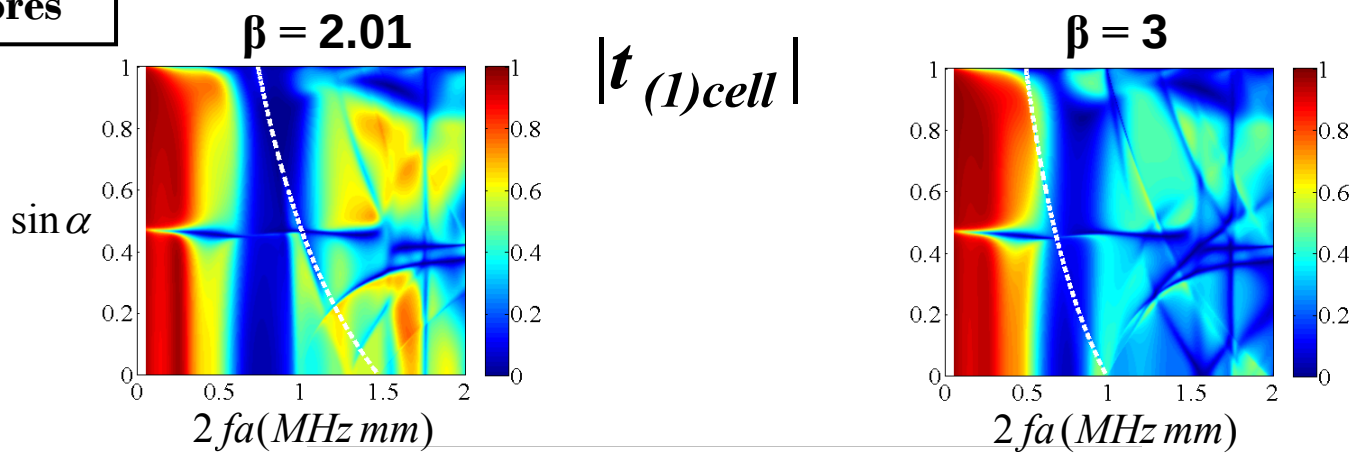
III.1. Finite periodic structures

III.1.1. Unit cell

Sealed pores



Open pores



III - Periodic systems of alternating plates and linear arrays of cylinders

III.1. Finite periodic structures

III.1.2. *N* cells *(N≥2)*

$$r_{(N)cell} = r_{(N-1)cell} + \frac{r_{(1)cell} t^2_{(N-1)cell} e^{-i\varphi_r}}{1 - r_{(1)cell} r_{(N-1)cell} e^{-i\varphi_r}}$$

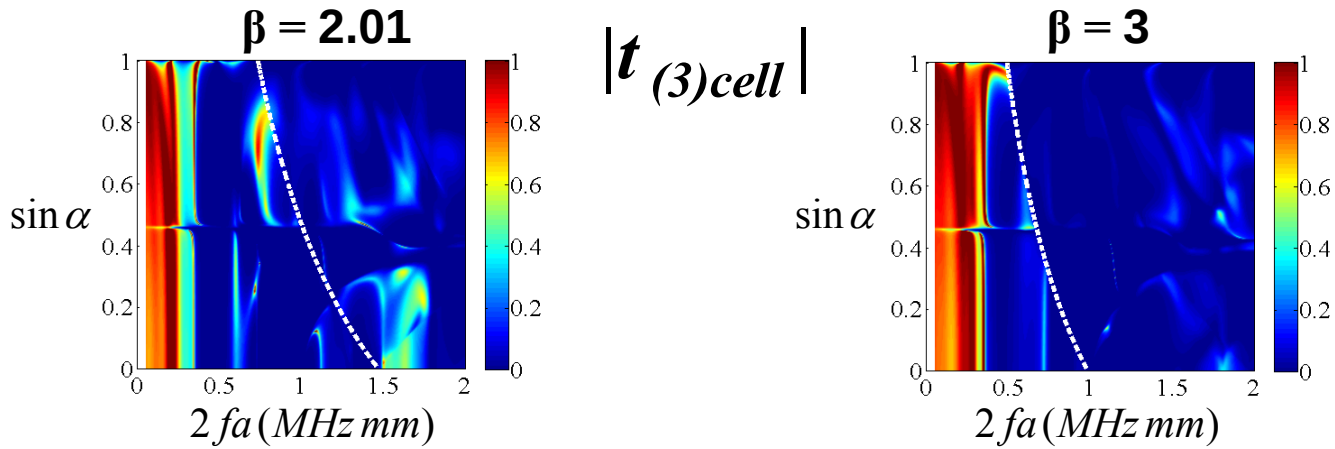
$$t_{(N)cell} = \frac{t_{(1)cell} \left(t_{(N-1)cell} \right) e^{-i\frac{\varphi_r}{2}}}{1 - r_{(1)cell} r_{(N-1)cell} e^{-i\varphi_r}}$$

III - Periodic systems of alternating plates and linear arrays of cylinders

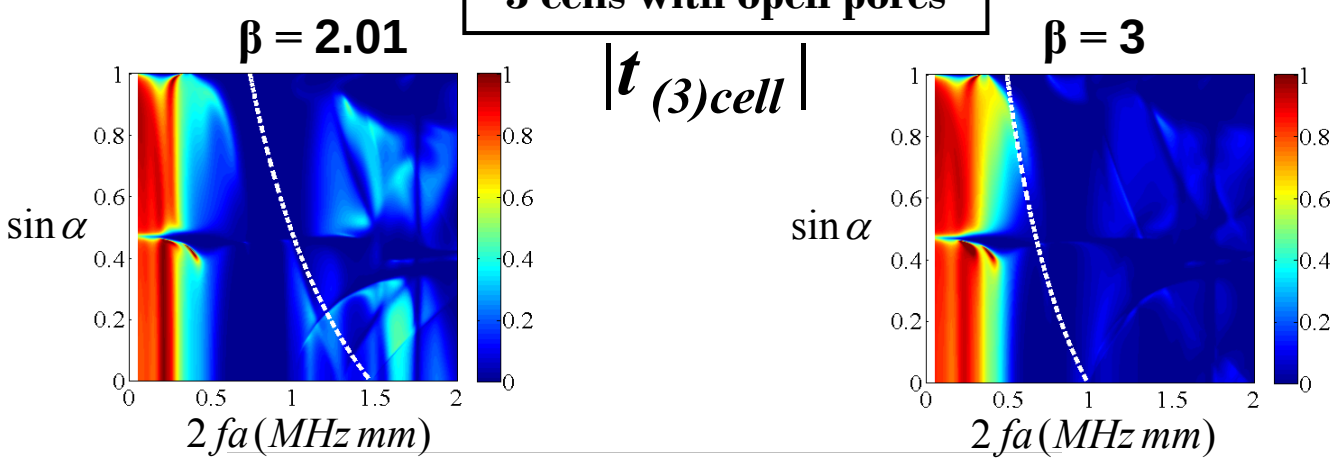
III.1. Finite periodic structures

III.1.2. *N* cells (*N*≥2)

3 cells with sealed pores



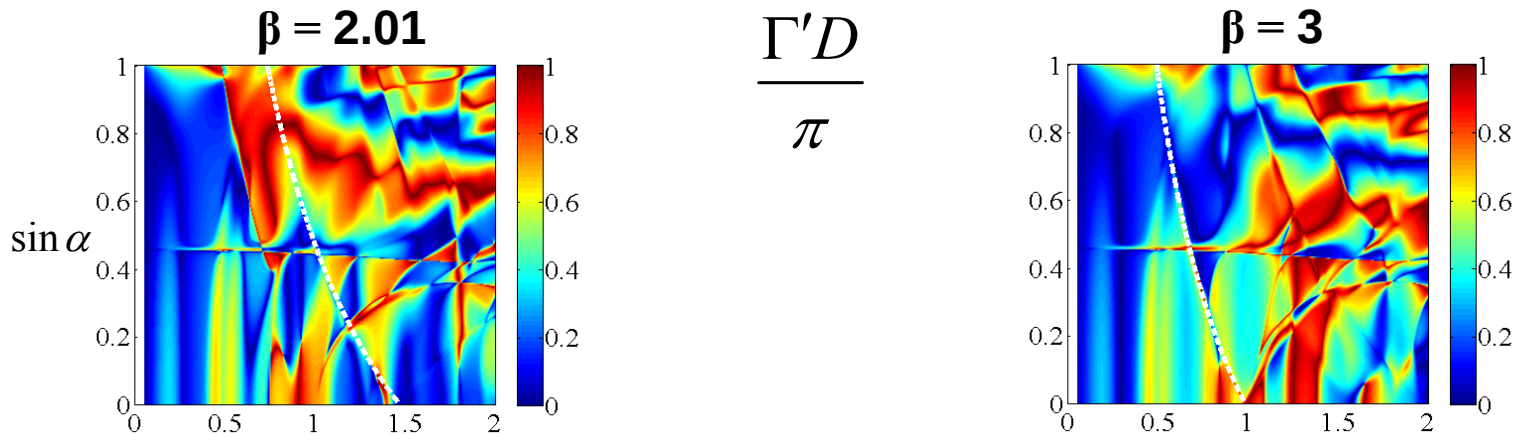
3 cells with open pores



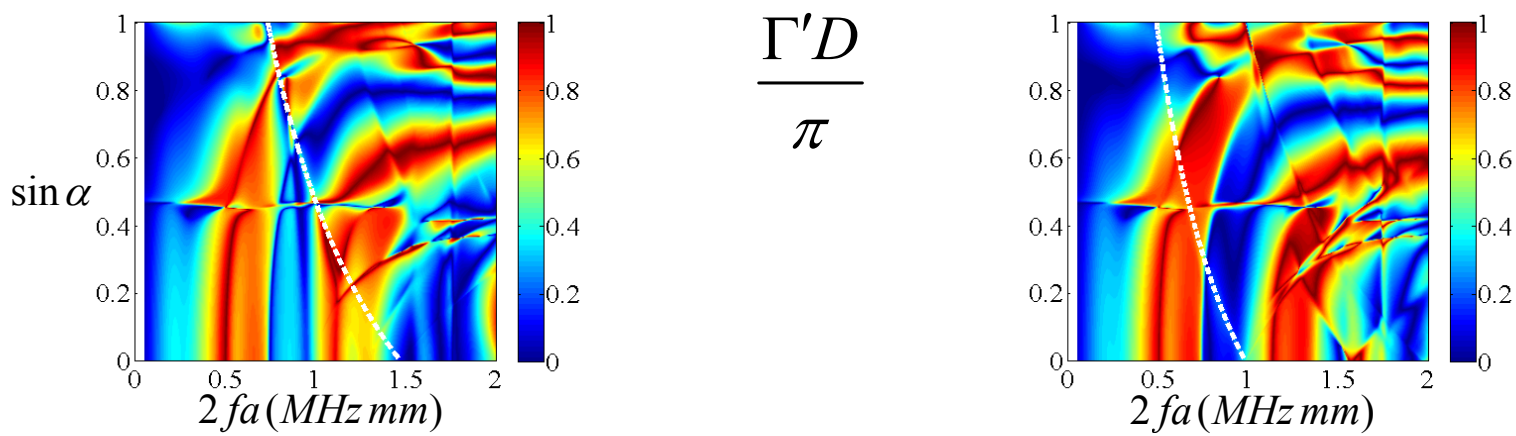
III - Periodic systems of alternating plates and linear arrays of cylinders

III.2. Infinite periodic structures

Sealed pores condition



Open pores condition

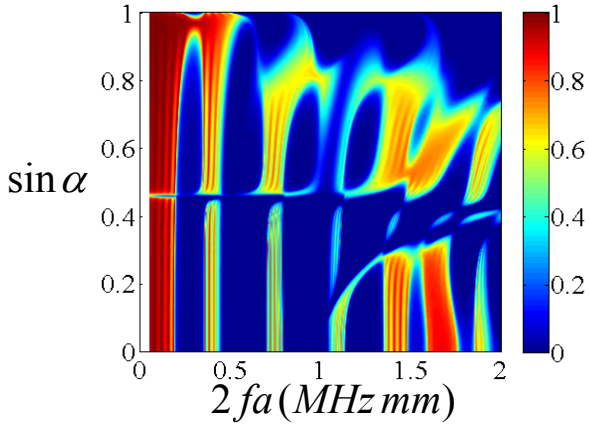


III - Periodic systems of alternating plates and linear arrays of cylinders

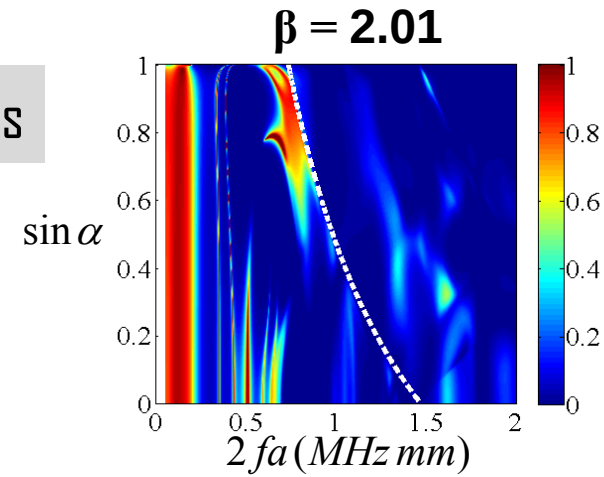
III.2. Finite periodic structures

Sealed pores condition

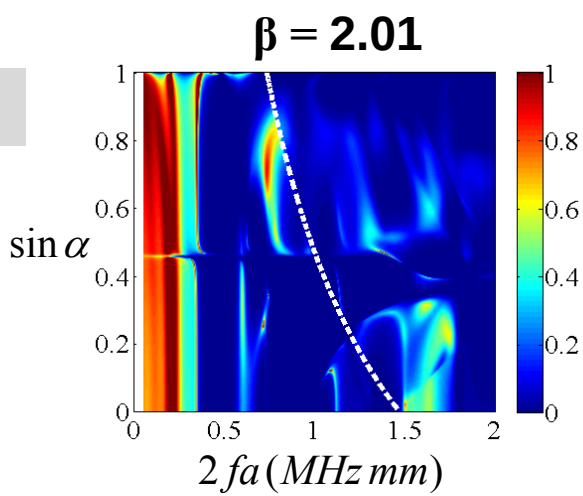
6 plates



6 gratings



3 cells

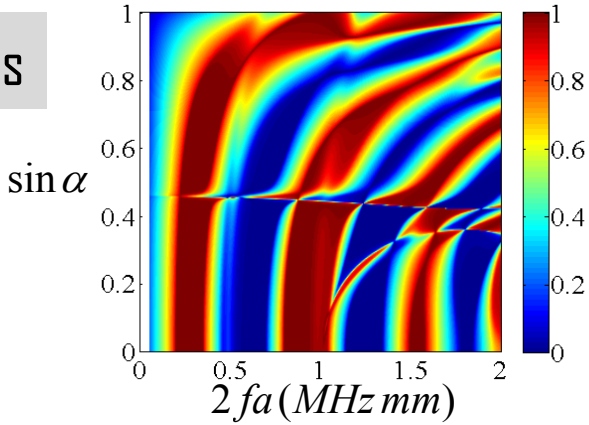


III - Periodic systems of alternating plates and linear arrays of cylinders

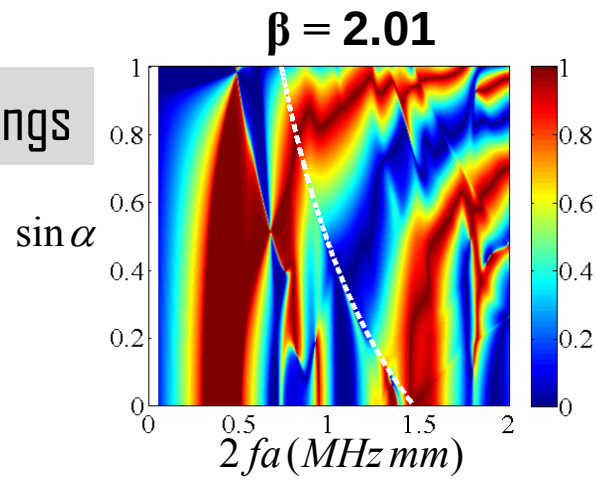
III.2. Infinite periodic structures

Sealed pores condition

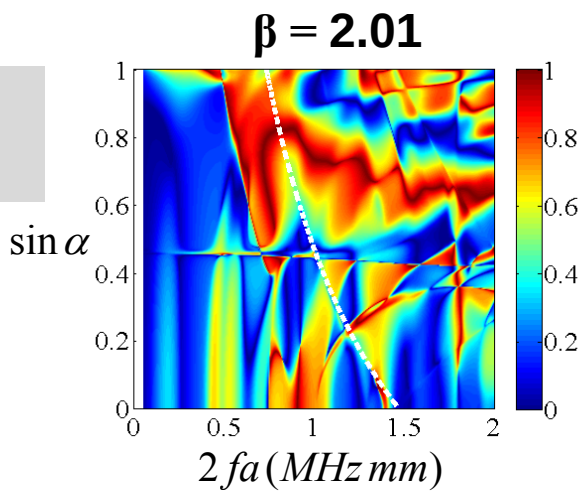
only plates



only gratings



Alternating plates and gratings



III - Conclusion

- In the case of infinite structure, the alternation causes a reduction in the width of the stop-bands.
- The surface condition of plates and cylinders also has a relevant effect on the bandgaps.
- The increase of the vertical spacing between two successive cylinders modifies substantially the stop-bands.
- The effects of porosity and saturating fluid have not been considered here. These can show other interesting results.


LE HAVRE
THANK YOU

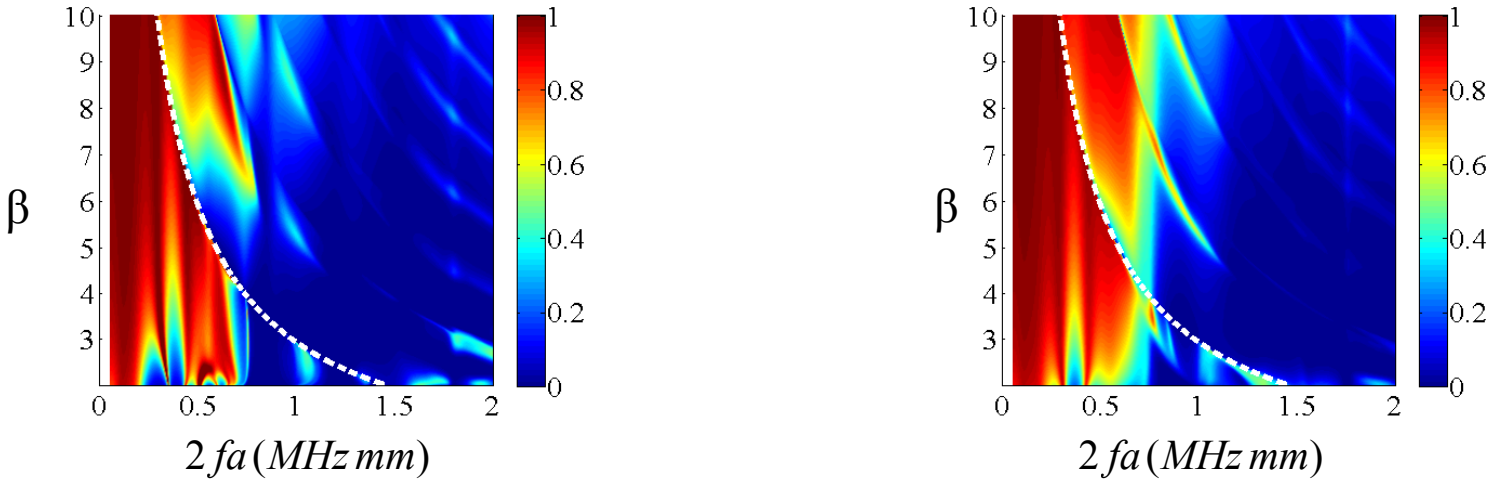
II - Periodic systems of linear arrays of porous cylinders

II.1. Finite periodic structures (normal incidence)

Sealed pores condition

$|t_{(6)G}|$

Open pores condition



Effect of β on $t_{(6)G}$

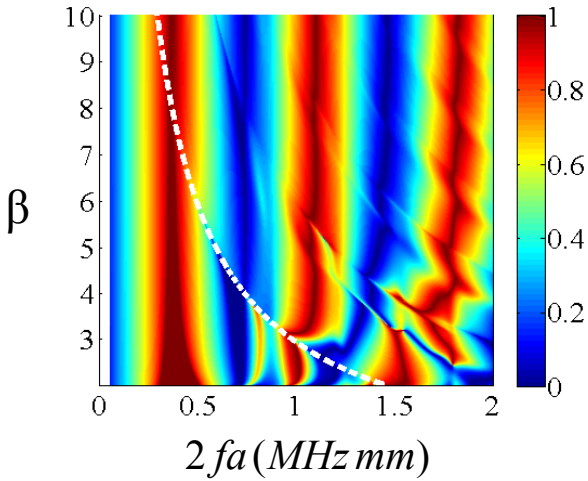
II - Periodic systems of linear arrays of porous cylinders

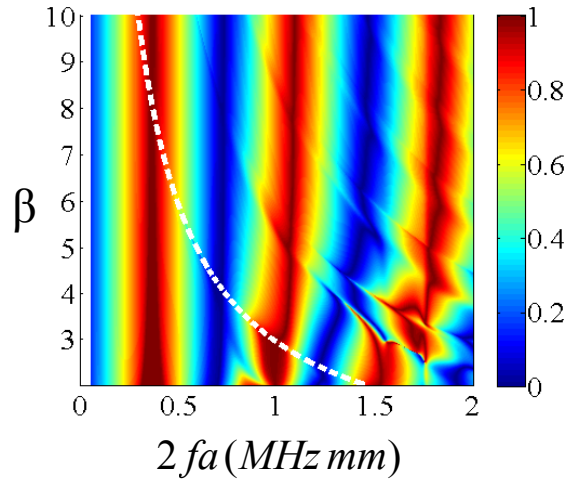
II.2. Infinite periodic structures (normal incidence)

Sealed pores condition

$$\frac{\Gamma' D}{\pi}$$

Open pores condition





Effect of β on $\frac{\Gamma' D}{\pi}$