Royal Institute of Technology

Symposium on the Acoustics of Poro-Elastic Materials

Acoustic porous media

Exploring multi-functionality

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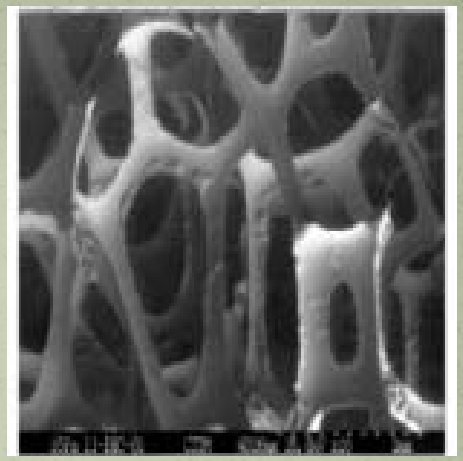
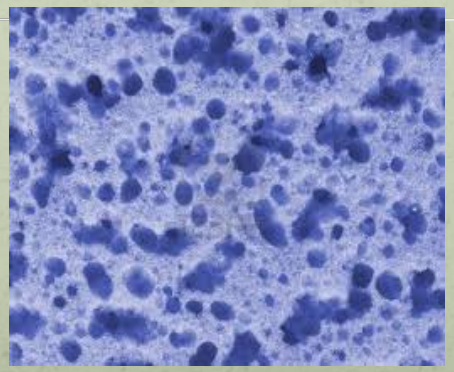
Measuring flow resistivity and thickness of porous materials via reflected waves at low frequencies.

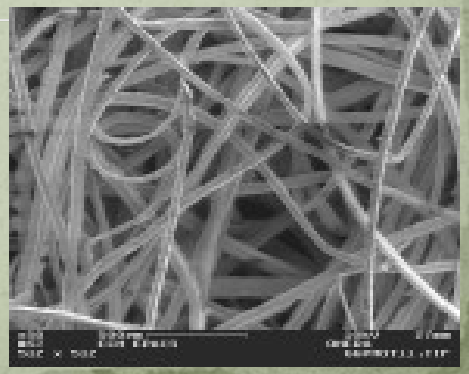
M. Sadouki[a], A. Berbiche[b,c], Z.E.A. Fellah[c] , M. Fellah[b] and C. Depollier[d]

- [a] Faculté des Sciences et de la Technologie, Université de Khemis-Miliana, Route de Thenia, Khemis Miliana BP 44225, Algeria
- [b] Laboratoire de Physique Théorique, Faculté de Physique, USTHB, BP 32 El Alia, Bab Ezzouar, 16111 Bab Ezzouar, Algeria
- [c] LMA – CNRS (UPR 7051) / Aix-Marseille Université, 31 chemin Joseph Aiguier, 13402 Marseille, France
- [d] Laboratoire d'acoustique de l'université du Maine, Bat. IAM - UFR Sciences Avenue Olivier Messiaen, 72085 LE MANS CEDEX 9



Air-saturated porous materials





Theories

General case → Vibration of the solid and fluid phases. → Biot theory.

Particulier case → Solid phase still. → Equivalent-fluid theory.

Equivalent Fluid Model

Free fluid

$$\rho_f \frac{\partial v}{\partial t} = -\nabla p,$$

$$K_a^{-1} \frac{\partial p}{\partial t} = -\nabla v,$$

Porous material

$$\rho(\omega) \frac{\partial v}{\partial t} = -\nabla p,$$

$$K^{-1}(\omega) \frac{\partial p}{\partial t} = -\nabla v$$

$$\rho(\omega) = \rho_f \alpha(\omega) \qquad K^{-1}(\omega) = K_a^{-1} \beta(\omega)$$

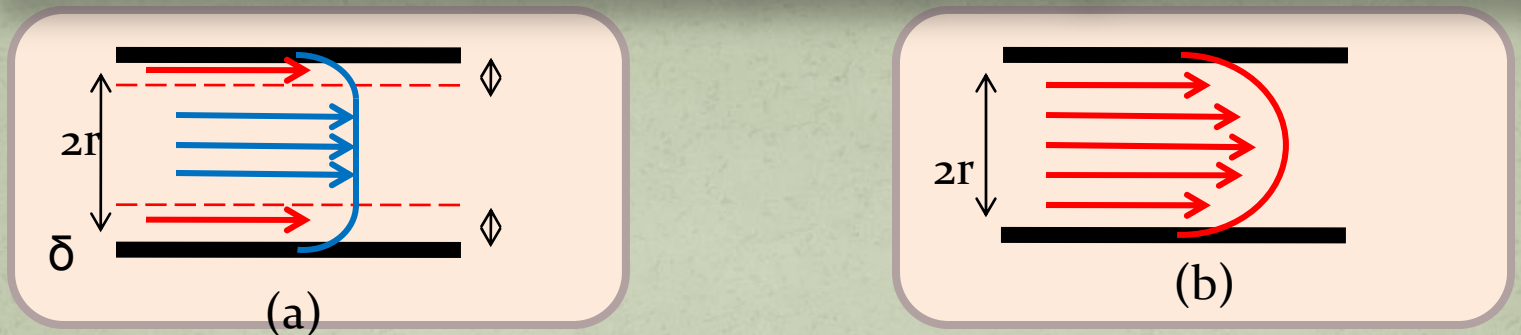
$\alpha(\omega)$: dynamic tortuosity [1]

$\beta(\omega)$: dynamic compressibility [2]

[1] D. L. Johnson, J. Koplik, and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media," J. Fluid. Mech. 176, 379–402 (1987).

[2] F. Allard, Propagation of Sound in Porous Media Modelling. Sound Absorbing Materials (Elsevier, London, UK, 1993), pp. 1–284.

Low frequency range



Schematic profiles of particle velocity in a cylindrical pore section:
(a) high frequency, (b) low frequency.

the viscous skin thicknesses: $\delta = \sqrt{2\eta/\omega\rho_f}$

radius of the pores r

$\delta \gg r$

Low frequency range

Darcy's regime (40-100)Hz

$$\alpha(\omega) = \frac{\sigma\phi}{\rho j\omega}, \qquad \beta(\omega) = \gamma \qquad [3]$$

Flow resistivity

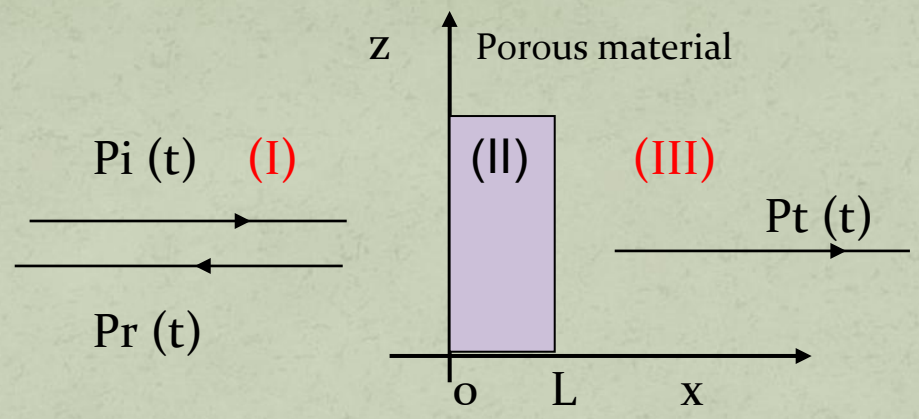
Porosity

σ: Flow resistivity

Φ: Porosity

[3] Johnson D.L. , Plona T.J. , Scala C., Pasierb F. , and Kojima H. "*Tortuosity and acoustic* slow waves. '' Phys. Rev. Lett., 49 : 1840, (1982).

Direct problem



$$p_r(x,t) = \int_0^t R(\tau)\, p_i\left(t - \tau + \frac{x}{c_0}\right) d\tau \quad = R(t) * p_i(t) * \delta\left(t + \frac{x}{c_0}\right)$$

$$R(\omega) = \frac{(1 - D^2)\sinh(sfL)}{2D\cosh(sfL) + (1 + D^2)\sinh(sfL)}$$ [4]

With,

$$D = \phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \qquad f = \sqrt{\frac{\rho_f}{K_a}\alpha(\omega)\beta(\omega)}$$

[4] Sadouki et al.: Measuring static viscous permeability of porous absorbing materials J. Acoust. Soc. Am., Vol. 135, No. 6, June 2014 y

Reflected coefficient

$$R_1^{TBF}(\omega) = \frac{\left(1 - j\omega C_1{}^2\right)\sinh\left(LC_2\sqrt{j\omega}\right)}{2C_1\sqrt{j\omega}\cosh\left(LC_2\sqrt{j\omega}\right) + \left(1 + j\omega C_1{}^2\right)\sinh\left(LC_2\sqrt{j\omega}\right)}$$

$$\Downarrow$$

$$R_1^{TBF} = \frac{1}{1 + \frac{2}{LB}}\left(1 - \frac{2C_1^2\left(1 + \frac{1}{3}LB + \frac{1}{LB}\right)}{1 + \frac{2}{LB}} j\omega + \cdots\right)$$

$$\Downarrow$$

$$\boxed{R_2^{TBF} = \frac{1}{1 + \frac{2}{LB}}, \qquad \omega \to 0}$$

$$B = \frac{C_2}{C_1} = \frac{\sigma}{\sqrt{K_a \rho_f}} \qquad C_2 = \sqrt{\frac{\gamma\sigma\phi}{K_a}}. \qquad C_1 = \sqrt{\frac{\gamma\rho_f\phi}{\sigma}}$$

R_2^{TBF} **depends on σ and L**

Inverse Problem

$$U(\sigma, L) = \sum_{i=1}^{i=N} \left[p^r_{exp}(x, t_i) - p^r(x, t_i) \right]^2$$

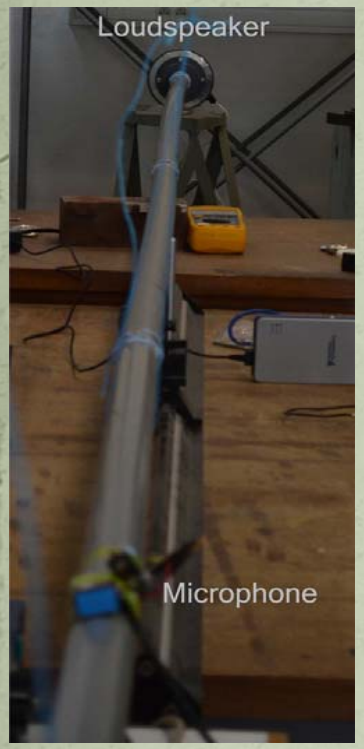
Where,

$p^r_{exp}(x, t_i)$ represents the discrete set of values of the experimental reflected signal.

$p^r(x, t_i)$ is the discrete set of values of the simulated reflected signal.

Experimental set up (40-200) Hz

Experiments are performed in a guide (pipe):



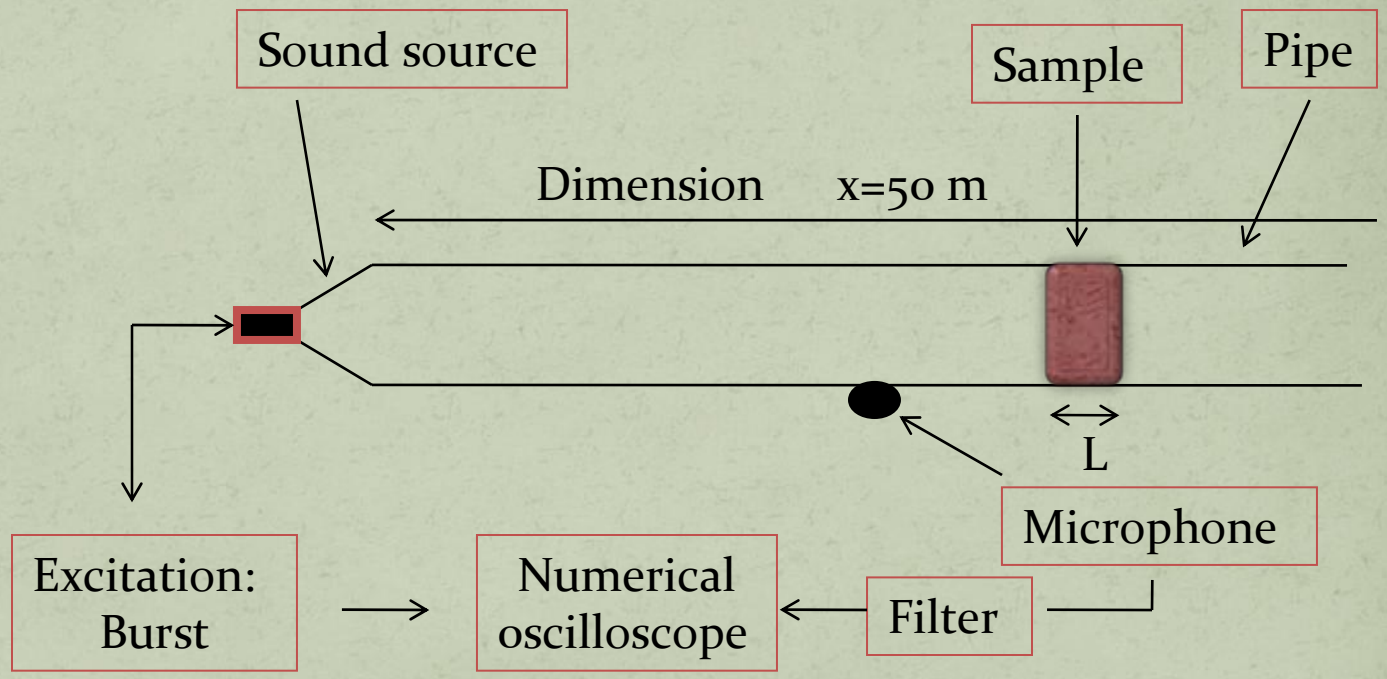


FIG. 2. Picture of the experimental setup.

Experimental data

Sample M: L = 2.6cm, $\sigma = (25.0 \pm 0.60)\ 10^{+3}\ Nm^{-4}s$,

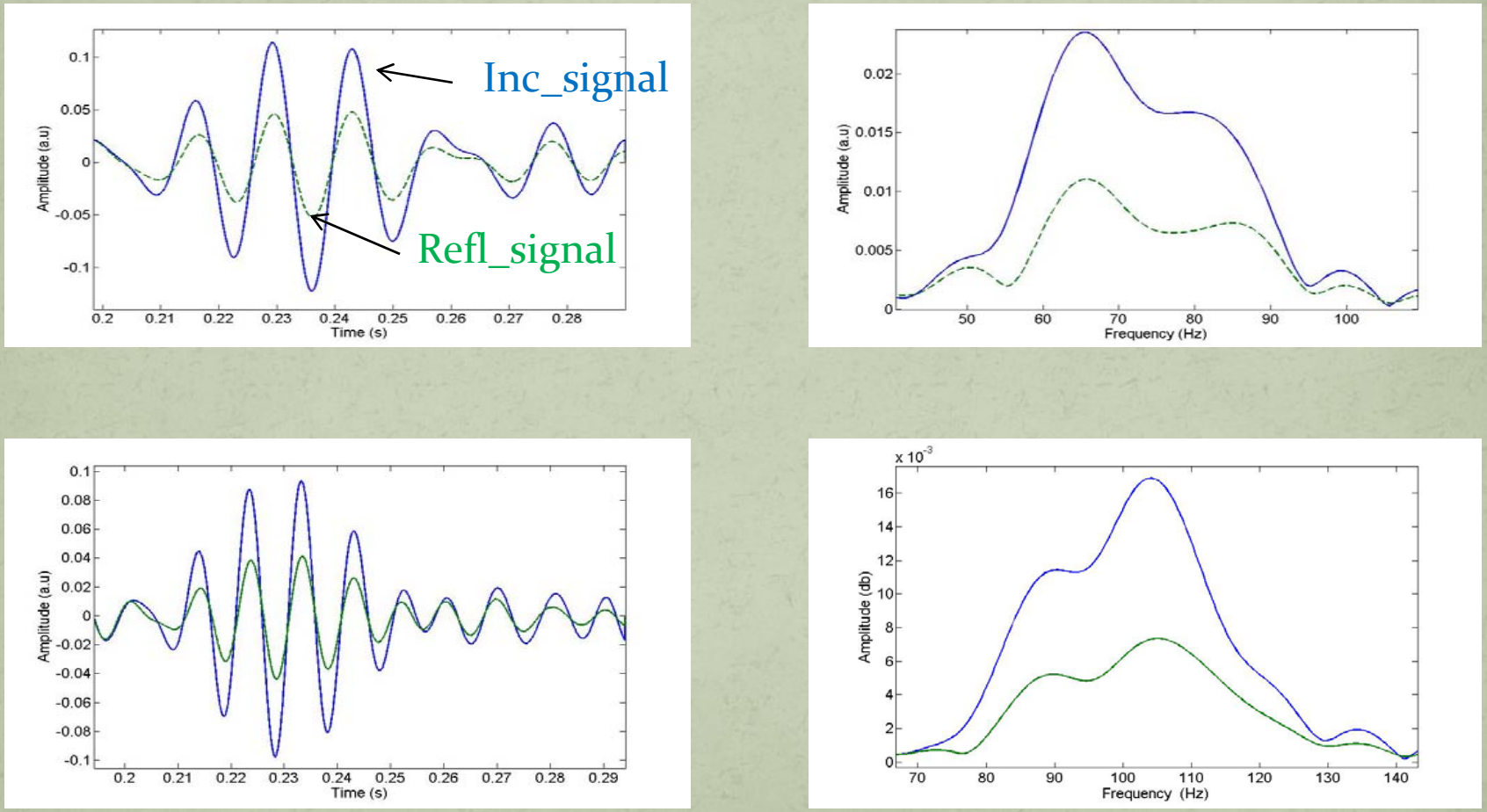


Fig.2. Experimental incident (solid line), reflected (dashed line) signals (left) and their spectra (right).

Measuring of the flow resistivity

First case: Determination of the flow resistivity σ

Sample M: L = 2.6cm, σ = 25.0 10^{+3} $Nm^{-4}s$

$$R^{TBF} = \frac{\frac{1}{2}BL}{1+\frac{1}{2}BL} \qquad B = \frac{\sigma}{\sqrt{\rho_f K_a}} \qquad U(\sigma) = \sum_{t_i}\left(R(t,\sigma) * e(t_i) - r_{exp}(t_i)\right)^2$$

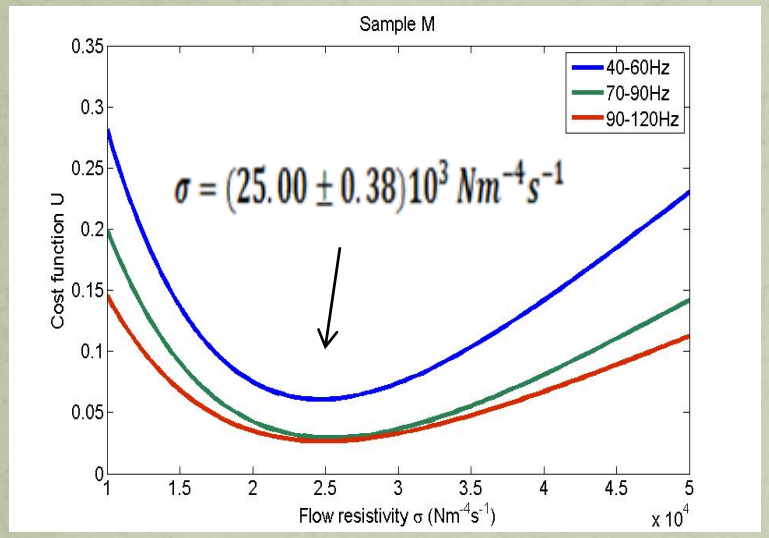


Fig.3. variation of the minimization function U with flow resistivity σ in frequencies bandwidth (40-60) Hz , (70-90)Hz and (90-120)Hz respectively.

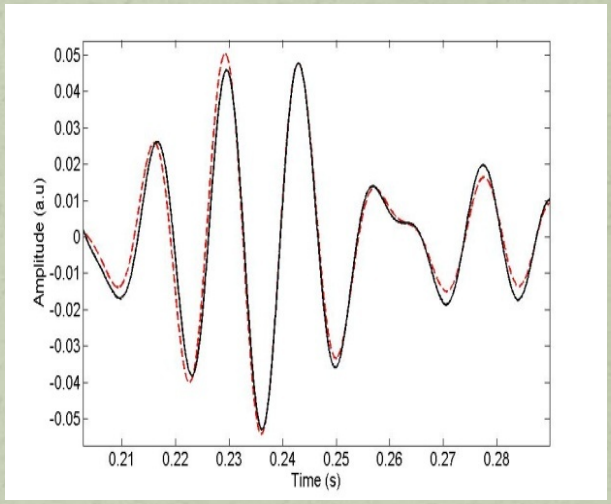


Fig.4. Comparison between experimental reflected signal (dashed line) and simulated reflected signal (solid line) for the sample M.

Measuring of the thickness

Second case: Determination of the thickness L

Sample M: L = 2.6cm, σ = 25.0 10^{+3} $Nm^{-4}s$

$$R^{TBF} = \frac{\frac{1}{2}BL}{1+\frac{1}{2}BL} \qquad B = \frac{\sigma}{\sqrt{\rho_f K_a}} \qquad U(L) = \sum_{t_i}\left(R(t,L) * e(t_i) - r_{exp}(t_i)\right)^2$$

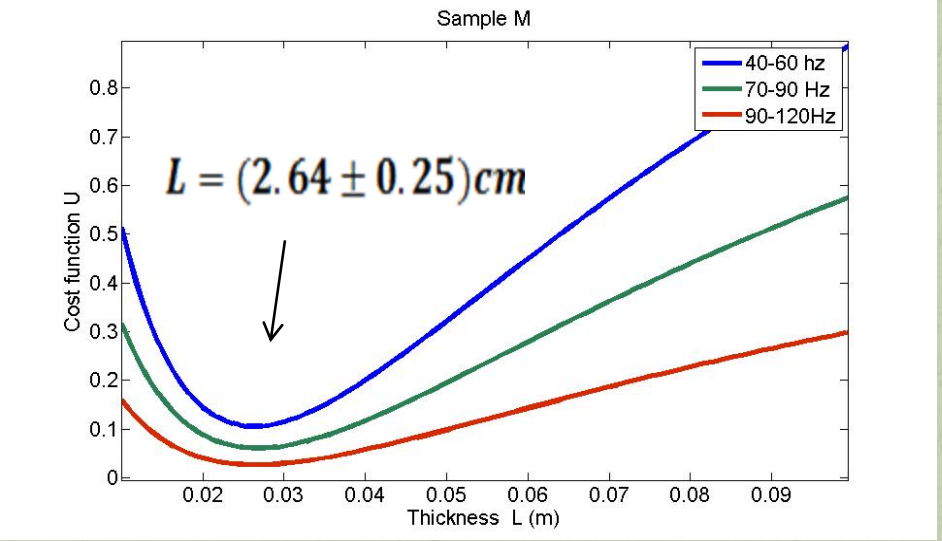
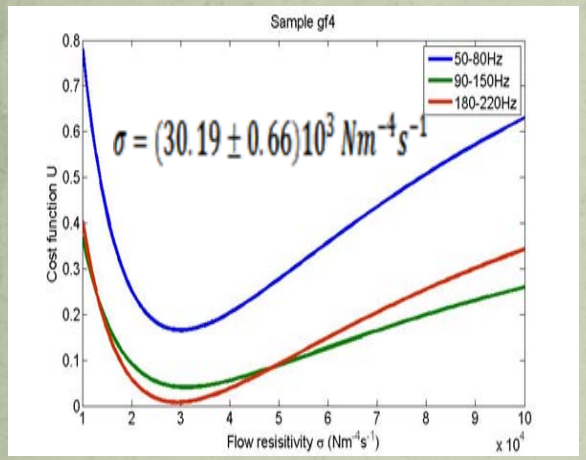
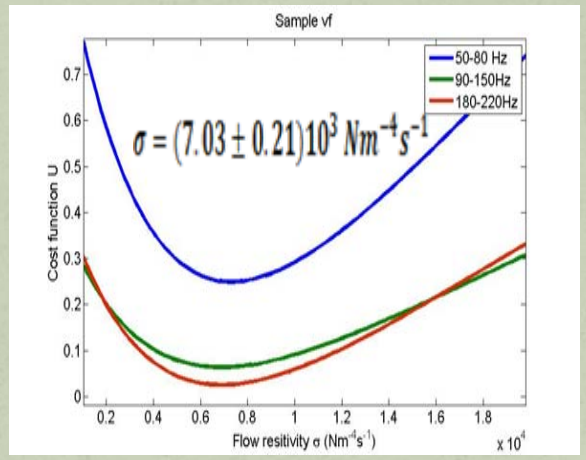


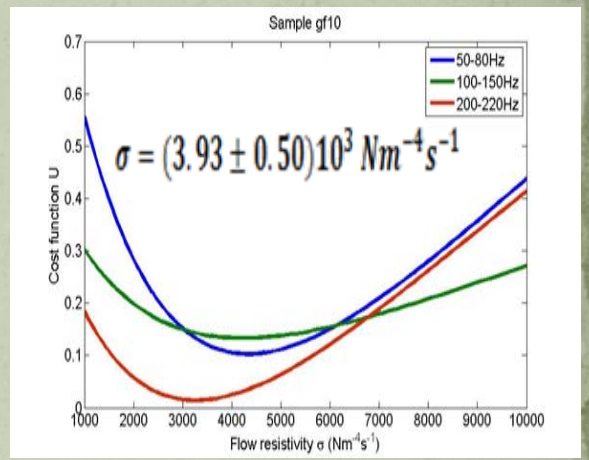
Fig.5. variation of the minimization function U with Thickness L in frequencies bandwidth (40-60) Hz (70-90)Hz and (90-120)Hz respectively.

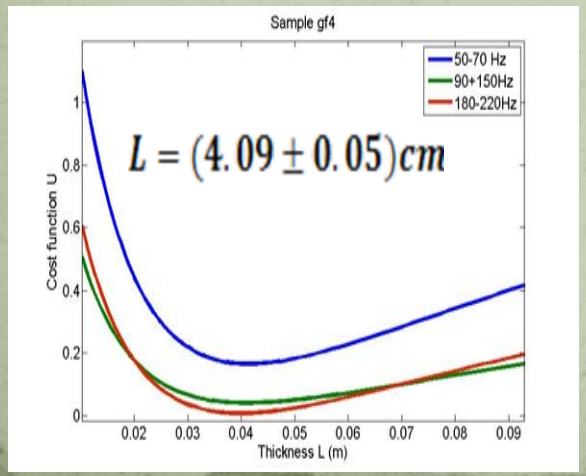
Extension to other samples

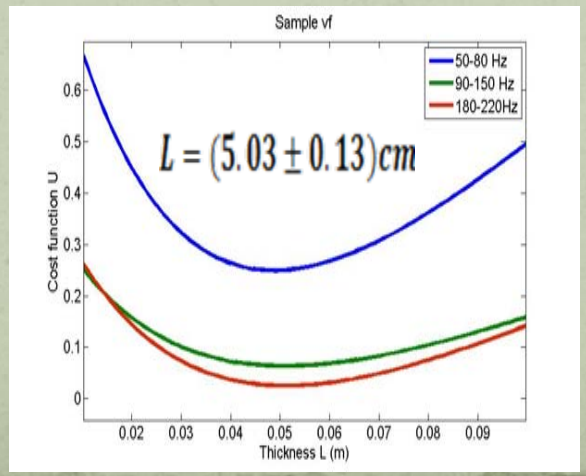
Sample gf4	Sample vf	Sample gf10
$\sigma = 30.0\ 10^3 Nm^{-4}s^{-1},\ L = 4.15cm$	$\sigma = 7.0\ 10^3 Nm^{-4}s^{-1},\quad L = 5.0cm$	$\sigma = 4.0\ 10^3\ Nm^{-4}s^{-1},\ L = (10.00 \pm 0.10)cm$

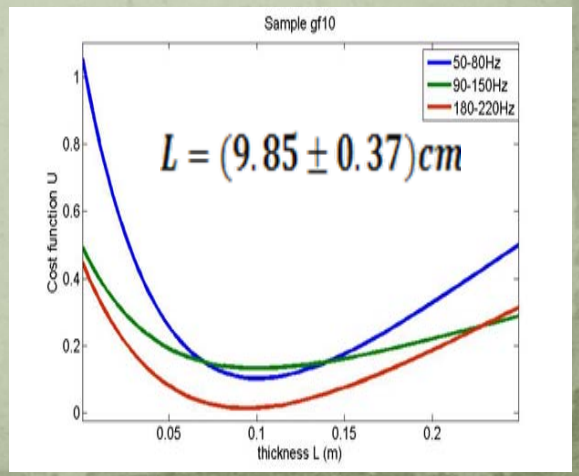












Conclusion

- Acoustic characterization of air saturated porous material in Low frequency range.

- A simple relation between flow resistivity, thickness and the reflected waves is obtained.

- The reconstructed values of flow resistivity and the thickness of the plastic foam are close to those using classical methods.

- The proposed experimental method has the advantage of being simple, rapid and efficient for estimating these parameters

Thank you