# Numerical simulation and experimental observation of the viscothermal acoustic effects in irregular geometries


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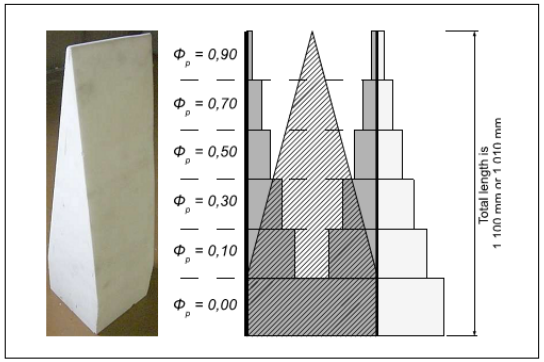
# Introduction




*[Becot et al. Acustica 2008]*

**Anechoic wedge,**
**Double porosity**

**Irregular Geometries**

**Increased Absorption**

?

**Localization**

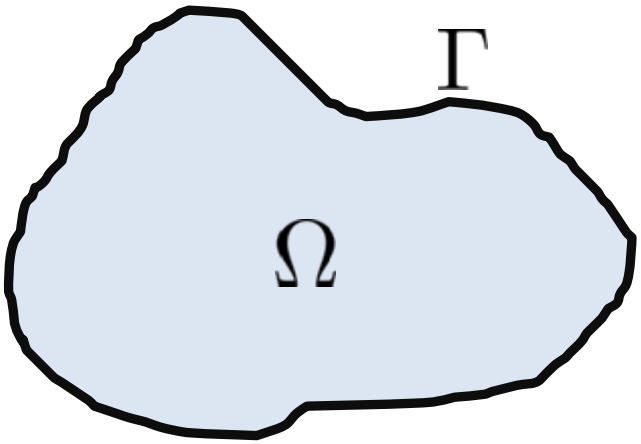
**Difficult experimental observation**

# Outline

- **Localization**
- **Localization and dissipation**
- **Pressure Measurement technique**

# I.1-Localization Acoustic field in a cavity

## Pressure field




$$(\Delta + k^2)p = 0 \qquad \Omega$$

$$\mathbf{n} \cdot \nabla p = -jk\epsilon p \qquad \Gamma$$

***Admittance***

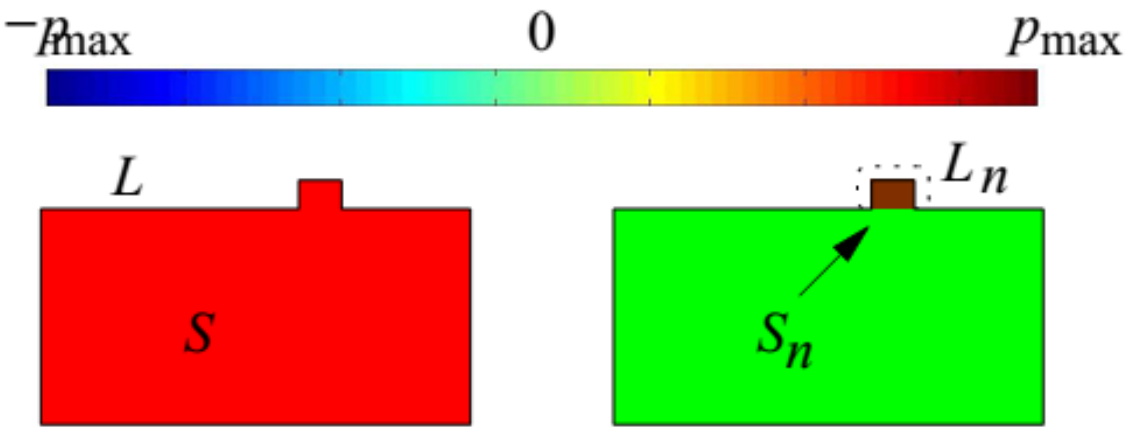
$$p(X,t) = \sum_{n=1}^{N} \Phi_n(X) e^{j\omega t}$$

- Rigid wall ($\epsilon \ll 1$)
- normal mode
- $\Delta \Phi_n(X) + k_n^2 \Phi_n(X) = 0 \qquad \Omega$

  $\mathbf{n} \cdot \nabla \Phi_n = 0 \qquad \Gamma$

# I.2-Localization

- **Localized modes**: vibrating energy of which is concentrated in a small region of the cavity




**Non-localized Mode** **Localized Mode**

# I.3-Localization - Characterization

- $\Phi_n(X)$: cavity mode
- $\phi_n(X)$ : normalized mode

$$\iiint_V |\phi_n(X)|^2 \, dV = 1$$

➢ ***Existence volume $V_n$***

$$V_n = \left[ \iiint_V |\phi_n(X)|^4 \, dV \right]^{-1}$$

➢ ***Relative existence volume***

$$V_{ER} = V_n / V$$

*[Thouless (1974), Sapoval, S. Felix (2006)]*

❖ ***Rectangular cavity***

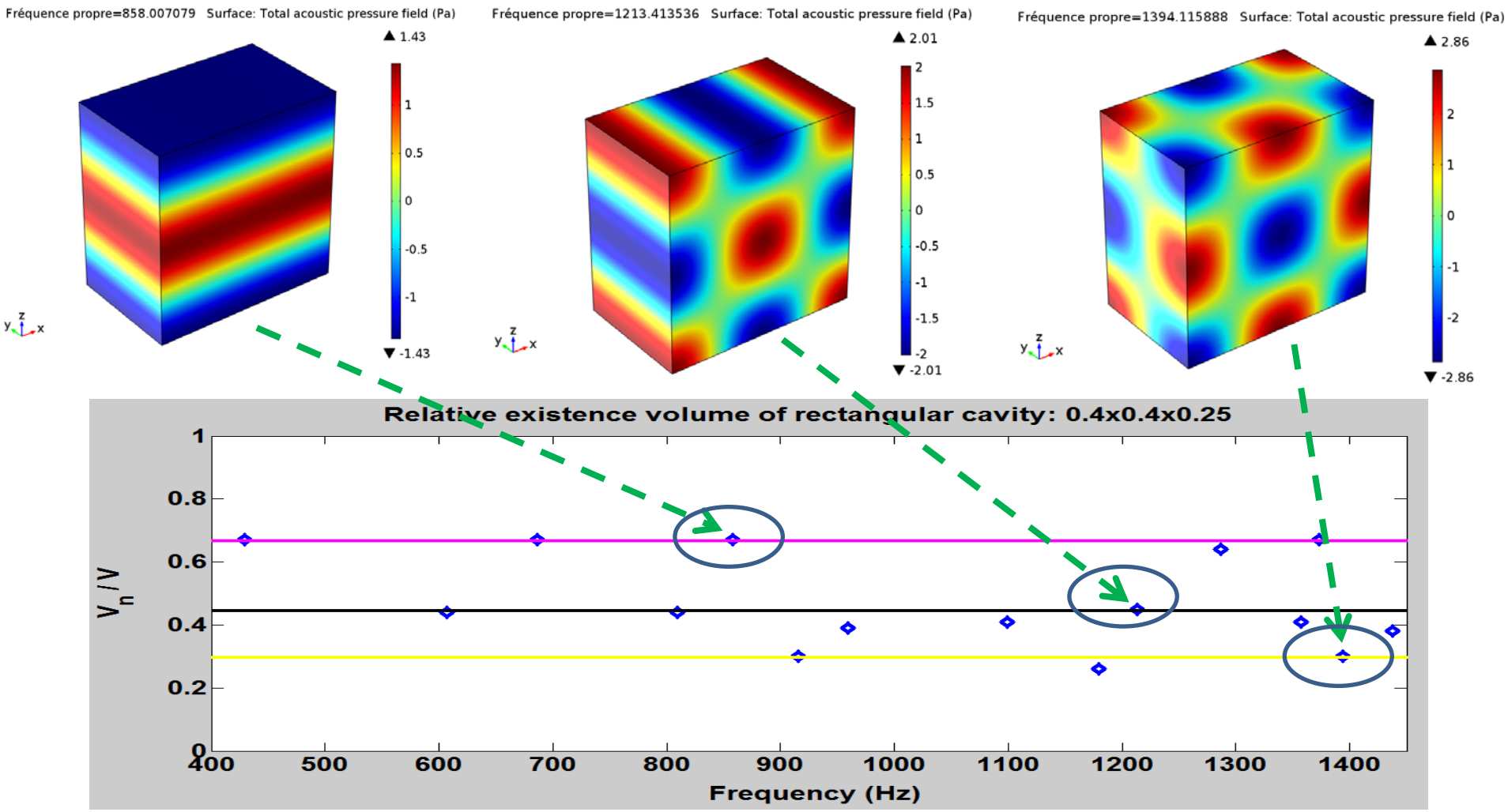
$$\Phi_n(X) = \Phi_{mnr}(X) = \cos\left(\frac{m\pi x}{L_x}\right) \cos\left(\frac{n\pi y}{L_y}\right) \cos\left(\frac{r\pi z}{L_z}\right)$$

$$V_n / V = (2/3)^n$$

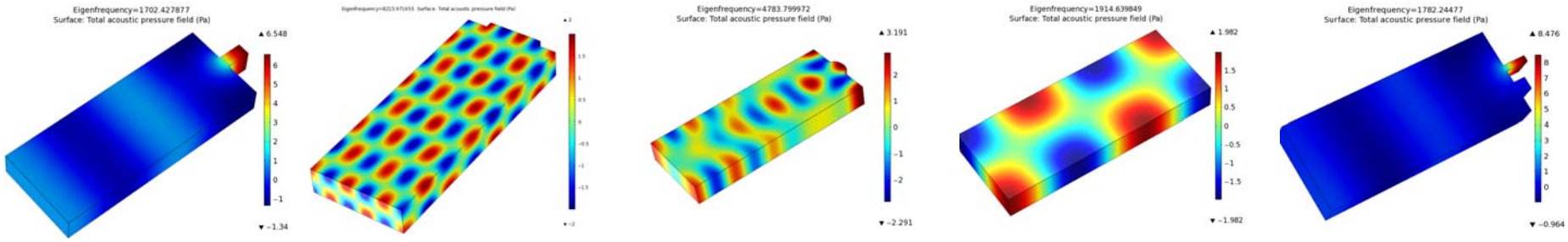
*n=Number of nonzeros index*

# I.3-Localization - Characterization

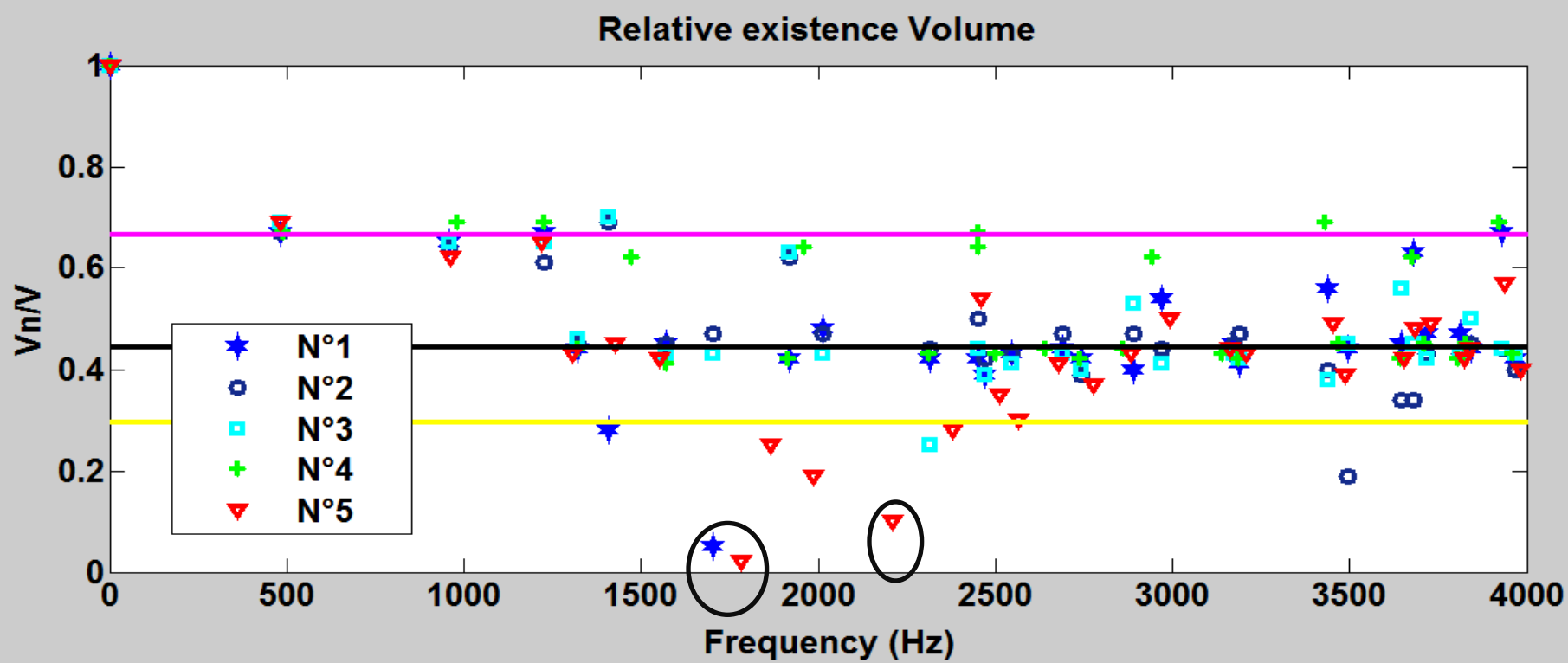
## ❖ *Rectangular cavity relative existence volume*



# I.3- Localization - few numerical results




N°1 N°2 N°3 N°4 N°5

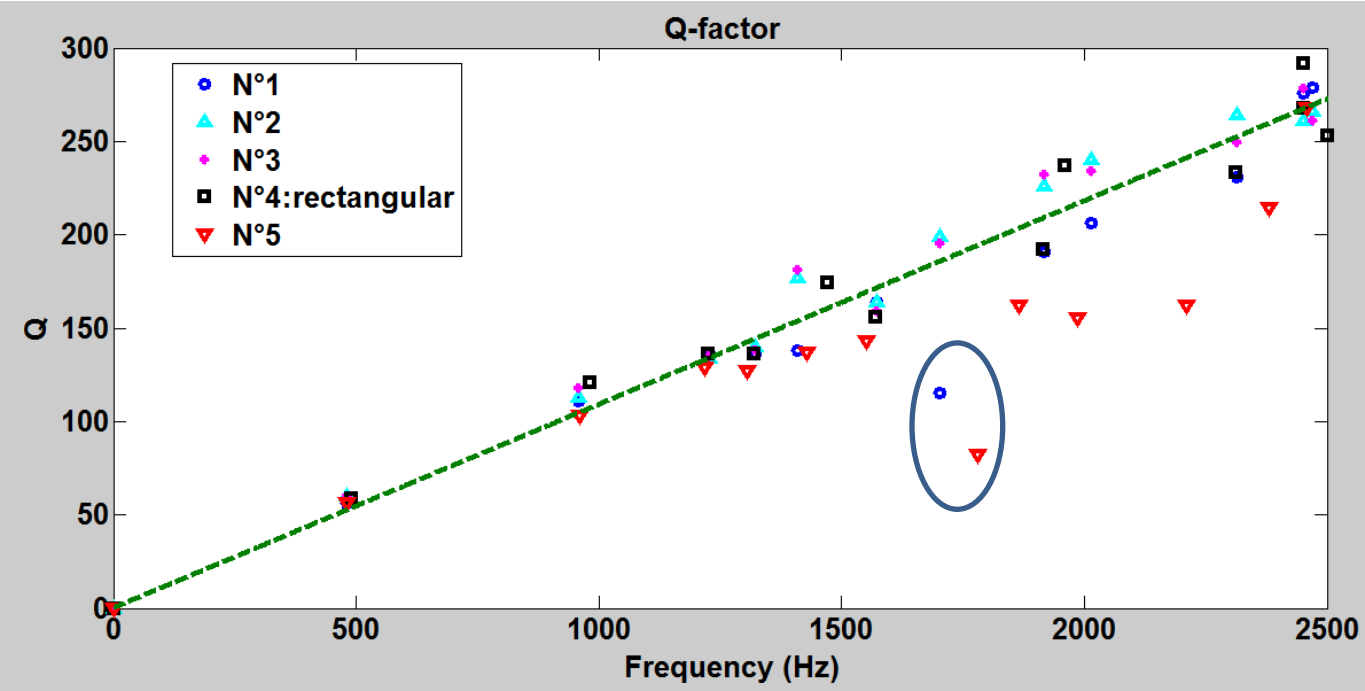


# II- Localization and dissipation

## Quality factor (Q)

$$Q = \frac{\omega E}{W} \Rightarrow Q_n = \frac{k_n \Lambda}{Re(\epsilon)} \Leftrightarrow \frac{1}{\Lambda} = \iint_S \phi_n(X)^2 \, dS$$

**E: Stored energy,** **W: average dissipated energy**




$$\epsilon = 0.002(1+i)$$

**Dissipation for:**
$Q$ **&** $\Lambda$ **small**
**High irregularity**

# II- Localization and dissipation

## Viscothermal effects

$$\delta_v = \sqrt{\frac{\mu}{\rho_0 \omega}}$$

Viscous and thermal boundary layers thicknesses

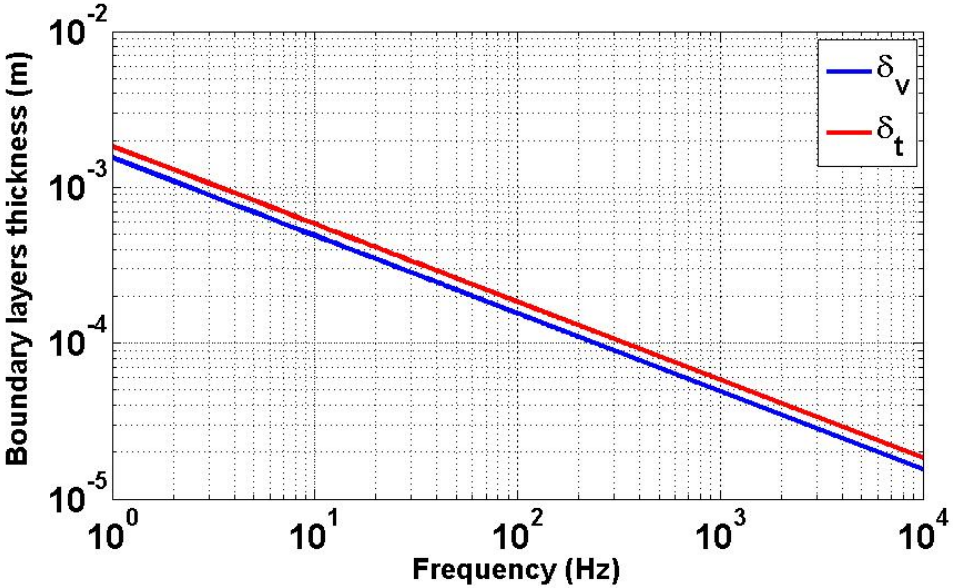
$$\delta_t = \sqrt{\frac{\kappa}{\rho_0 C_p \omega}}$$

$$\boxed{\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p} + \frac{1}{3}\mu \nabla(\nabla \cdot \mathbf{v}) + \mu \Delta \mathbf{v}$$

$$\boxed{\rho_0 \nabla \cdot \mathbf{v} + \frac{\partial \rho}{\partial t} = 0}$$

$$\rho_0 C_p \frac{\partial T}{\partial t} = \kappa \Delta T + \frac{\partial p}{\partial t}$$

$$\boxed{p = \rho R_0 T}$$



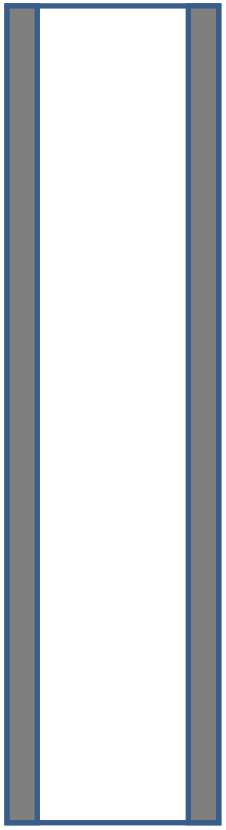

$$\begin{cases} \mu : \text{dynamic viscosity} \\ C_p : \text{specific heat at constant pressure} \\ \kappa : \text{thermal conductivity} \\ \rho_0 : \text{static density} \end{cases}$$

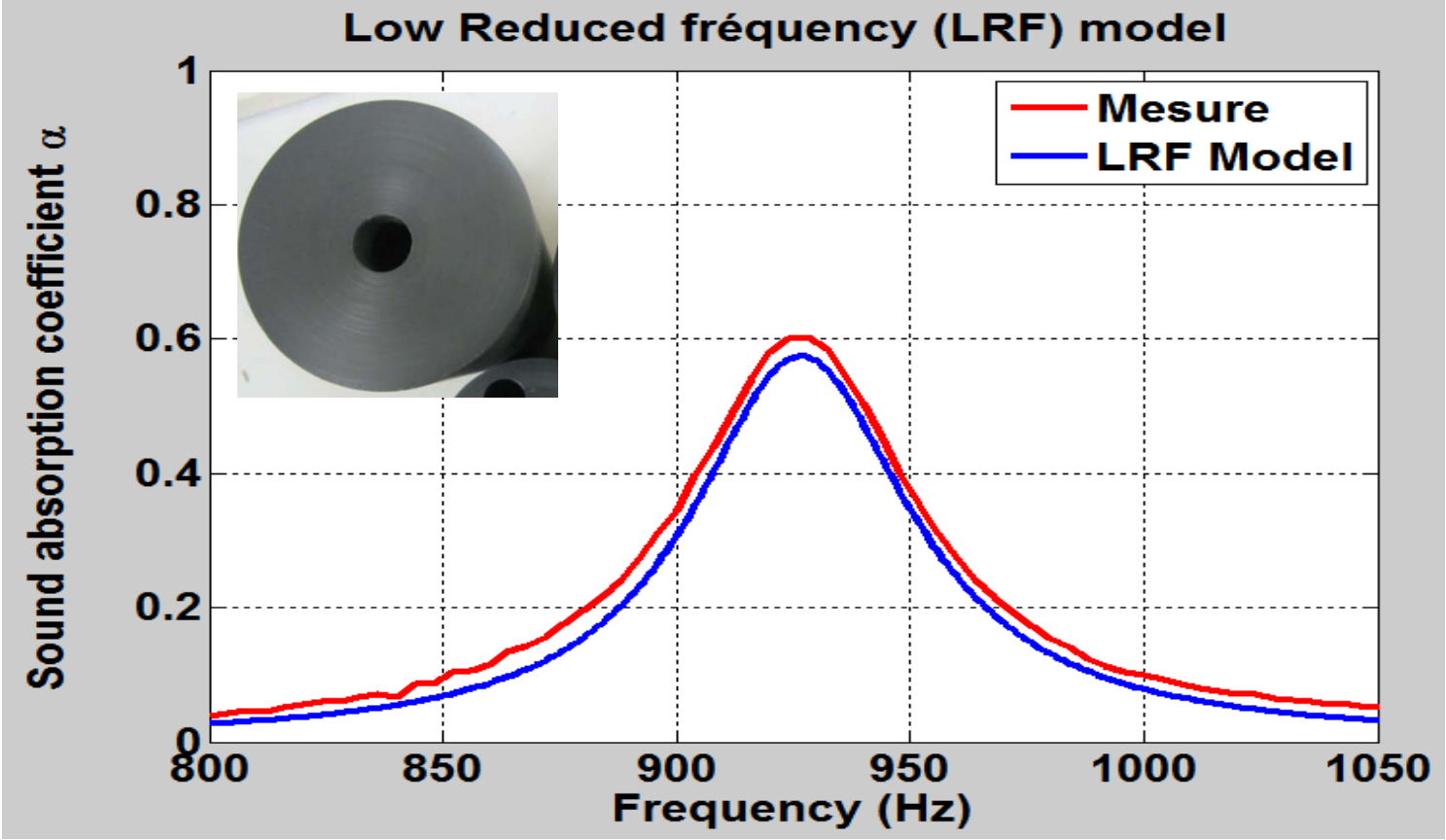
# II- Localization and dissipation

## Viscothermal effects: LRF Model results

**Tubes,…**

*[Zwikker & Kosten, 1949]*



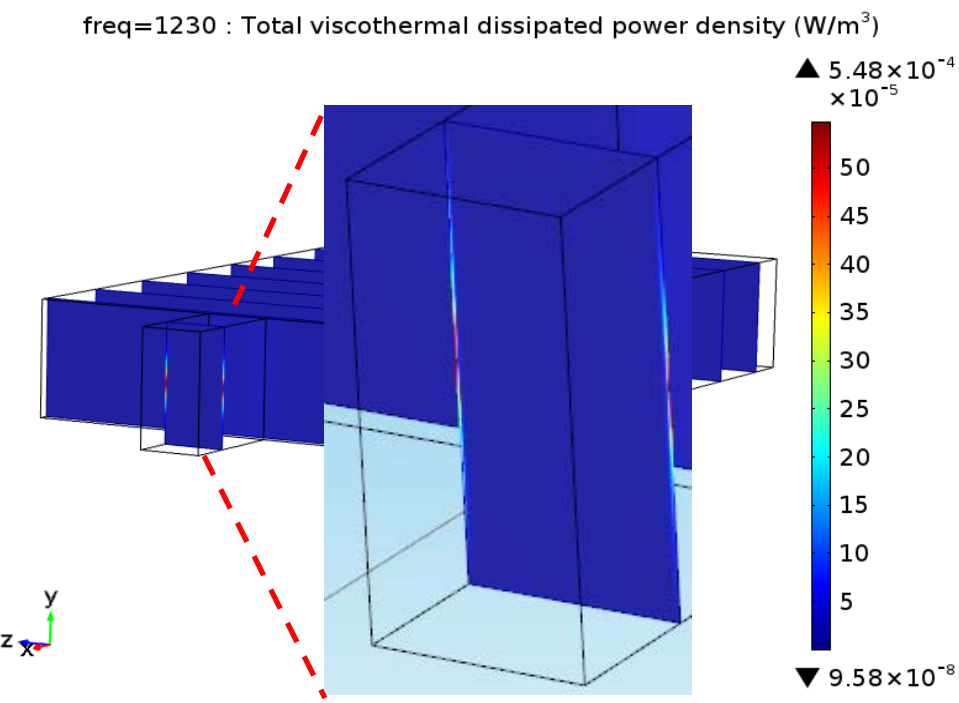


# II- Localization and dissipation

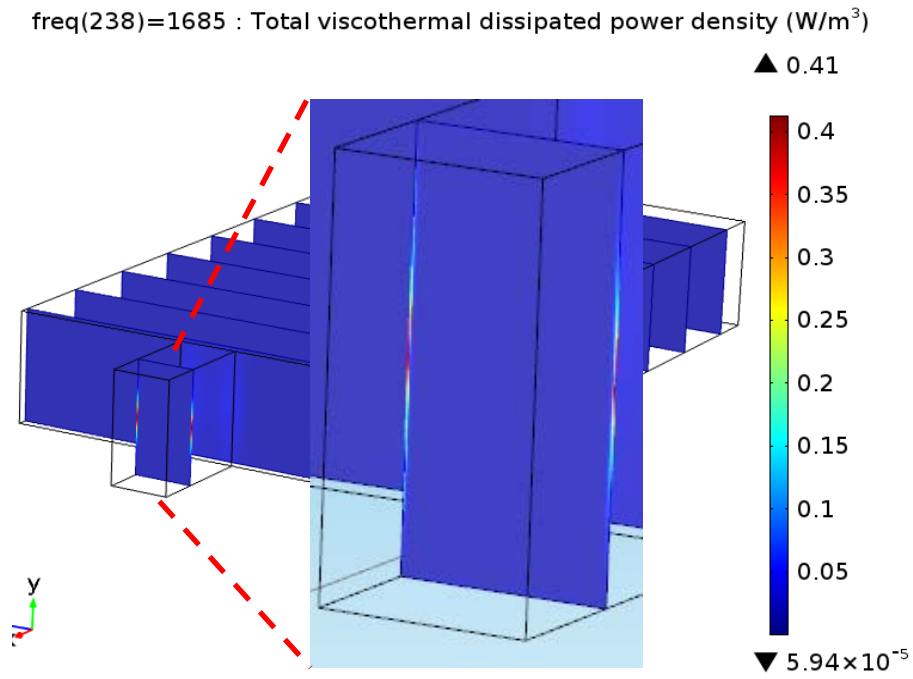
## Viscothermal effects: dissipated power density

$$\Delta = \boldsymbol{\sigma} : \nabla \mathbf{v} + \frac{\kappa}{T_0}(\nabla T)^2 = \Delta_v + \Delta_t$$

**Viscous stress**




***Non-localized mode***




***Localized mode***

# III-Measurement system - Laser vibrometer

## Laser vibrometer principle

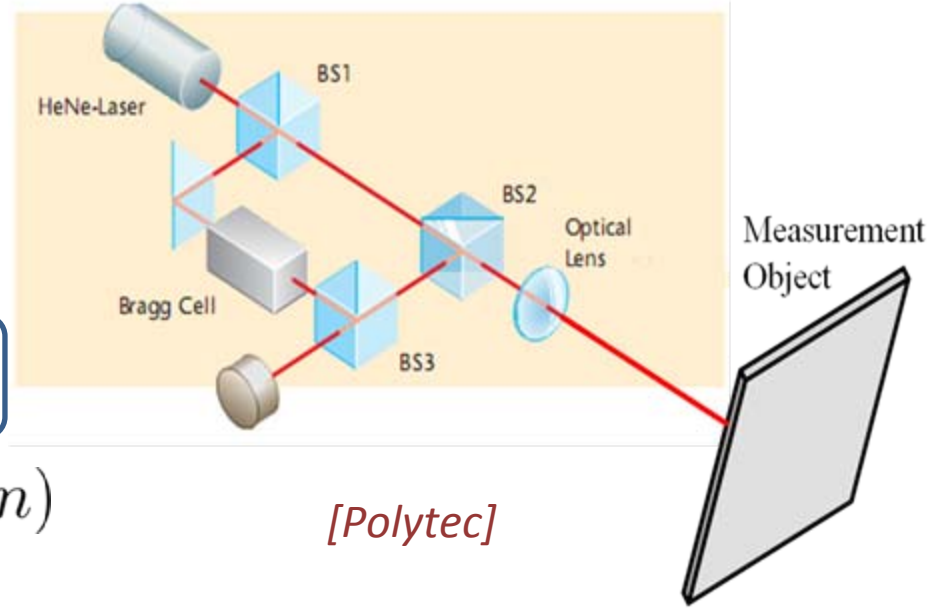
- Doppler effect

$$\Delta f = f_r - f_e = \pm 2\frac{v}{\lambda_e}$$

- Interferometry

$$I = I_1 + I_2 + \boxed{2\sqrt{I_1 I_2}cos(\Delta\phi)}$$

$$\Delta\phi = \Delta(knL) = k(n\Delta L + L\Delta n)$$



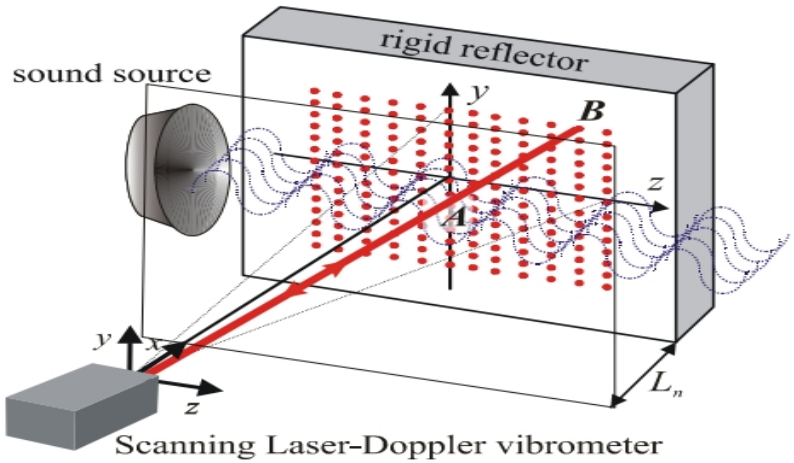

*[Polytec]*

- $k$ :optical wave number; $n$ :refractive index
- In general: $\Delta\phi_g = kn\Delta L \gg \Delta\phi_n = kL\Delta n$
- Pressure fluctuation $\Delta P(X,t)$ ⟹ $\Delta n(X,t) = n(X,t) - n_0$
- Velocity contribution: $v = v_g + v_a$

# III-Measurement system - Laser vibrometer

## Pressure measurement

- Rigid reflector
- Transparent medium
- Acoustic field limited in a volume of thickness $L_n$




*[Nils-Erick, Lothar Zipser (2004)]*

$$v = v_a = \int_{L_n} \dot{n}(X,t)\ dx$$

- For $\dot{n}(X,t)$ constant along x, $v_a = \dot{n}(X,t)L_n$
- Gladstone-Dale equation: $n = \alpha\rho + 1$
- Acoustic relation: $p = c^2\rho$
- $$v_a = \frac{\alpha}{c^2}\dot{p}(X,t)L_n = \kappa\dot{p}(X,t)L_n$$

# III-Measurement system - Laser vibrometer

## Pressure measurement

$$v_a = \frac{\alpha}{c^2}\dot{p}(X,t)L_n = \kappa\dot{p}(X,t)L_n$$

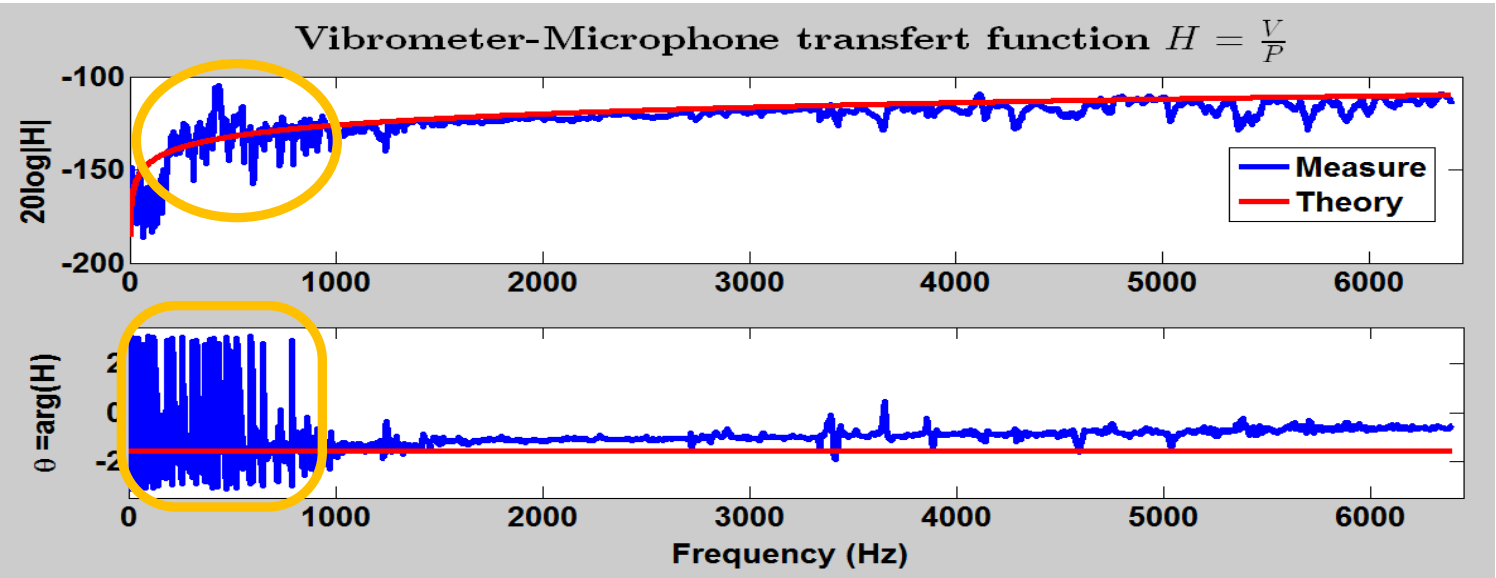
- Harmonic time dependence:

$$v_a = Ve^{j\omega t}$$

$$p = Pe^{j\omega t}$$

$$H = \frac{P}{V} = \frac{-j}{\omega\kappa L_n}$$

$$\kappa = 2.1e^{-9}Pa^{-1} \quad L_n = 0.04m$$



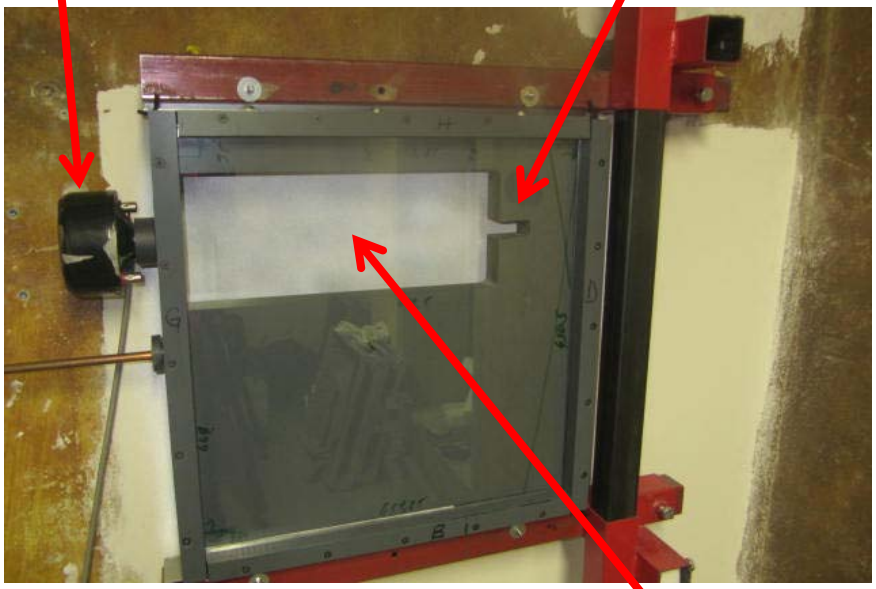
# III-Experimental Results

## Experimental set-up

*Excitation*

***4x2x4cm Resonator***

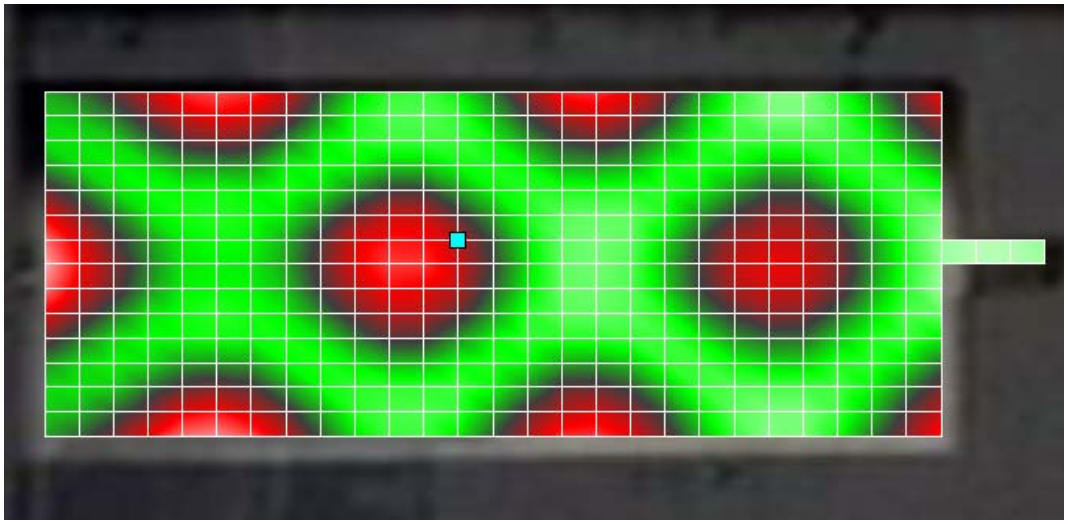
**Laser Vibrometer**



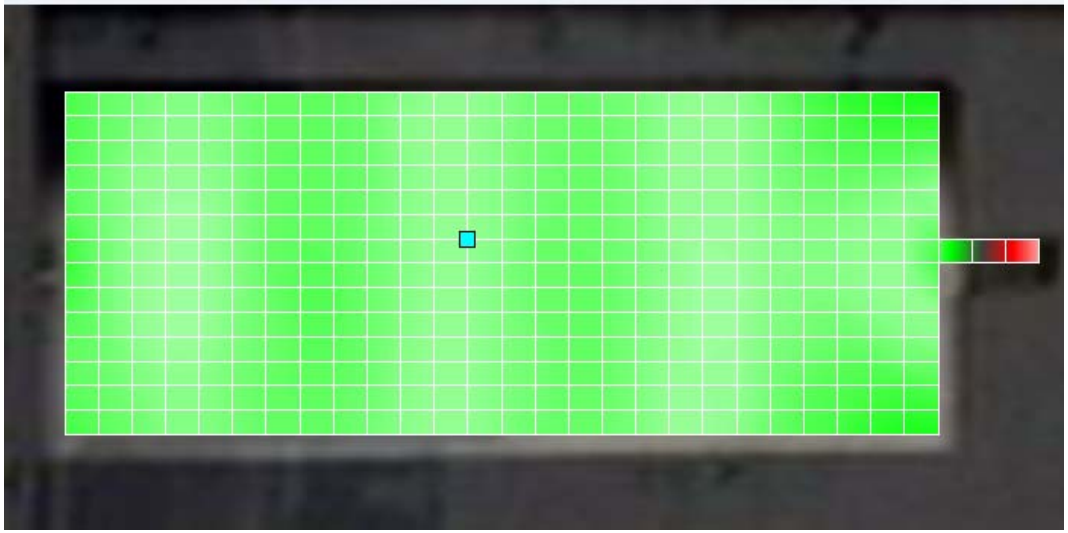

**Simple irregular cavity**
**35x14x4cm+resonator**

# III-Experimental Results

## Modes visualization



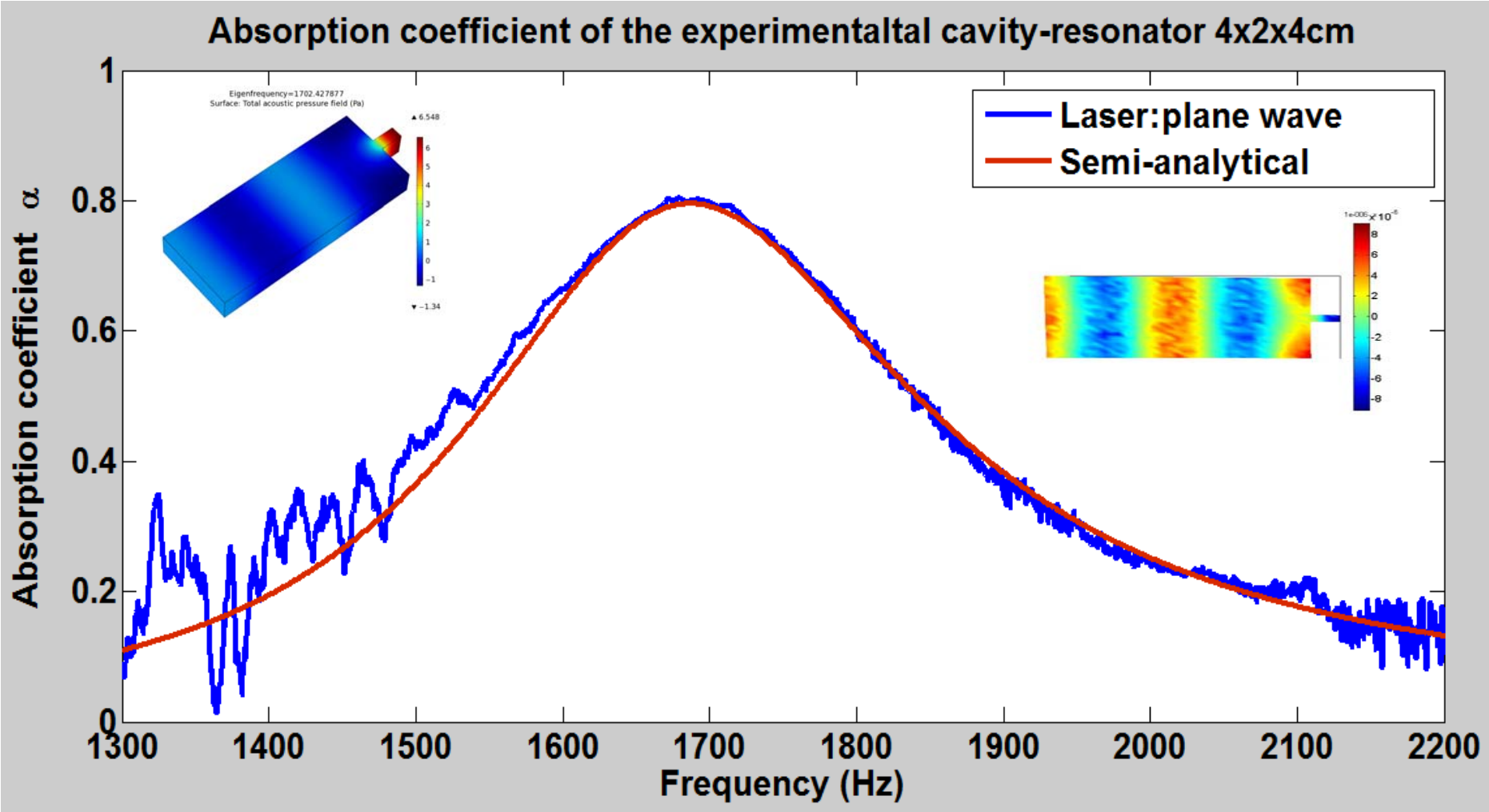
***Non-localized Mode***
***freq= 2465 Hz***



***Localized Mode***
***freq= 1702 Hz***

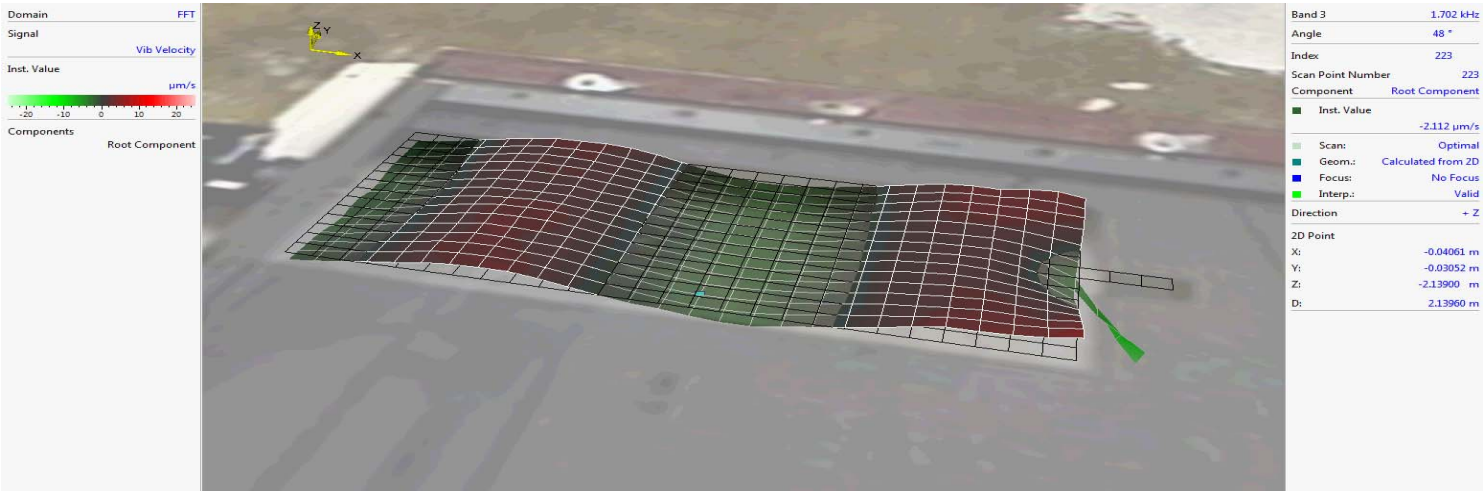
# III-Experimental Results

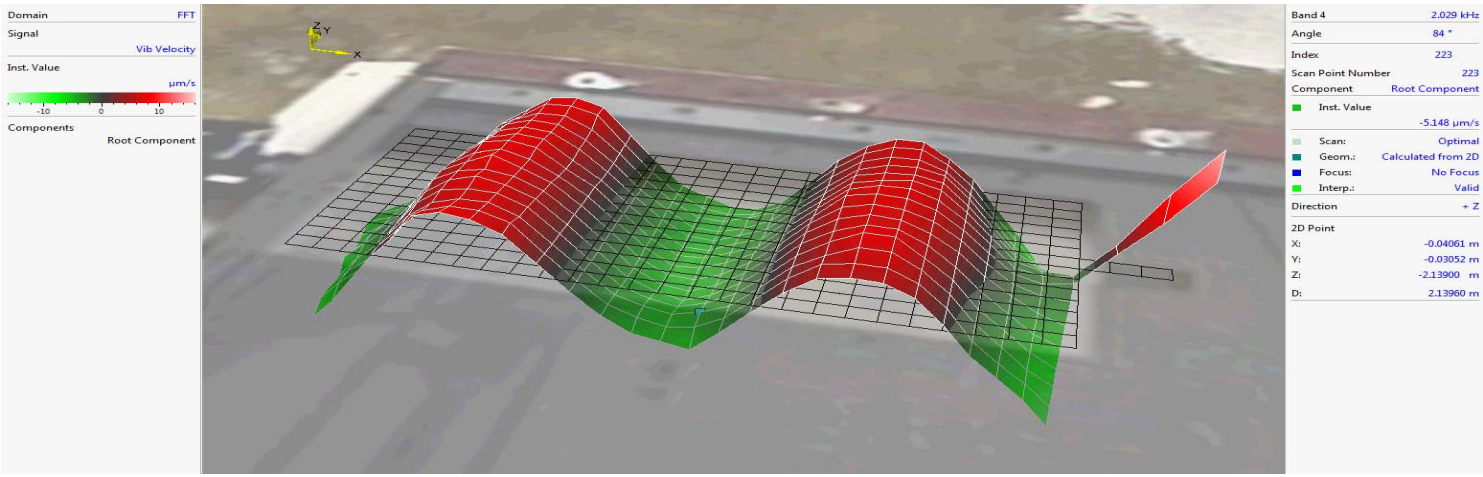
## Absorption Coefficient



# III-Experimental Results

## Absorption Coefficient



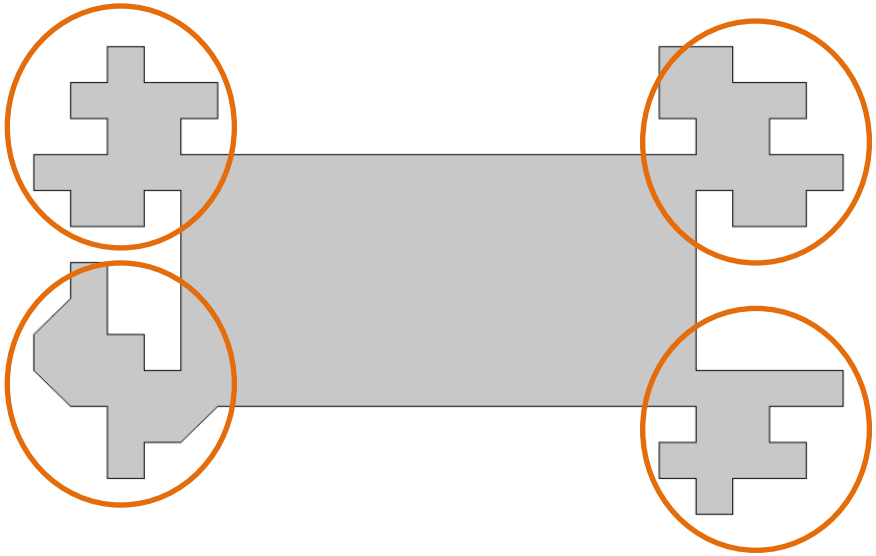



# Conclusion & perspectives

- Geometrical irregularities can localize acoustic energy
- Localization can enhance dissipation in favorable conditions;
- Laser vibrometer allow experimental observation of localization

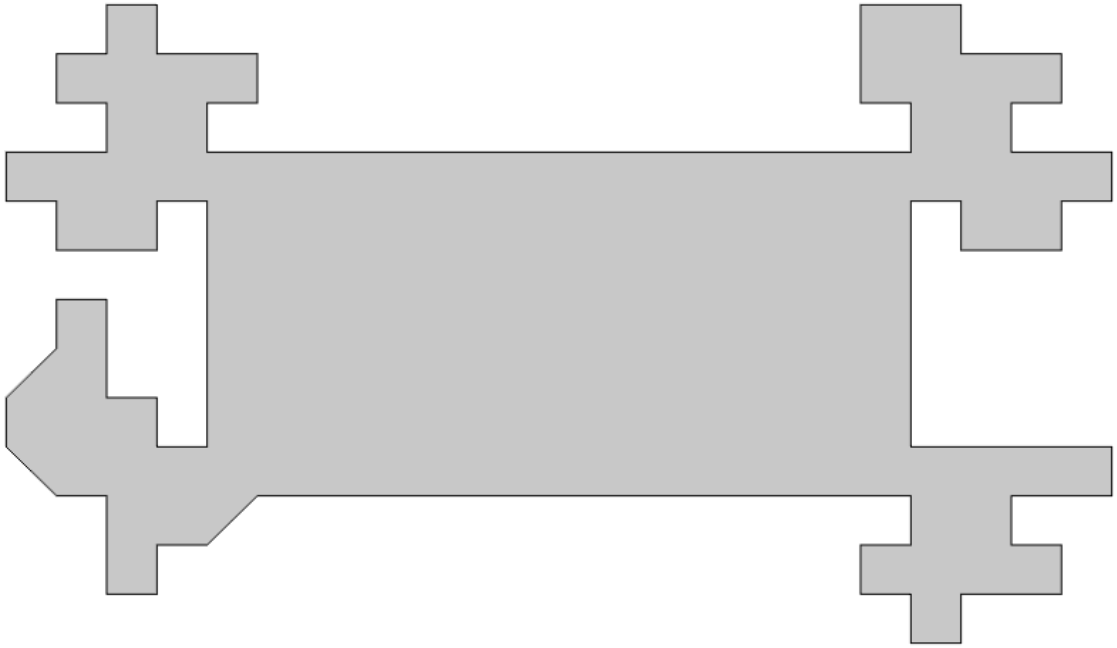
➢ **Future Work**

Can localization enhance porous materials performance and reduce their amount?



# My hope

**I'm looking for a new dissipation indicator!!!!**



# THANKS!

# II.2- Localization and dissipation

## Viscothermal effects: Absorption coefficient