Homogenization of sound propagation in a deformable porous material based on microscopic viscous-thermal effects

Kun Gao[1]
J.A.W. van Dommelen[1] P. Göransson[2] M.G.D. Geers[1]

[1]Mechanics of Materials, Department of Mechanical Engineering
Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands

[2]Marcus Wallenberg Laboratory, Department of Aeronautical and Vehicle Engineering
KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden

Symposium on the Acoustics of Poro-elastic Materials, Dec. 16, 2014

Outline

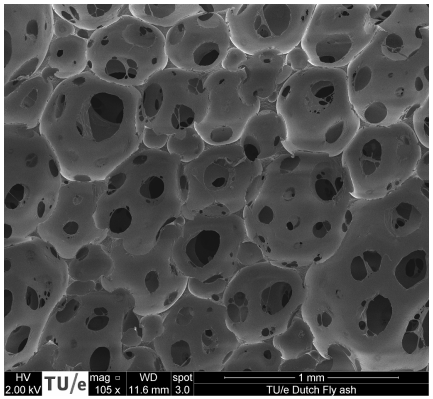
Cleanroom in ASML

MRI scanner from Philips Healthcare

- To reduce low-mid frequency noises
- Acoustic porous materials for shielding

Acoustic porous materials

- Good absorption performance
- Light weighted: high porosity ϕ
- Complex microstructure



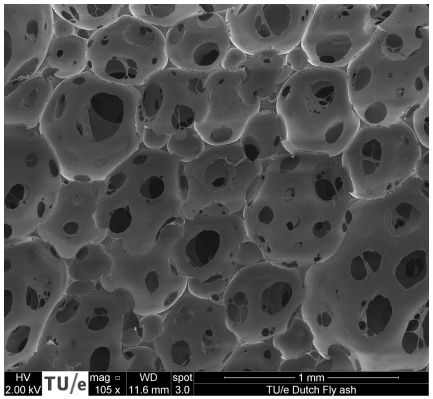
SEM photo of an acoustic foam from Huntsman (www.huntsman.com)

Absorption mechanisms

- Viscous losses: $\delta = \mathrm{O}(10^{-4})\ \mathrm{m}$
- Thermal losses
- Helmholtz-type resonator
- Mechanical damping
- * Vortex shedding

Acoustic porous materials

- Good absorption performance
- Light weighted: high porosity ϕ
- Complex microstructure



SEM photo of an acoustic foam from Huntsman (www.huntsman.com)

Objective

Build a multiscale model of acoustic porous materials

- Microscopic fluid-solid interaction
- Macroscopic acoustic properties

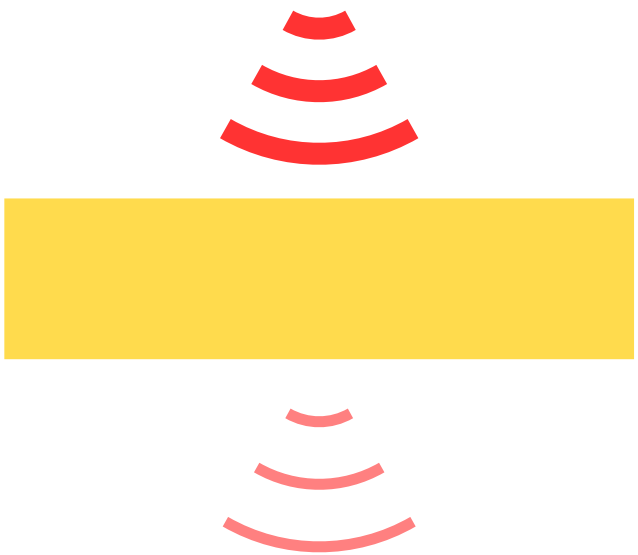
Absorption mechanisms

- Viscous losses: $\delta = \mathrm{O}(10^{-4})$ m
- Thermal losses
- Helmholtz-type resonator
- Mechanical damping
- * Vortex shedding

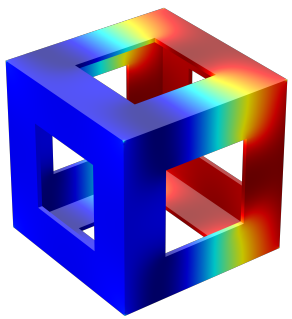
Scale separation principle

$$\lambda \geq L_M^* \gg R$$

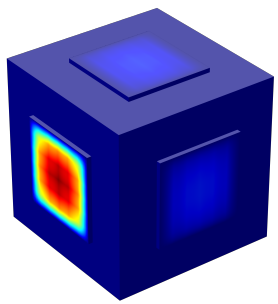
Macroscopic Problem



Microscopic RVE



Solid



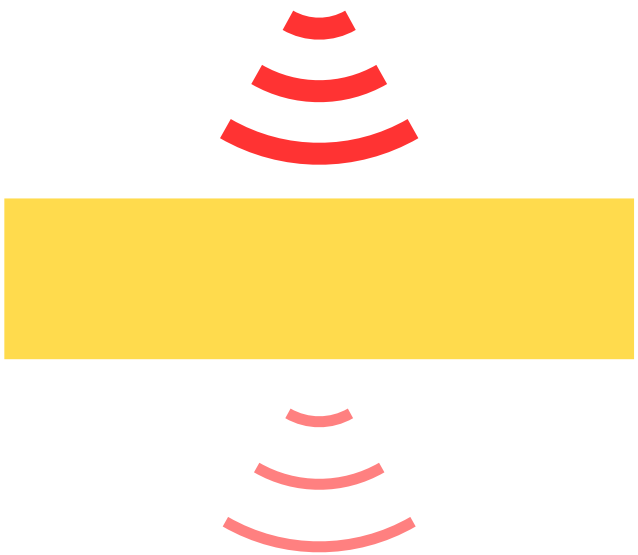
Fluid

Scale separation principle

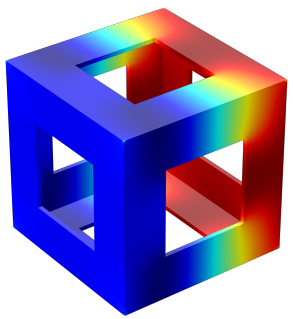
$$\lambda \geq L_M^* \gg R$$

Macroscopic Problem

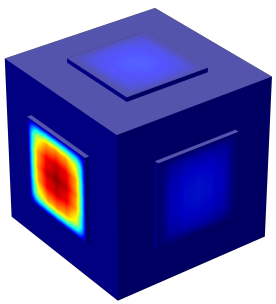
$$\begin{aligned} \boldsymbol{f}_M^s - \boldsymbol{\nabla}_M \cdot (\boldsymbol{\sigma}_M^s)^{\mathrm{T}} &= \mathbf{0} \\ \epsilon_M^f - \boldsymbol{\nabla}_M \cdot \boldsymbol{u}_M^f &= 0 \end{aligned}$$



Microscopic RVE



Solid



Fluid

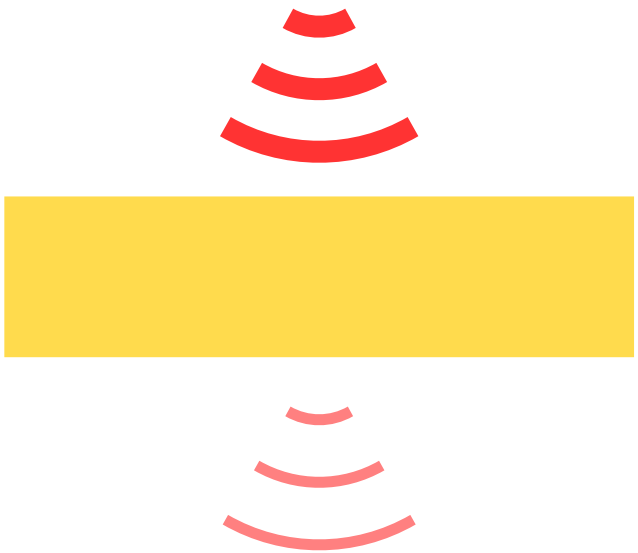
Technische Universiteit Eindhoven University of Technology

Scale separation principle

$$\lambda \geq L_M^* \gg R$$

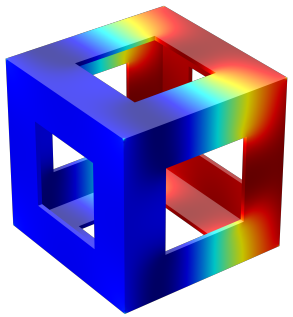
Macroscopic Problem

$$\boldsymbol{f}_M^s - \boldsymbol{\nabla}_M \cdot (\boldsymbol{\sigma}_M^s)^{\mathrm{T}} = \boldsymbol{0}$$
$$\epsilon_M^f - \boldsymbol{\nabla}_M \cdot \boldsymbol{u}_M^f = 0$$



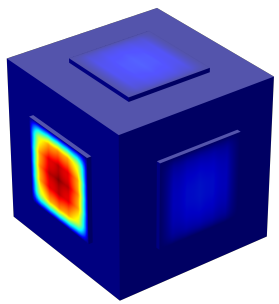
Microscopic RVE

$$-\omega^2 \rho_0^s \boldsymbol{u}_m^s = \boldsymbol{\nabla}_m \cdot \boldsymbol{\sigma}_m^s$$



Solid

$$-\omega^2 \rho_0^f \boldsymbol{u}_m^f = -\boldsymbol{\nabla}_m p_m^f + \boldsymbol{\nabla}_m \cdot \boldsymbol{\tau}_m^f$$



Fluid

TU/e Technische Universiteit Eindhoven University of Technology

Scale separation principle

$$\lambda \geq L_M^* \gg R$$

Macroscopic Problem

$$\boldsymbol{f}_M^s - \boldsymbol{\nabla}_M \cdot (\boldsymbol{\sigma}_M^s)^{\mathrm{T}} = \boldsymbol{0}$$
$$\epsilon_M^f - \boldsymbol{\nabla}_M \cdot \boldsymbol{u}_M^f = 0$$

$\boldsymbol{\nabla}_M \boldsymbol{u}_M^s$

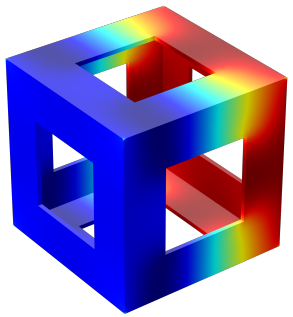
$\boldsymbol{u}_M^s$

p_M^f

$\boldsymbol{\nabla}_M p_M^f$

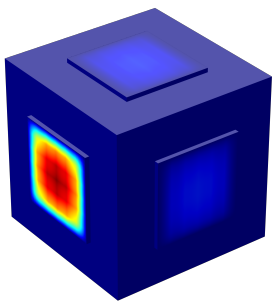
Microscopic RVE

$$-\omega^2 \rho_0^s \boldsymbol{u}_m^s = \boldsymbol{\nabla}_m \cdot \boldsymbol{\sigma}_m^s$$



Solid

$$-\omega^2 \rho_0^f \boldsymbol{u}_m^f = -\boldsymbol{\nabla}_m p_m^f + \boldsymbol{\nabla}_m \cdot \boldsymbol{\tau}_m^f$$



Fluid

Technische Universiteit Eindhoven University of Technology

Scale separation principle

$$\lambda \geq L_M^* \gg R$$

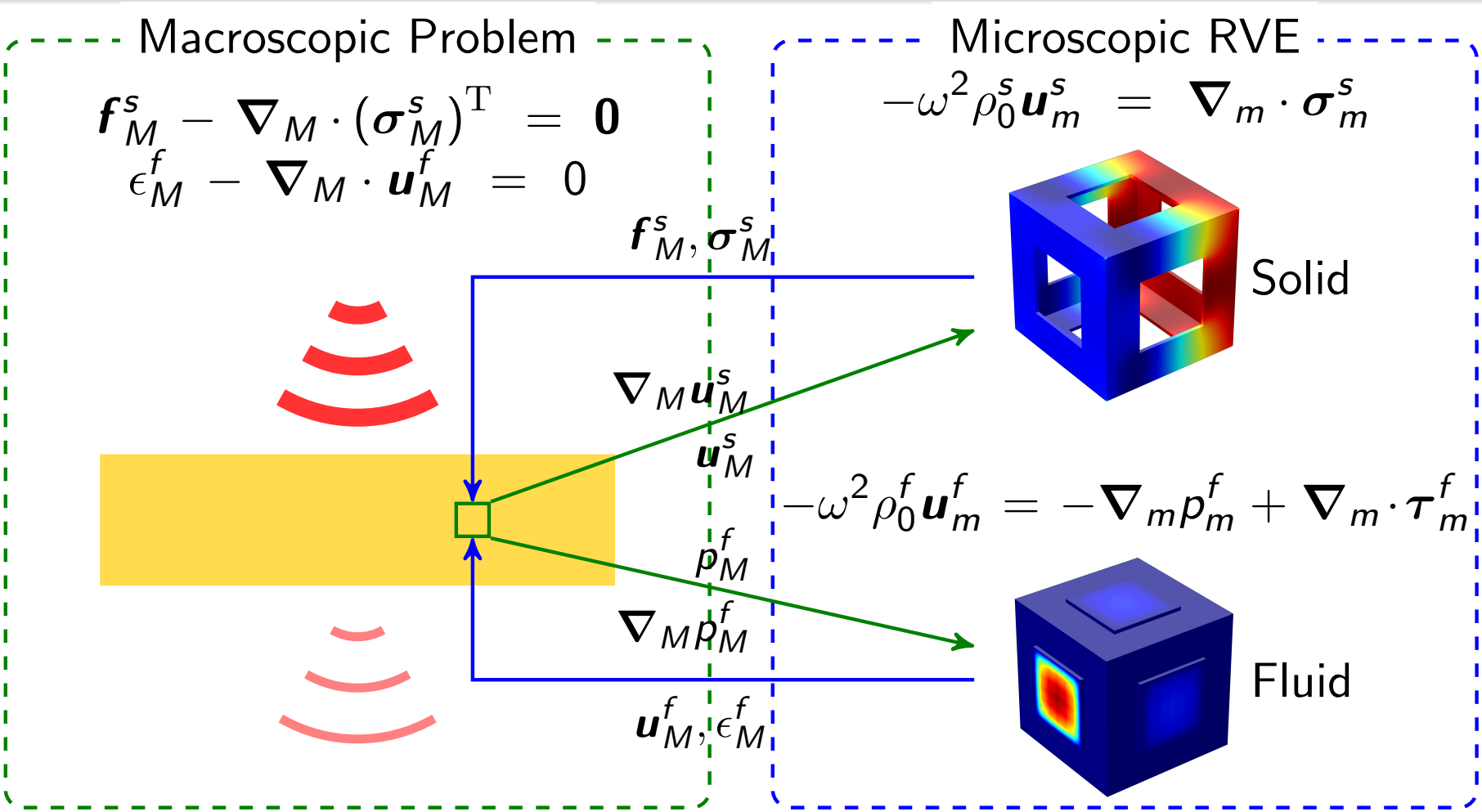
Macroscopic Problem

$$\boldsymbol{f}_M^s - \boldsymbol{\nabla}_M \cdot (\boldsymbol{\sigma}_M^s)^{\mathrm{T}} = \boldsymbol{0}$$
$$\epsilon_M^f - \boldsymbol{\nabla}_M \cdot \boldsymbol{u}_M^f = 0$$

Microscopic RVE

$$-\omega^2 \rho_0^s \boldsymbol{u}_m^s = \boldsymbol{\nabla}_m \cdot \boldsymbol{\sigma}_m^s$$

$$-\omega^2 \rho_0^f \boldsymbol{u}_m^f = -\boldsymbol{\nabla}_m p_m^f + \boldsymbol{\nabla}_m \cdot \boldsymbol{\tau}_m^f$$



B.C. (Macro-to-micro)

$$\begin{aligned} \boldsymbol{u}_m^s &= \boldsymbol{u}_M^s + (\boldsymbol{\nabla}_M \boldsymbol{u}_M^s)^{\mathrm{T}} \cdot \boldsymbol{x}_m^s + \tilde{\boldsymbol{u}}_m^s \\ p_m^f &= p_M^f + (\boldsymbol{\nabla}_M p_M^f) \cdot \boldsymbol{x}_m^f + \tilde{p}_m^f \end{aligned} \Longrightarrow \begin{aligned} \tilde{\boldsymbol{u}}_m^{s+} &= \tilde{\boldsymbol{u}}_m^{s-} \\ \boldsymbol{\sigma}_m^f \cdot \boldsymbol{n} &= -\left[p_M^f + \boldsymbol{\nabla}_M p_M^f \cdot \boldsymbol{x}_m^f \right] \boldsymbol{n} \end{aligned}$$

B.C. (Macro-to-micro)

$$\begin{aligned} \boldsymbol{u}_m^s &= \boldsymbol{u}_M^s + (\boldsymbol{\nabla}_M \boldsymbol{u}_M^s)^{\mathrm{T}} \cdot \boldsymbol{x}_m^s + \tilde{\boldsymbol{u}}_m^s \\ p_m^f &= p_M^f + (\boldsymbol{\nabla}_M p_M^f) \cdot \boldsymbol{x}_m^f + \tilde{p}_m^f \end{aligned} \Longrightarrow \begin{aligned} \tilde{\boldsymbol{u}}_m^{s+} &= \tilde{\boldsymbol{u}}_m^{s-} \\ \boldsymbol{\sigma}_m^f \cdot \boldsymbol{n} &= -\left[p_M^f + \boldsymbol{\nabla}_M p_M^f \cdot \boldsymbol{x}_m^f \right] \boldsymbol{n} \end{aligned}$$

Energy consistency (Micro-to-macro)

$$\delta E_M = \frac{1}{V} \int_V \delta E_m dV$$

$$\delta E_M = (1-\phi) \boldsymbol{f}_M^s \cdot \delta \boldsymbol{u}_M^s + (1-\phi) \boldsymbol{\sigma}_M^s : \delta(\boldsymbol{\nabla}_M \boldsymbol{u}_M^s) - \phi \epsilon_M^f \delta p_M^f - \phi \boldsymbol{u}_M^f \cdot \delta(\boldsymbol{\nabla}_M p_M^f)$$

$$\frac{1}{V} \int_V \delta E_m dV = \left[\frac{1}{V} \int_{S_e^s} (\boldsymbol{n} \cdot \boldsymbol{\sigma}_m^s) dA \right] \cdot \delta \boldsymbol{u}_M^s + \left[\frac{1}{V} \int_{S_e^s} (\boldsymbol{n} \cdot \boldsymbol{\sigma}_m^s) \boldsymbol{x}_m^s dA \right] : \delta(\boldsymbol{\nabla}_M \boldsymbol{u}_M^s)$$

$$- \delta p_M^f \left[\frac{1}{V} \int_{S_e^f} \boldsymbol{n} \cdot \boldsymbol{u}_m^f dA \right] - \delta \boldsymbol{\nabla}_M p_M^f \cdot \left[\frac{1}{V} \int_{S_e^f} \boldsymbol{x}_m^f (\boldsymbol{n} \cdot \boldsymbol{u}_m^f) dA \right]$$

Inputs and outputs

$$\begin{bmatrix} (1-\phi)\boldsymbol{f}_M^s \\ \phi \boldsymbol{u}_M^f \\ (1-\phi)\boldsymbol{\sigma}_M^s \\ \phi \epsilon_M^f \end{bmatrix} = \mathbf{M} \begin{bmatrix} \boldsymbol{u}_M^s \\ \boldsymbol{\nabla}_M p_M^f \\ \boldsymbol{\nabla}_M \boldsymbol{u}_M^s \\ p_M^f \end{bmatrix}$$

Inputs and outputs

$$\begin{bmatrix} (1-\phi)\boldsymbol{f}_M^s \\ \phi \boldsymbol{u}_M^f \\ (1-\phi)\boldsymbol{\sigma}_M^s \\ \phi \epsilon_M^f \end{bmatrix} = \mathbf{M} \begin{bmatrix} \boldsymbol{u}_M^s \\ \boldsymbol{\nabla}_M p_M^f \\ \boldsymbol{\nabla}_M \boldsymbol{u}_M^s \\ p_M^f \end{bmatrix}$$

Balance equations

$$\begin{bmatrix} (1-\phi)\boldsymbol{f}_M^s \\ \phi \boldsymbol{u}_M^f \end{bmatrix} = \begin{bmatrix} -\omega^2 \rho_0^s \boldsymbol{K}^s & \boldsymbol{K}^{sf} \\ \boldsymbol{K}^{sf} & (\omega^2 \rho_0^f)^{-1} \boldsymbol{K}^f \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_M^s \\ \boldsymbol{\nabla}_M p_M^f \end{bmatrix}$$

Stress-strain relation

$$\begin{cases} (1-\phi)\boldsymbol{\sigma}_M^s = {}^4\boldsymbol{C}^s : \boldsymbol{\nabla}_M \boldsymbol{u}_M^s - \boldsymbol{\Psi}^{sf} p_M^f \\ \phi \epsilon_M^f = -\boldsymbol{\Psi}^{sf} : \boldsymbol{\nabla}_M \boldsymbol{u}_M^s - S^f p_M^f \end{cases}$$

Homogenization result

$$\begin{bmatrix} (1-\phi)\boldsymbol{f}_M^s \\ \phi \boldsymbol{u}_M^f \end{bmatrix} = \begin{bmatrix} -\omega^2 \rho_0^s \boldsymbol{K}^s & \boldsymbol{K}^{sf} \\ \boldsymbol{K}^{sf} & (\omega^2 \rho_0^f)^{-1} \boldsymbol{K}^f \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_M^s \\ \boldsymbol{\nabla}_M p_M^f \end{bmatrix}$$

Biot's equations (Biot, 1962)

$$\begin{cases} (1-\phi)\boldsymbol{f}_M^s - \phi \boldsymbol{\nabla}_M p_M^f = -\omega^2 (1-\phi)\rho_0^s \boldsymbol{u}_M^s - \omega^2 \phi \rho_0^f \boldsymbol{u}_M^f \\ -\phi \boldsymbol{\nabla}_M p_M^f + \omega^2 \phi \rho_0^f \boldsymbol{u}_M^f = -\omega^2 \boldsymbol{\rho}_c \cdot \left(\boldsymbol{u}_M^f - \boldsymbol{u}_M^s \right) \end{cases}$$

Relation with Biot's theory

Homogenization result

$$\begin{bmatrix}(1-\phi)\boldsymbol{f}_M^s \\ \phi \boldsymbol{u}_M^f\end{bmatrix} = \begin{bmatrix}-\omega^2\rho_0^s \boldsymbol{K}^s & \boldsymbol{K}^{sf} \\ \boldsymbol{K}^{sf} & (\omega^2\rho_0^f)^{-1}\boldsymbol{K}^f\end{bmatrix} \cdot \begin{bmatrix}\boldsymbol{u}_M^s \\ \boldsymbol{\nabla}_M p_M^f\end{bmatrix}$$

$$\Longleftrightarrow \begin{cases}(1-\phi)\boldsymbol{f}_M^s = -\omega^2\rho_0^s\boldsymbol{\eta}^s \cdot \boldsymbol{u}_M^s + \omega^2\boldsymbol{\rho}_c \cdot \left(\boldsymbol{u}_M^f - \boldsymbol{u}_M^s\right) \\ -\phi\boldsymbol{\nabla}_M p_M^f = -\omega^2\rho_0^f\boldsymbol{\eta}^f \cdot \boldsymbol{u}_M^f - \omega^2\boldsymbol{\rho}_c \cdot \left(\boldsymbol{u}_M^f - \boldsymbol{u}_M^s\right)\end{cases}$$

Biot's equations (Biot, 1962)

$$\begin{cases}(1-\phi)\boldsymbol{f}_M^s = -\omega^2(1-\phi)\rho_0^s\boldsymbol{u}_M^s + \omega^2\boldsymbol{\rho}_c \cdot \left(\boldsymbol{u}_M^f - \boldsymbol{u}_M^s\right) \\ -\phi\boldsymbol{\nabla}_M p_M^f = -\omega^2\phi\rho_0^f\boldsymbol{u}_M^f - \omega^2\boldsymbol{\rho}_c \cdot \left(\boldsymbol{u}_M^f - \boldsymbol{u}_M^s\right)\end{cases}$$

Homogenization result

$$\begin{bmatrix}(1-\phi)\boldsymbol{f}_M^s\\ \phi\boldsymbol{u}_M^f\end{bmatrix}=\begin{bmatrix}-\omega^2\rho_0^s\boldsymbol{K}^s & \boldsymbol{K}^{sf}\\ \boldsymbol{K}^{sf} & (\omega^2\rho_0^f)^{-1}\boldsymbol{K}^f\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{u}_M^s\\ \boldsymbol{\nabla}_M p_M^f\end{bmatrix}$$

$$\Longleftrightarrow\begin{cases}(1-\phi)\boldsymbol{f}_M^s=-\omega^2\rho_0^s\boldsymbol{\eta}^s\cdot\boldsymbol{u}_M^s+\omega^2\boldsymbol{\rho}_c\cdot\left(\boldsymbol{u}_M^f-\boldsymbol{u}_M^s\right)\\ -\phi\boldsymbol{\nabla}_M p_M^f=-\omega^2\rho_0^f\boldsymbol{\eta}^f\cdot\boldsymbol{u}_M^f-\omega^2\boldsymbol{\rho}_c\cdot\left(\boldsymbol{u}_M^f-\boldsymbol{u}_M^s\right)\end{cases}$$

Biot's equations (Biot, 1962)

$$\begin{cases}(1-\phi)\boldsymbol{f}_M^s=-\omega^2(1-\phi)\rho_0^s\boldsymbol{u}_M^s+\omega^2\boldsymbol{\rho}_c\cdot\left(\boldsymbol{u}_M^f-\boldsymbol{u}_M^s\right)\\ -\phi\boldsymbol{\nabla}_M p_M^f=-\omega^2\phi\rho_0^f\boldsymbol{u}_M^f-\omega^2\boldsymbol{\rho}_c\cdot\left(\boldsymbol{u}_M^f-\boldsymbol{u}_M^s\right)\end{cases}$$

Relation (ignoring the micro inertial effects)

$$\boldsymbol{\eta}^s=(1-\phi)\mathbf{I}\,,\quad \boldsymbol{\eta}^f=\phi\mathbf{I}\quad\text{and}\quad \boldsymbol{\rho}_c=\phi\rho_0^f\boldsymbol{K}^{sf}\cdot(\boldsymbol{K}^f)^{-1}$$

Full homogenization

$$\begin{cases} (1-\phi)\nabla_M \cdot \boldsymbol{\sigma}_M^s = -\omega^2 \rho_0^s \boldsymbol{\eta}^s \cdot \boldsymbol{u}_M^s + \omega^2 \boldsymbol{\rho}_c \cdot (\boldsymbol{u}_M^f - \boldsymbol{u}_M^s) \\ -\phi \nabla_M p_M^f = -\omega^2 \rho_0^f \boldsymbol{\eta}^f \cdot \boldsymbol{u}_M^f - \omega^2 \boldsymbol{\rho}_c \cdot (\boldsymbol{u}_M^f - \boldsymbol{u}_M^s) \end{cases}$$

$$\text{and} \quad \begin{cases} (1-\phi)\boldsymbol{\sigma}_M^s = {}^4\boldsymbol{D} : \nabla_M \boldsymbol{u}_M^s + Q \nabla_M \cdot \boldsymbol{u}_M^f \mathbf{I} \\ -\phi p_M^f = Q \nabla_M \cdot \boldsymbol{u}_M^s + R \nabla_M \cdot \boldsymbol{u}_M^f \end{cases}$$

Biot-Homogenization

$$\boldsymbol{\eta}^s = (1-\phi)\mathbf{I}\ , \quad \boldsymbol{\eta}^f = \phi \mathbf{I} \quad \text{and} \quad \boldsymbol{\rho}_c = \phi \rho_0^f (\phi \mathbf{I} - \boldsymbol{K}^f) \cdot (\boldsymbol{K}^f)^{-1}$$

Biot-JCAL (Johnson-Champoux-Allard-Lafarge) + Perrot et al., 2012

$$\rho_{\text{eff}}^f = \phi \rho_0^f \alpha_\infty + \frac{\phi^2 \mu^f}{j\omega q_0} G(\omega; \Lambda) \quad \text{and} \quad K_{\text{eff}}^f = \gamma P_0 / \left[\gamma - \frac{\gamma - 1}{1 + F(\omega; \Lambda')} \right]$$

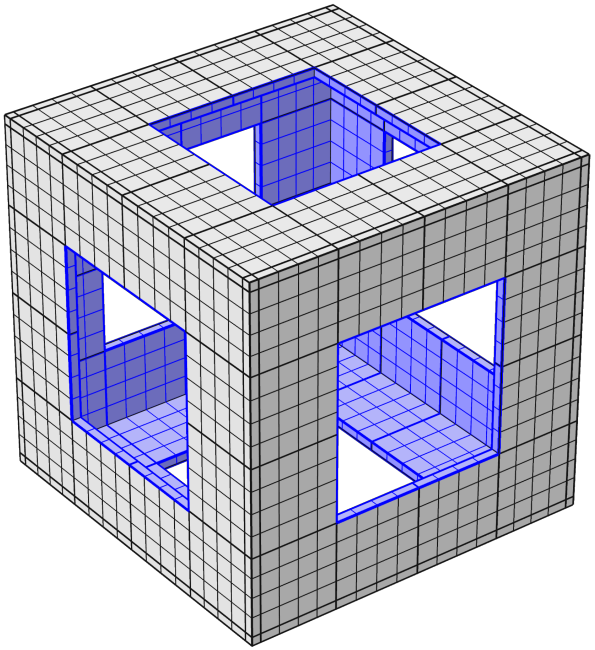
$$\rho_c = \rho_{\text{eff}}^f - \phi \rho_0^f\ , \quad R = \frac{\phi^2 K^s}{(1 - \phi - K_b/K^s + \phi K^s / K_{\text{eff}}^f)} \quad \text{and} \quad Q = \left(\phi - \frac{R}{K_{\text{eff}}^f} \right) K^s$$

Microscopic RVE

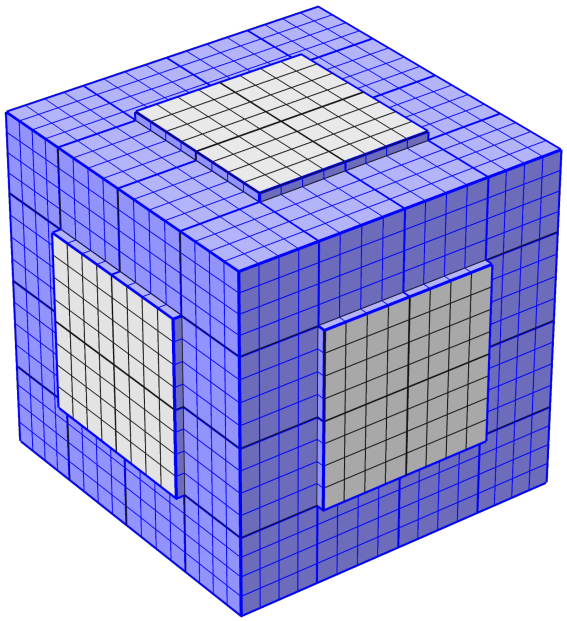
- Cubic semi-open cell: 1mm
- Hole: $0.5 \times 0.5\ \mathrm{mm}^2$
- Wall thickness: 50 μm
- Porosity $\phi = 0.895$

ρ_0^s [kg m^{-3}]	1100
K^s [GPa]	0.22
G^s [GPa]	0.083
C^s [J kg^{-1} K^{-1}]	1800
k^s [W m^{-1} K^{-1}]	0.022

Solid domain

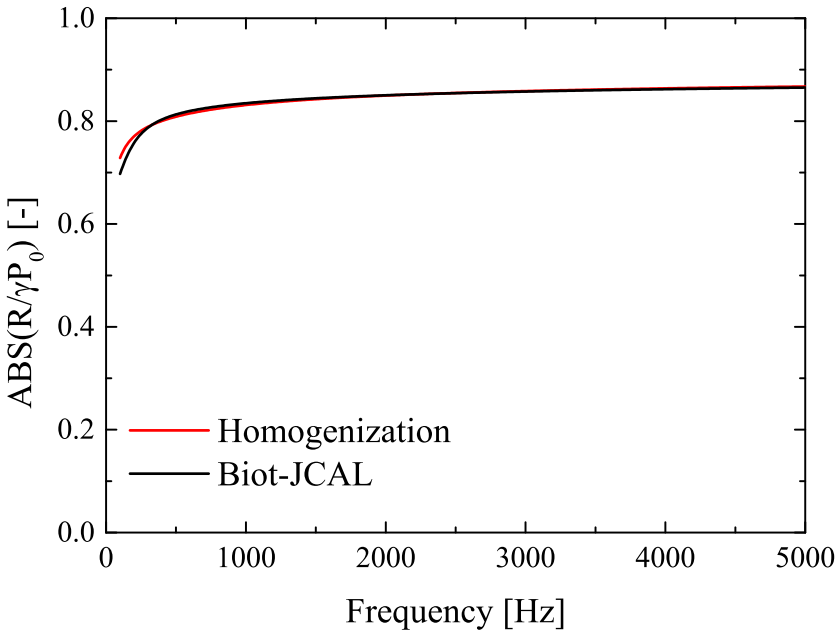


Fluid domain

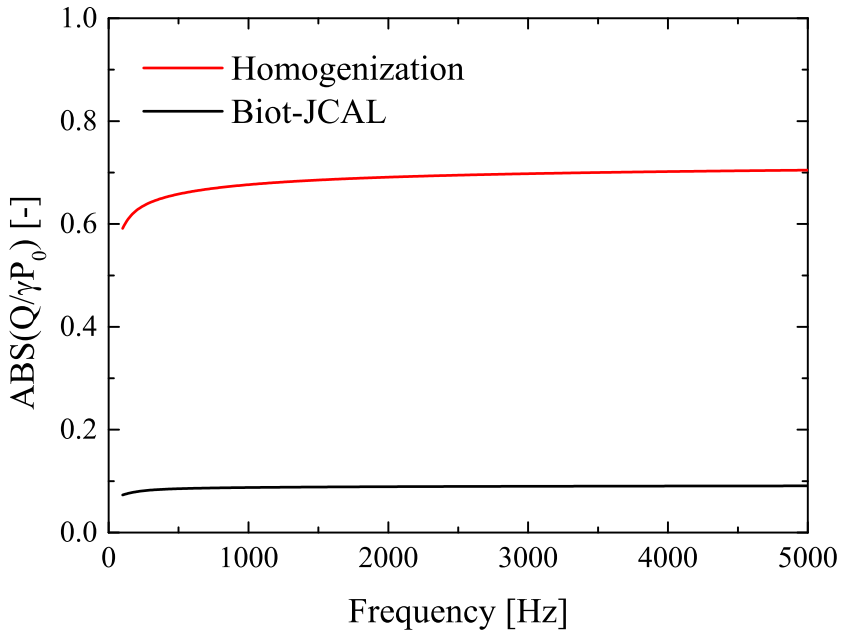


Microscopic RVE - effective properties

R - effect of $\nabla_M \cdot \boldsymbol{u}_M^f$

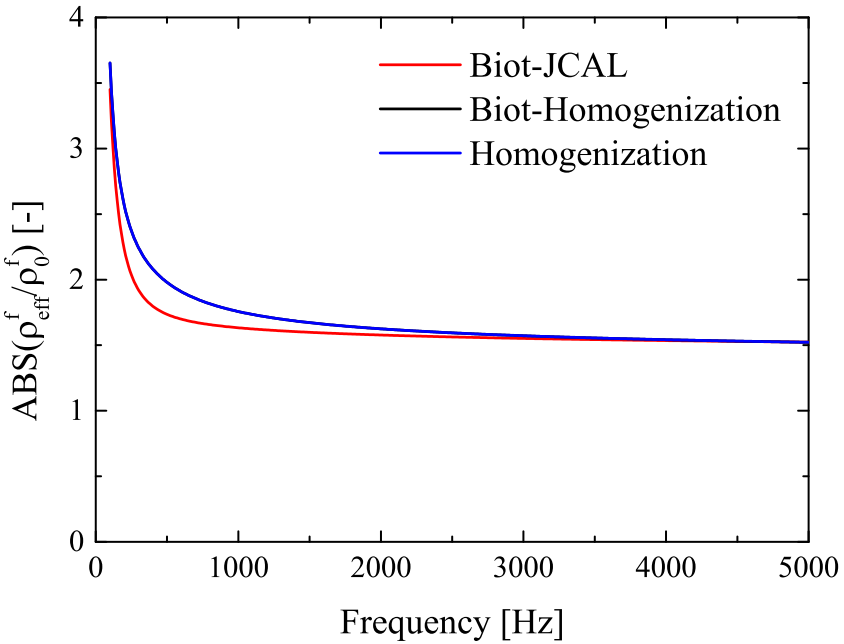


Q - effect of $\nabla_M \cdot \boldsymbol{u}_M^s$

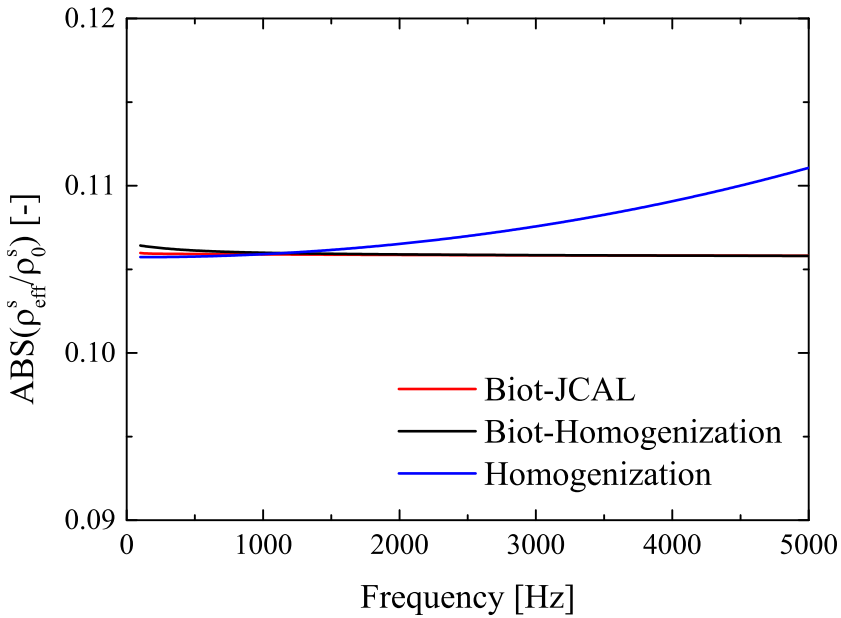


Microscopic RVE - effective properties

$$\rho^f_{\text{eff}} = \eta^f \rho^f_0 + \rho_c$$

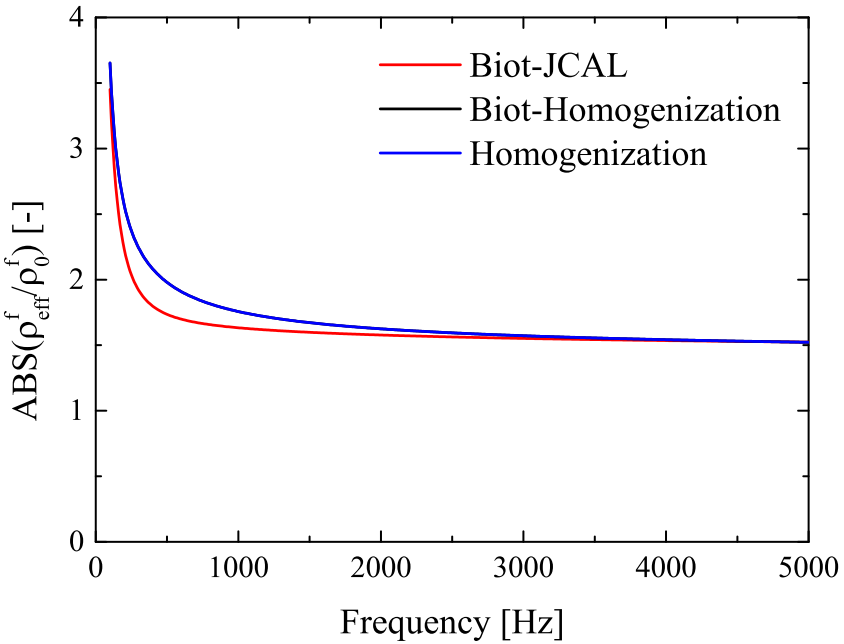


$$\rho^s_{\text{eff}} = \eta^s \rho^s_0 + \rho_c$$

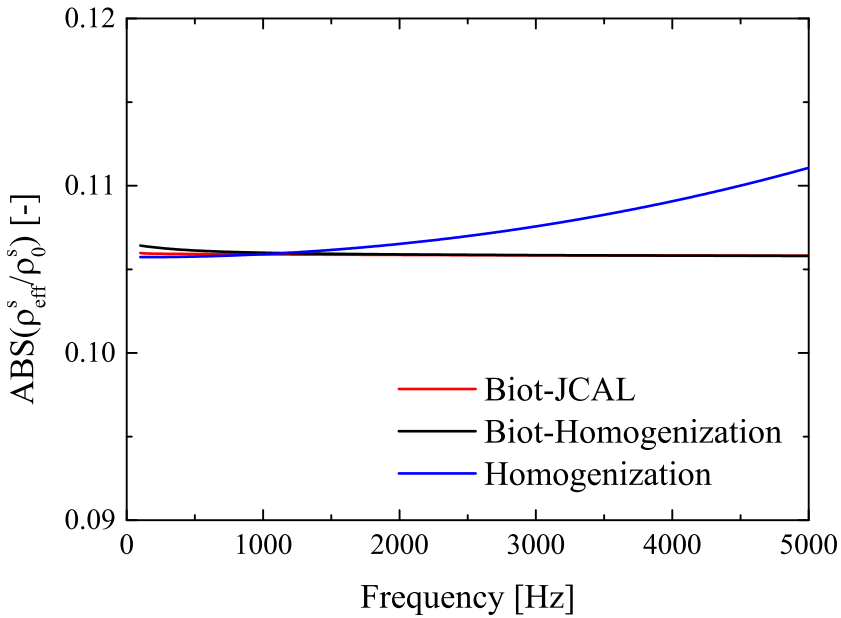


Microscopic RVE - effective properties

$$\rho^f_{\text{eff}} = \eta^f \rho^f_0 + \rho_c$$



$$\rho^s_{\text{eff}} = \eta^s \rho^s_0 + \rho_c$$

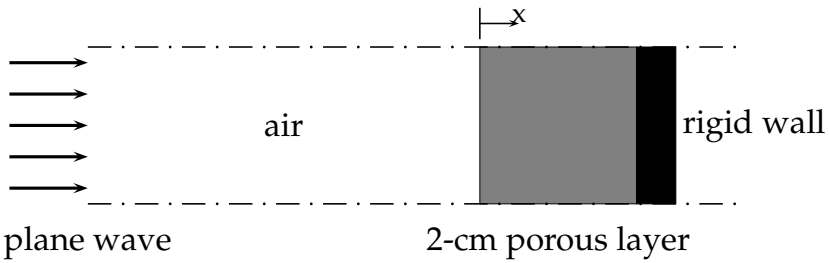


Conclusions

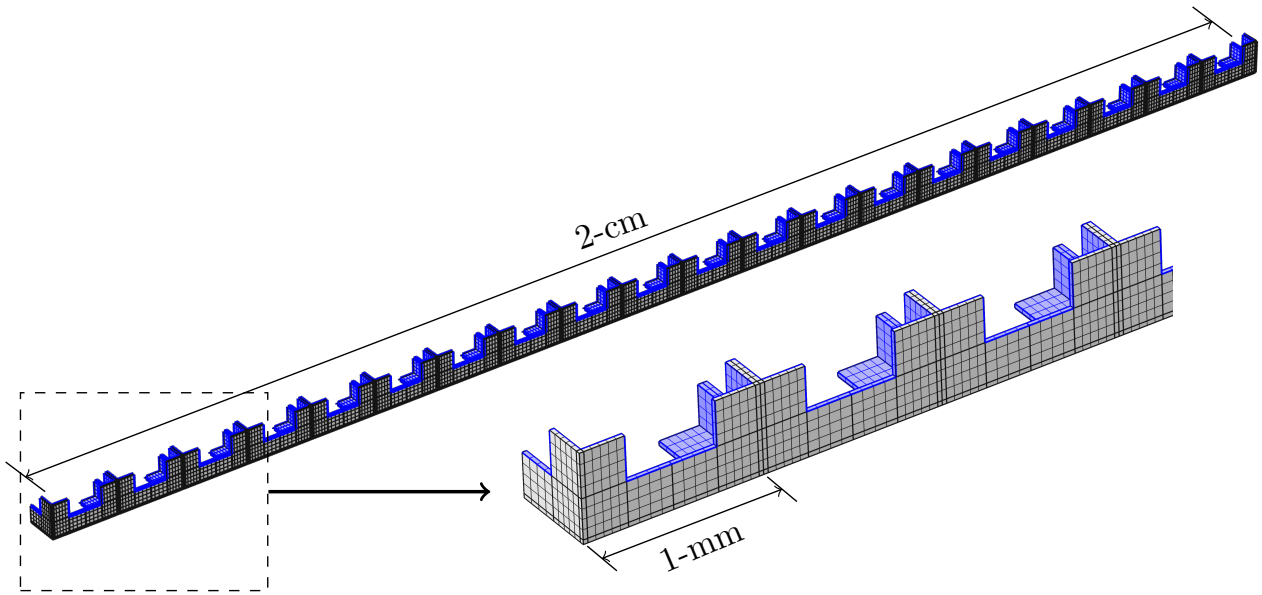
- the fluid properties qualitatively agree with the JCAL model
- $|\rho^f_{\text{eff}}|_{\text{H}} = |\rho^f_{\text{eff}}|_{\text{B}} > |\rho^f_{\text{eff}}|_{\text{J}}$ and $|\rho^s_{\text{eff}}|_{\text{H}} > |\rho^s_{\text{eff}}|_{\text{B}} \approx |\rho^s_{\text{eff}}|_{\text{J}}$
- the micro inertial effects could be significant

Macroscopic simulation

- Macroscopic sound absorption test (100 - 5000 Hz, stepsize 10 Hz)



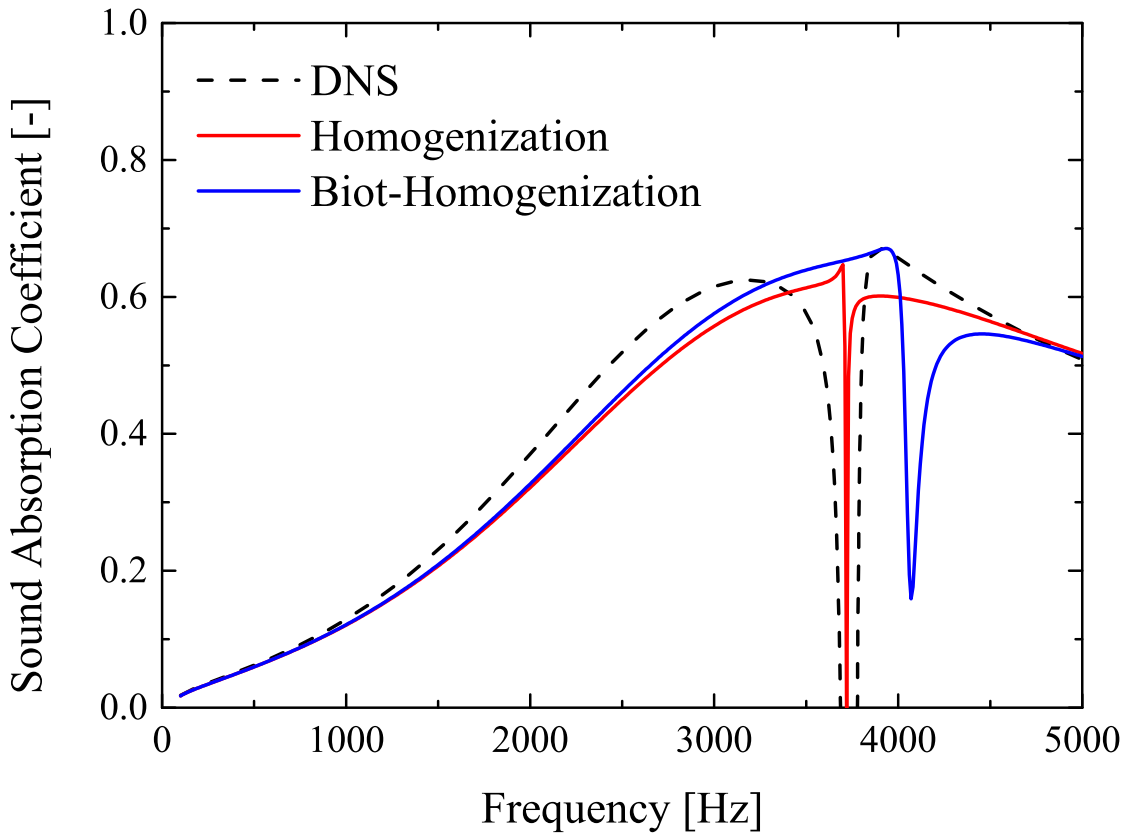
- Air: Helmholtz equation (sound speed 343 m/s)
- Porous: Homogenization vs. Biot-Homogenization, DNS (reference)



Remark

- $|\rho^f_{\text{eff}}|_{\text{H}} = |\rho^f_{\text{eff}}|_{\text{B}} \Longrightarrow$ the results are very close.
- $|\rho^s_{\text{eff}}|_{\text{H}} \geq |\rho^s_{\text{eff}}|_{\text{B}} \Longrightarrow$ the resonance frequencies are different.

$$\alpha = 1 - \left| \frac{p_{\text{ref}}}{p_{\text{inc}}} \right|^2$$

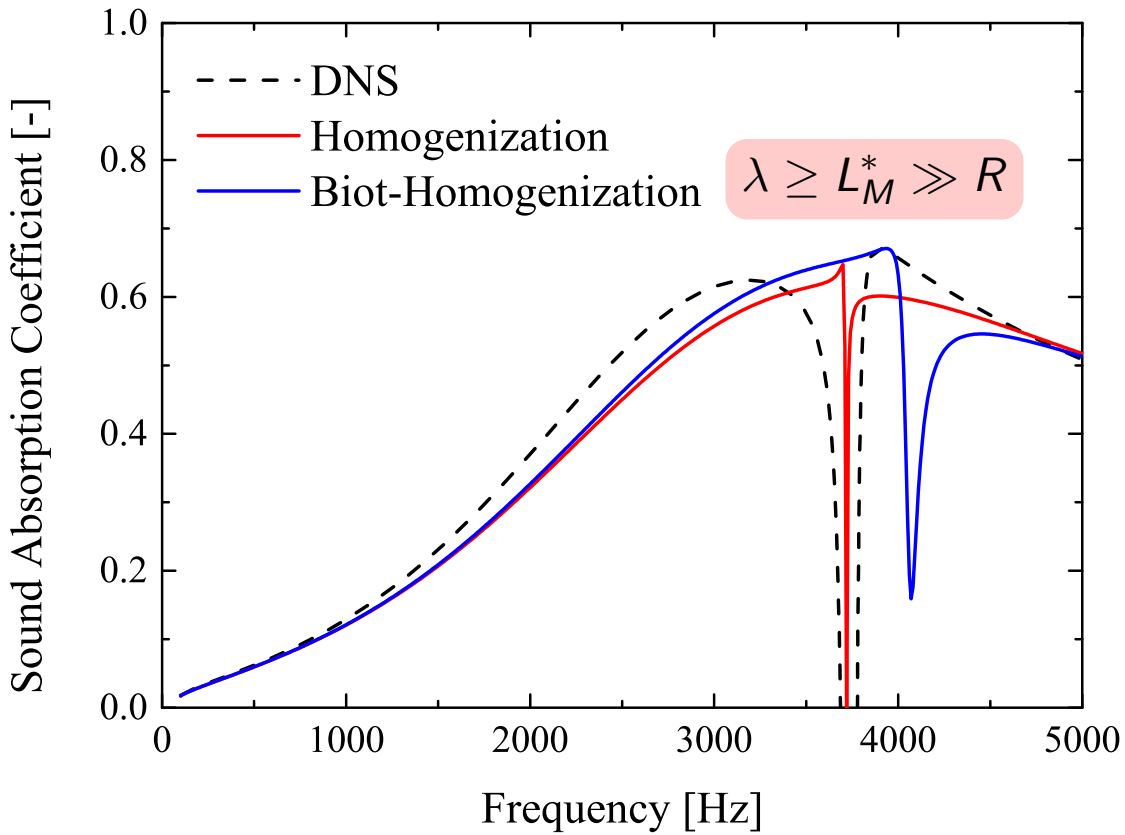


Homogenization vs. Biot's theory

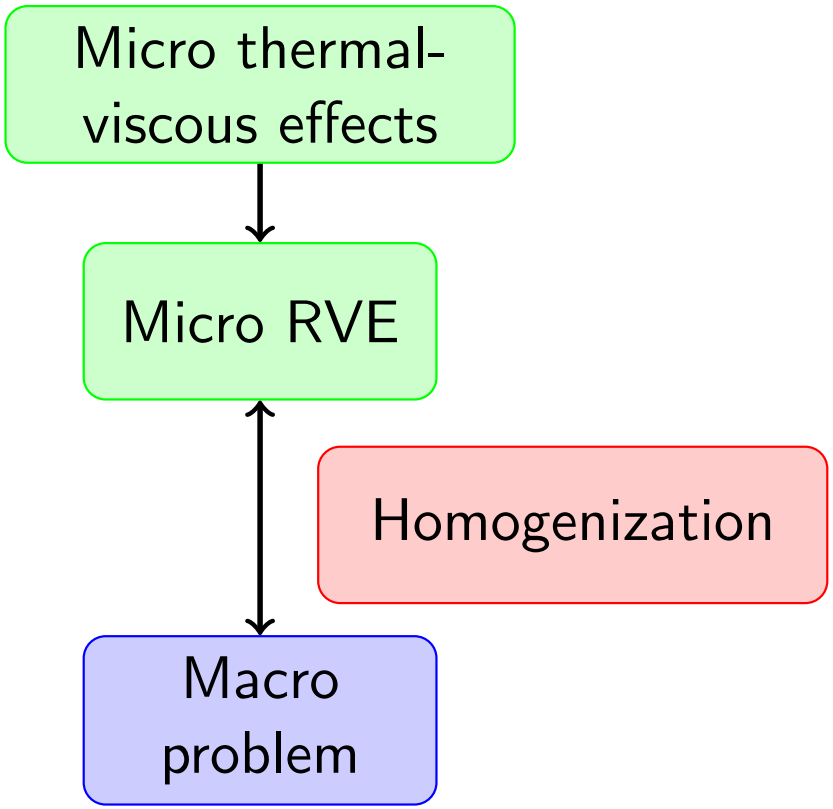
Remark

- $|\rho^f_{\text{eff}}|_{\text{H}} = |\rho^f_{\text{eff}}|_{\text{B}} \Longrightarrow$ the results are very close.
- $|\rho^s_{\text{eff}}|_{\text{H}} \geq |\rho^s_{\text{eff}}|_{\text{B}} \Longrightarrow$ the resonance frequencies are different.

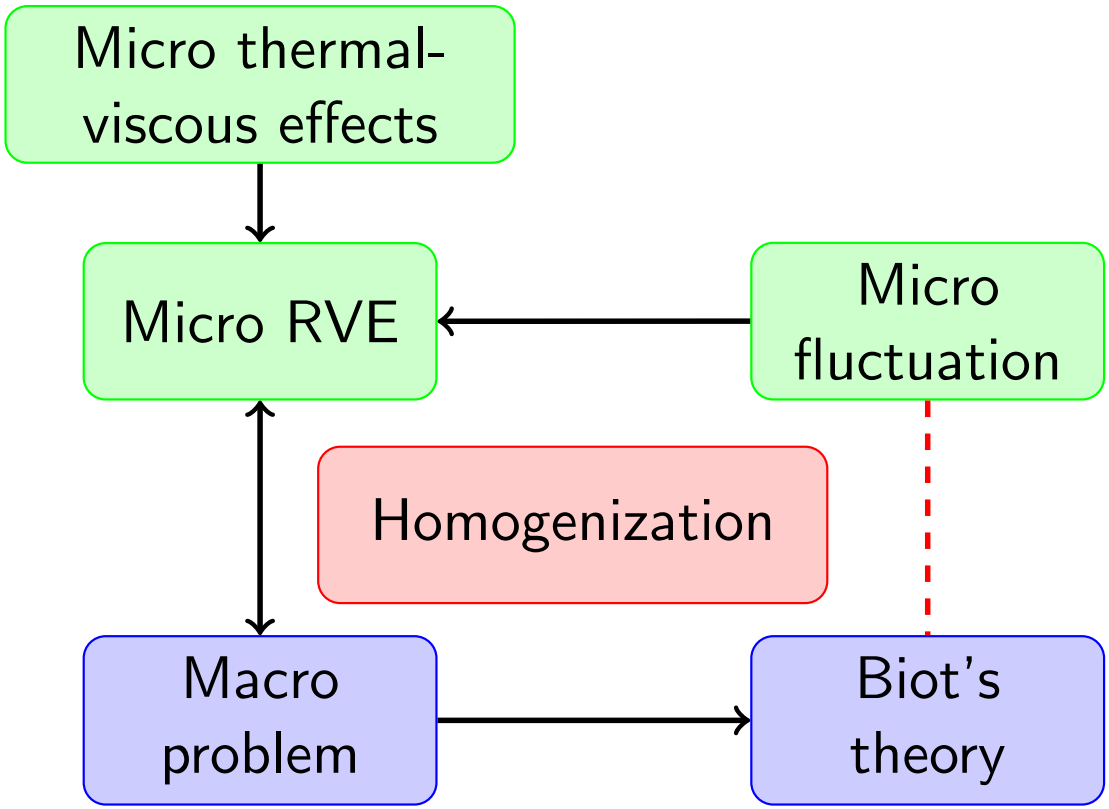
$$\alpha = 1 - \left| \frac{p_{\text{ref}}}{p_{\text{inc}}} \right|^2$$

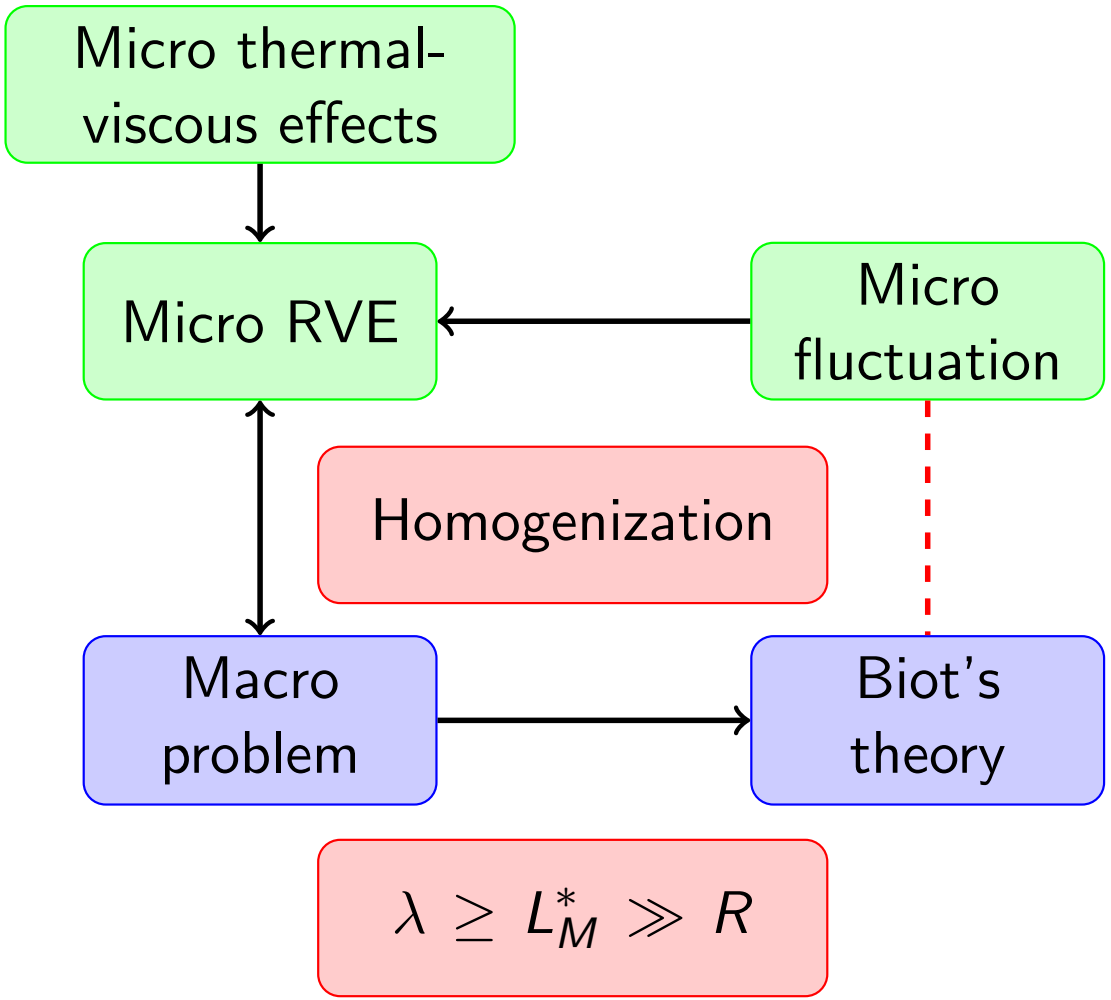


Conclusions

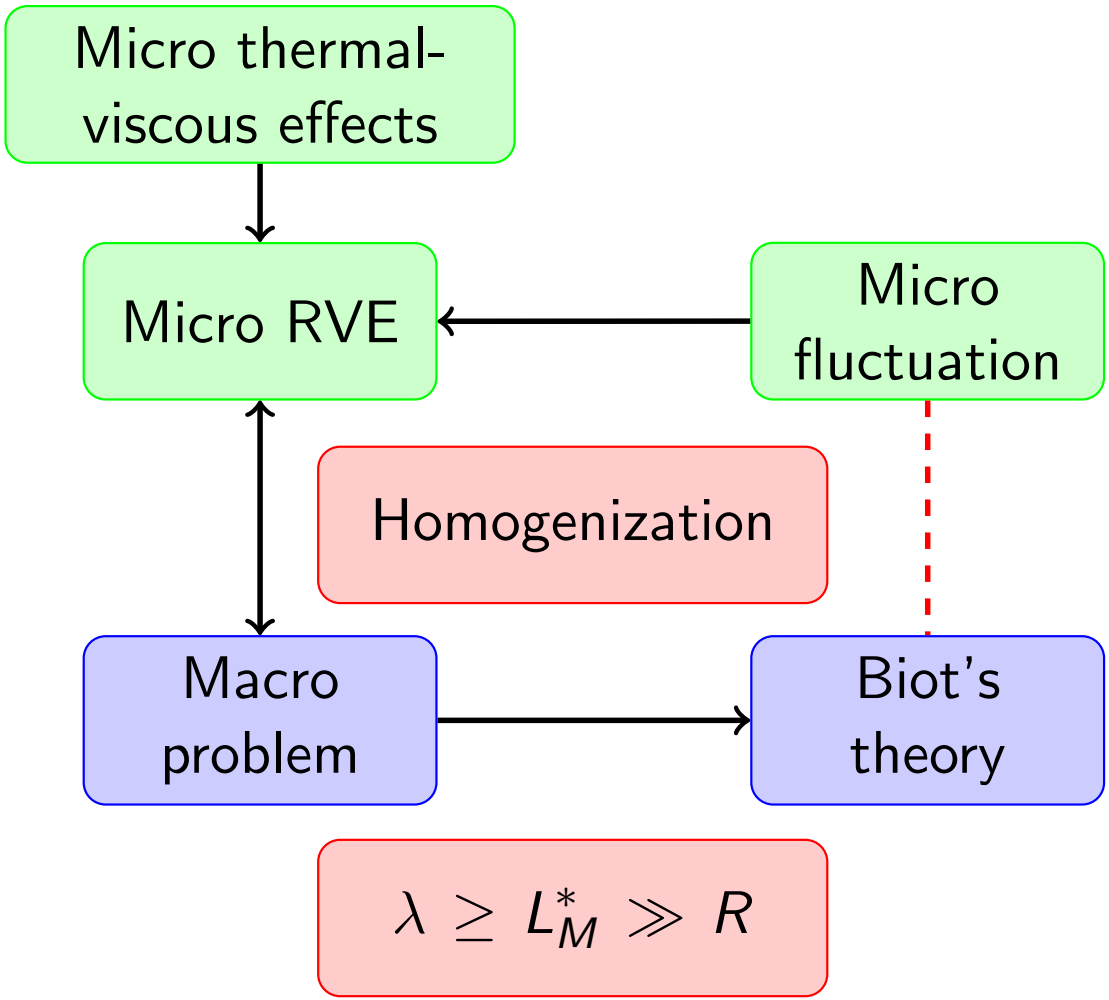


Conclusions



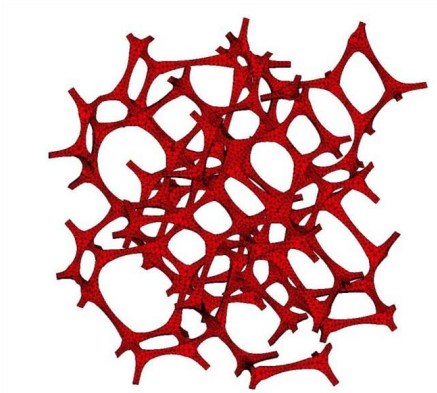

Micro thermal-viscous effects
Micro RVE
Micro fluctuation
Homogenization
Macro problem
Biot's theory
$\lambda \geq L_M^* \gg R$

Conclusions



Future

Num. vs. Exp.



Thank you!