Double porosity model applied to parallel baffle silencers

B. Nennig[1], N. Dauchez[2], R. Binois[3], E. Perrey-Debain[2], J.-M. Ville[2]

[1]Laboratoire d'Ingénierie des Systèmes Mécaniques et des Matériaux EA 2336 (LISMMA)
Institut Supérieur de Mécanique de Paris - SUPMECA
93400 Saint-Ouen, France

[2]Laboratoire Roberval UMR 7337, Département Acoustique et Vibration, Centre de Recherches de Royallieu
Université de Technologie de Compiègne (UTC)
BP 20529, 60205 Compiègne cedex, France

[3]Alhyange Acoustique
60 rue du Faubourg Poissonnière, 75010 Paris, France

SAPEM 2014

December 16-18 2014, Stockholm (Sweden).

Context

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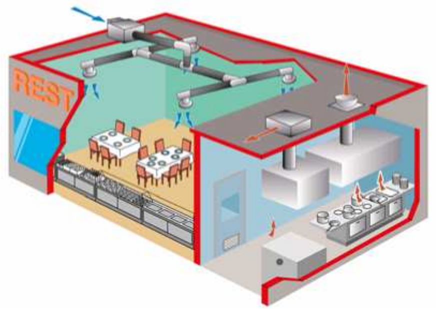
HVAC Silencers are most often made of parallel baffles of porous material (rockwool)



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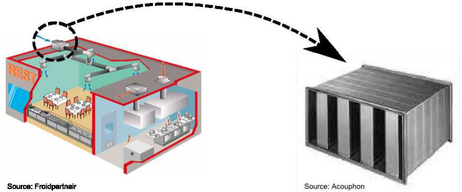

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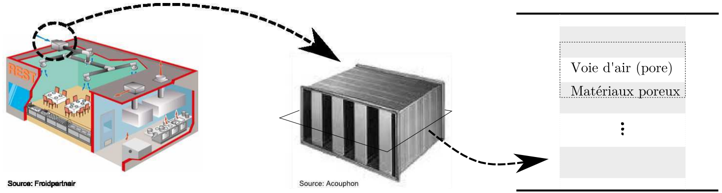

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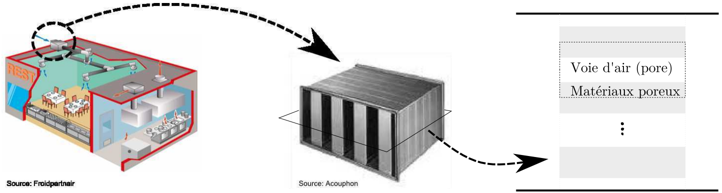

- Weak attenuation at low frequency [63 Hz - 500 Hz].
- For this frequency range fundamental mode dominates
- Fundamental mode range depends on both the duct height h and of the number of baffle N : $n = n^{inc} + 2qN$. [Mechel, 1990]

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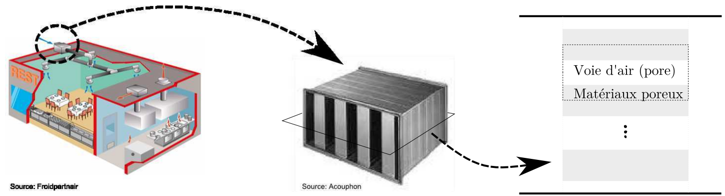

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Objectives

1. Propose a **simple model** for silencer design
2. Propose a new **framework** to explain the behaviour of such systems
3. Get **closed form solution** for the resistivity which maximise attenuation

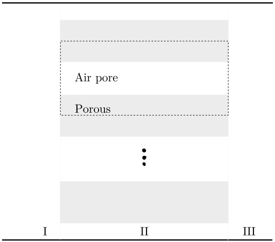
Context

Parallel baffle silencers are often described by 2 ways :

Global FEM models

Full mesh and resolution

- [Mehdizadeh et al., 2005]



Cross-section Eigenmode expansion

$\Longrightarrow$ 2 steps :
1) **Compute the guided eigenmode** then 2) **mode matching**

1. Direct dispersion relation solving
   - [Ko, 1975]
   - [Cummings-Sormaz, 1993]
   - [Aurégan et al., 2001]
2. Periodic arrangement
   - [Mechel, 1990]
   - [Tam-Fahy, 1991]
3. Finite element method
   - [Kirby, 2005]

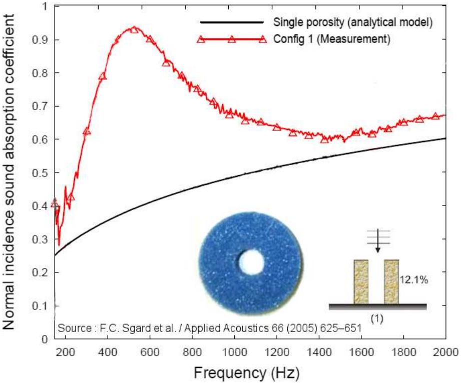
If the fundamental mode dominates the solution,

Parallel baffle silencer can be described as an effective medium with an equivalent density and an equivalent wavenumber.

Context

This effective media is well known and corresponds to the double porosity model (DP) !

- Acoustics in DP material have been established through the **homogenization method** by [Auriault et al., 1994].
- Extended by [Olny et al., 2003] to clarify the influence of the permeabilities contrast between the macro-pores and the micro-pores arising in **sound absorbing materials**.
- DP material have been extensively used to enhanced **normal incidence absorption** of porous materials [Atalla et al., 2001 ; Sgard et al., 2005 ; Bécot, 2008].



[From Sgard, 2005]

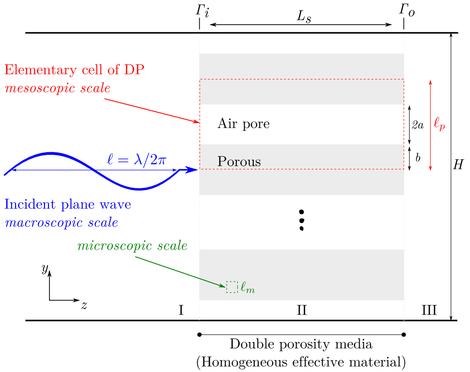
To the authors knowledges, no attempt was made to use DP in the silencer context

Contents

1. Simplified silencer model : Transfer matrix approach + DP
2. Accuracy
3. Derivation of optimal design parameter

1. Simplified silencer model

Problem statement



In each domain, only the fundamental mode is taken into account. The pressure field fulfils the 1D Helmholtz equation according to z direction ($\mathrm{e}^{\mathrm{i}\omega t}$).

$$\partial_z^2 p_i + k_i^2 p_i = 0, \text{ with } i = \text{I, II, III},$$

with the wavenumber k_i ($i =$ air, dp, air).

This propagation model is very basic, but the difficulty now relies on the determination of the effective wavenumber k_{dp} and the effective density ρ_{dp} of region II [Olny, 2003].

Transfer matrix approach

Propagation is driven by the transfer matrix which couple the pressure p and the velocity v at the input (i) to those of the output (o). For a duct of length L, we get

$$\begin{pmatrix} p \\ v \end{pmatrix}_{\mathrm{i}} = \underbrace{\begin{bmatrix} \cos k_i L & \mathrm{i} Z_i \sin k_i L \\ (\mathrm{i}/Z_i) \sin k_i L & \cos k_i L \end{bmatrix}}_{\mathbf{T}_e} \begin{pmatrix} p \\ v \end{pmatrix}_{\mathrm{o}},$$

Where $Z_i = \rho_i c_i$, ρ_i and c_i are the characteristic impedance, the density, the speed of sound in the domain i.

At each interface input (i) or output (o), the continuity of the pressure p and of the velocity v must be satisfied. The global transfer matrix $\mathbf{T}$ of the whole silencer is obtained by multiplying together the transfer matrix of each domain$\mathbf{T}_i$.

Metallic fairing can be included by discontinuity impedance matrix.

For an incident fundamental mode, the reflection coefficient R and the transmission coefficient T can be extracted from $\mathbf{T}$.
Transmission loss is defined as,

$$\mathrm{TL} = -20 \log_{10} |T|.$$

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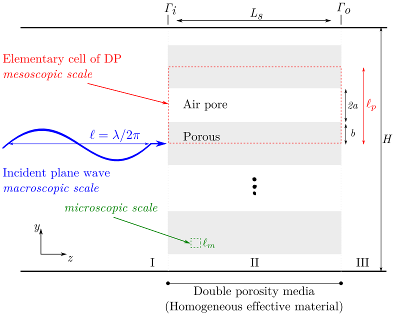
Double porosity model

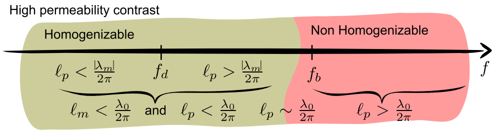
Three characteristic lengths are required to describe each scale in a DP material

1. **At macroscopic scale**, the length $\ell \sim \lambda/2\pi$, is given by the wavelength λ
2. **At mesoscopic scale**, the length ℓ_p is the size of the airway
3. **At microscopic scale**, the length $\ell_m \sim \sqrt{\frac{8\eta}{\sigma\phi}}$ corresponds to the micro-pore size

Subscripts p, m and dp are used, for the pore (=Airway), the micro-pores (porous baffle) and the DP material (silencer).



To ensure scale separation and applied homogenization, $\ell_m/\ell_p \ll 1$, $\ell_p/\ell \ll 1$ are required.

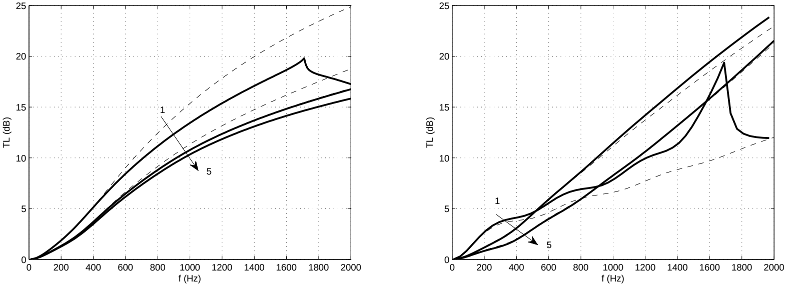


- The validity of DP model depends on the frequency, the material and the geometry.

2. Accuracy

Comparison with a reference solution

Comparison with the DP model (− −) and the reference solution (——) [Binois, 2013]. $H = 20$ cm, $L_s = 30$ cm, percentage of airway 50% with 1 to 5 baffles.

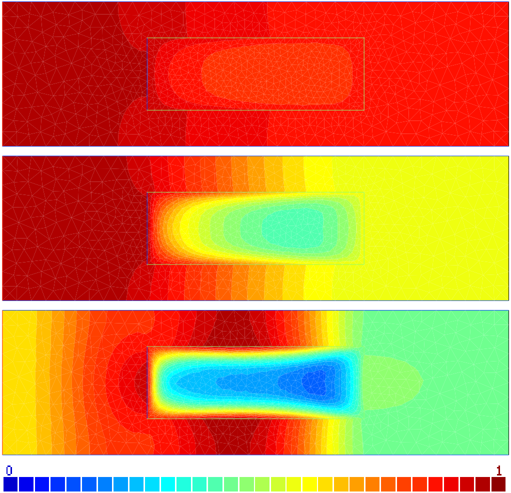


(a) $\sigma = 14066$ Nm^{-4}s (b) $\sigma = 135000$ Nm^{-4}s

- When the number of baffle N increase, the DP model become more accurate. Indeed, the assumption $\ell_p/\ell \ll 1$ is easier to satisfy.
- When several baffle are present,it can be shown [Mechel, 1990] that the fundamental mode propagates alone even above cut-off.
- In this case $\omega_d = \frac{3P_0}{b^2\sigma\phi} \to \infty$ and the pressure is constant across the section

Pressure field at different regimes

Normalized pressure field, obtained by a reference solution [Binois,2013]



$\omega = \frac{\omega_d}{3}$, Pressure is nearly *uniform*

$\omega = \omega_d$, $\frac{\lambda_m}{2} \approx 2b$, $\frac{|p_{max}|}{2}$

$\omega = 3\omega_d$, Boundary layer effect

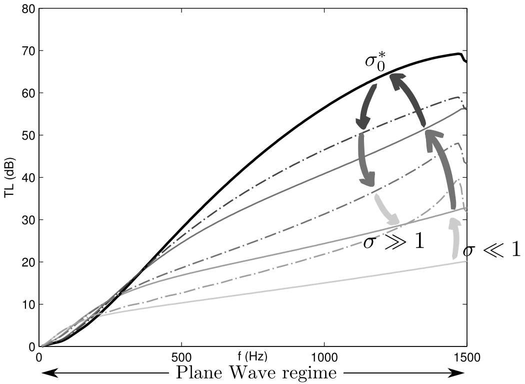
$\sigma = 135000$ Nm^{-4}s, $H = 20$ cm, $L_s = 30$ cm, percentage of airway 50%, 1 baffle.

▶ Diffusion frequency is a usefull parameter !

3. Design

First observation

It can be observed for a large number of configurations that if the resistivity is changed (other parameter are modify accordingly to keep the shape factor c, the ratio of the Characteristic length $r_\Lambda = \Lambda/\Lambda'$, and α_∞ constant) :



An optimal value exists and corresponds to the best compromise for the fundamental mode attenuation.

- How to find this value ?

Optimal resistivity determination

Assumptions

- Weak reflection at the inlet and the outlet of the silencer
- Attenuation is given only by the imaginary part of k_{dp} and $\mathrm{TL} \approx 8.66\,\mathfrak{Im}\,k_{dp} L_s$

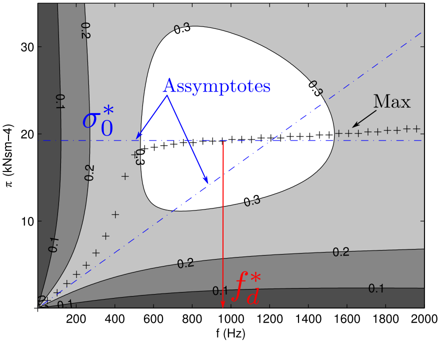
Main steps

1. Let be $\epsilon_d = \frac{\omega}{\omega_d}$ and $\epsilon_b = \frac{\omega}{\omega_b}$ two small parameters, the Biot frequency $\omega_b = \frac{\sigma \phi_m}{\rho_0 \alpha_\infty}$ and the diffusion frequency $\omega_d = \frac{3P_0}{b^2 \sigma \phi}$.
2. Compute the low frequency approximation of the effective parameters $(m,\ p,\ dp)$
3. Get low frequency approximation of $\mathfrak{Im}\,k_{dp}$
4. Solve $\frac{\partial\, \mathfrak{Im}\, k_{dp}}{\partial \sigma} = 0$ to find some envelops curves of the TL

$$\sigma_0^* = \sqrt{\frac{3\rho_0 P_0}{b^2 \phi^2} \left(\frac{1}{\gamma \alpha_\infty} + \frac{\bar{\phi}_p \phi}{\phi_p \alpha_\infty} + \Upsilon \right)},$$

with $\Upsilon = \frac{\gamma - 1}{\gamma} \frac{\mathrm{Pr}}{(r_\Lambda\ c)^2}$, c square root of the shape factor and $r_\Lambda = \Lambda / \Lambda'$.

Isovalue of $\mathfrak{Im}\, k_{dp}/k_0$ according to the frequency and the resistivity.
Geometry of the silencer $H = 0.2$ m, $\phi_p = 0.5$, $\sigma = 14066$ Nm^{-4}s.

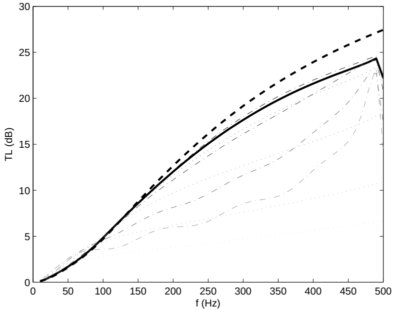


$$\sigma_0^* = \sqrt{\frac{3\rho_0 P_0}{b^2\phi^2}\left(\frac{1}{\gamma\alpha_\infty} + \frac{\bar{\phi}_p\phi}{\phi_p\alpha_\infty} + \Upsilon\right)}$$

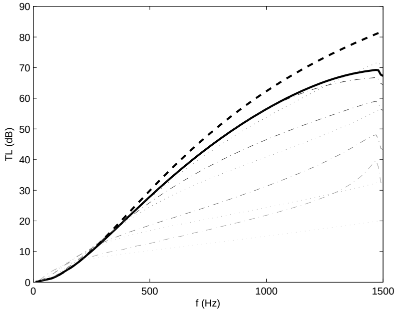
- The Low frequency asymptote leads to a good approximation of the optimal resistivity
- The best attenuation arises close to f_d^*, the diffusion frequency obtained with $\sigma = \sigma_0^*$
- Check on a lots of configurations

Examples and robustness of σ_0^*

Transmission loss $\sigma \in [0.1\sigma_0^*, \ldots, 10\sigma_0^*]$. Obtained with a reference solution [Binois, 2013] on realistic silencers. If $\sigma < \sigma_0^*$, (· · ·), If $\sigma > \sigma_0^*$,(— · —), If $\sigma = \sigma_0^*$, (▬▬). Geometry $H = 0.70$ m and $L_s = 1.20$ m.



(a) $\phi_p = 0.5$, $N = 1$



(b) $\phi_p = 0.5$, $N = 3$

Even if the DP model is not used to obtained the TL (less stringent assumption), σ_0^* yields always to the best attenuation in the frequency band.

Conclusions and prospects

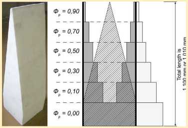
Conclusions and prospects

Going back to the objectives, the DP models

- ✓ Leads to good results, for very small computational effort for a large variety of silencers.
- ✓ Leads to a new insight of the two dynamical regime of the silencer thank to ω_d (transversal resonance inside the baffle).
- ✓ Leads to a closed form expression for the resistivity that maximizes the attenuation.

Prospects :

1. What happens for absorption at normal incidence $\sigma = \sigma_0^*$?
2. Extend this approach to more complex configuration
   - see for instance [Jaouen et al.,2008]



   - Take into account metallic fairings

– Thank you for your attention –

Annexe : Materials

TABLE: Material properties used in numerical tests. With the porosity ϕ_m, flow resistivity σ, the tortuosity α_∞, the viscous and thermal characteristic lengths Λ and Λ'.

Material	ϕ_m	σ	α_∞	Λ	Λ'
	-	[$Nm^{-4}s$]	-	[μm]	[μm]
GW1	0.954	14 066	1.0	91.2	182.4
GW2	0.94	135 000	2.1	49	166

Appendix : DP model details

The effective wavenumber

$$k_{dp} = \omega\sqrt{\rho_{dp}/K_{dp}}. \tag{1}$$

The effective density,

$$\rho_{dp} = \frac{\eta}{\mathrm{i}\omega\Pi_{dp}}, \tag{2}$$

is related to the dynamic permeability in the normal direction $\Pi_{dp} = (1-\phi_p)\Pi_m + \Pi_p$, where Π_m can be deduced from (2) and (**??**) by changing the subscript $dp \rightleftarrows m$, and with dynamic permeability in the meso pore $\Pi_p = \frac{\phi_p}{\mathrm{i}}\delta_v^2 F(\mu_v)$, with the function

$$F(\mu) = \left(1 - \frac{\tanh \mu\sqrt{\mathrm{i}}}{\mu\sqrt{\mathrm{i}}}\right), \tag{3}$$

the viscous boundary layer thickness $\delta_v = \sqrt{\eta/(\rho_0\omega)}$ and the ratio $\mu_v = a/\delta_v$ between the air gap and the viscous boundary layer.

The bulk modulus of the DP material is a combination of the bulk modulus of the porous media K_m (see (**??**)) and the bulk modulus of the air gapK_p given by the simplified Lafarge's model [**?**] :

$$K_p = \frac{\gamma P_0/\phi_p}{\gamma - \mathrm{i}(\gamma-1)\frac{\Theta_p}{\delta_t^2\phi_p}}, \tag{4}$$

where

$$\Theta_p = \frac{\phi_p}{\mathrm{i}}\delta_t^2 F(\mu_t) \tag{5}$$

Appendix : DP model details (cont.)

with, $\delta_t = \sqrt{\kappa/(\rho_0 C_p \omega)}$, the thermal boundary layer thickness and the ratio $\mu_t = a/\delta_t$. This yields

$$K_{dp} = \left[\frac{1}{K_p} + (1-\phi_p) \frac{F_d\left(\omega \frac{P_0}{\phi_m K_m}\right)}{K_m} \right]^{-1} \tag{6}$$

with

$$F_d(\omega) = 1 - \mathrm{i} \frac{\omega}{\omega_d} \frac{D(\omega)}{D(0)}, \tag{7}$$

the diffusion frequency $\omega_d = \frac{\bar{\phi}_p P_0}{\sigma \phi D_0}$. For slits, Olny [?] gives $D(0) = (1-\phi_p) b^2/3$ and

$$D(\omega) = -\mathrm{i}(1-\phi_p)\delta_d^2 F(\mu_d). \tag{8}$$

Where the pressure diffusion skin depth $\delta_d = \sqrt{\frac{P_0}{\sigma \phi_m \omega}}$, gives an estimation of the boundary layer where take place strong fluctuation of the micro pore pressure and $\mu_d = b/\delta_d$. In this case the diffusion frequency reads

$$\omega_d = \frac{3P_0}{b^2 \sigma \phi_m}. \tag{9}$$

Appendix : DP model details (cont.)

The function F_d relates the mean pressure in the micro pore to the average diffused pressure in the air gap. In the case of slit, the expression of F_d can be simplified into

$$F_d(\omega) = \frac{\tanh \mu_d \sqrt{\mathrm{i}}}{\mu_d \sqrt{\mathrm{i}}}, \tag{10}$$

$\delta_d = b\sqrt{\omega_d/(3\omega)}$.