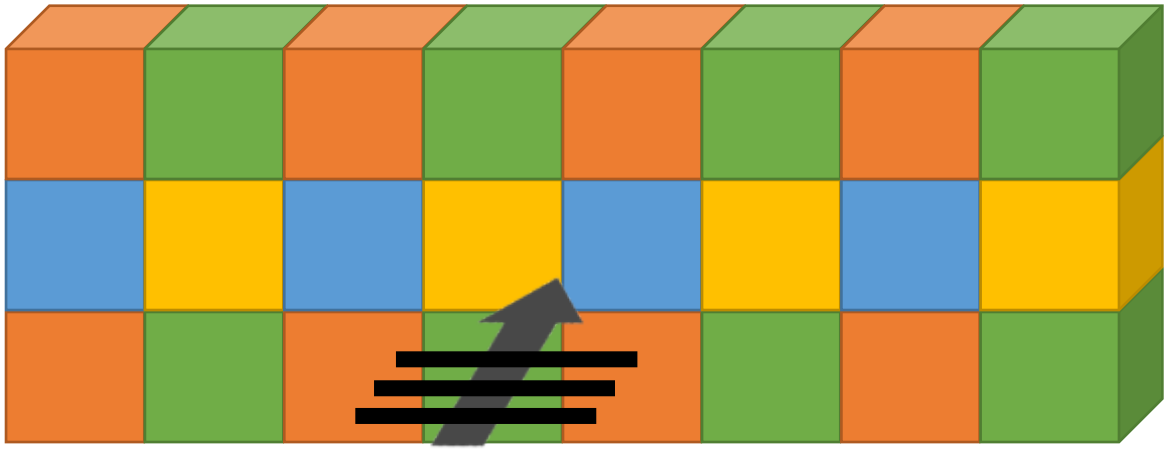
Recent highlights on the parallel transfer matrix method (PTMM)



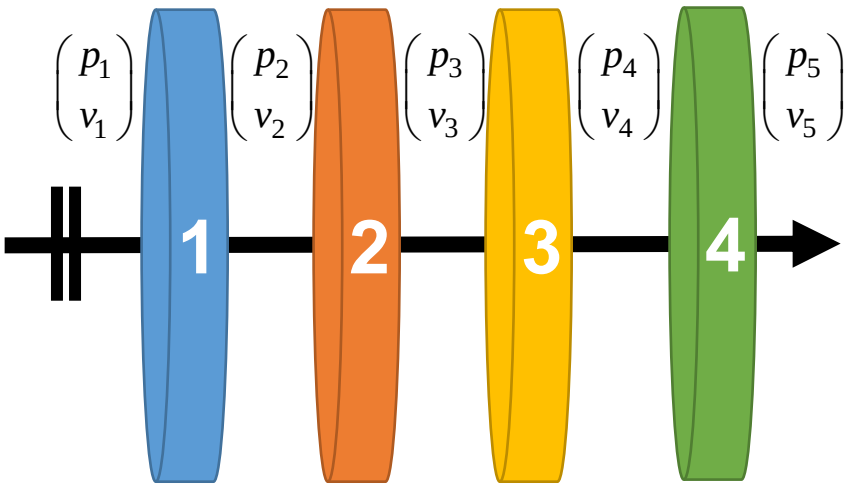
Kévin Verdière[1]
Raymond Panneton[1]
Saïd Elkoun[1]

Thomas Dupont[2]
Philippe Leclaire[2]

[1] GAUS, Université de Sherbrooke (Qc, Canada)
[2] DRIVE, ISAT, Université de Bourgogne (Nevers, France)

PRESENTATION OUTLINE

1. Series Transfer Matrix Method (STMM)
2. Parallel Transfer Matrix Method (PTMM)
3. PTMM versus Admittance Sum Method (ASM)
4. PTMM and lateral interactions
5. PTMM versus Mixing Rules Method (MRM)
6. PTMM and diffusion phenomenon
7. Homogeneity (pressure uniformity)
8. Conclusion



$\tilde{k}_i$: Complex wave number of medium i

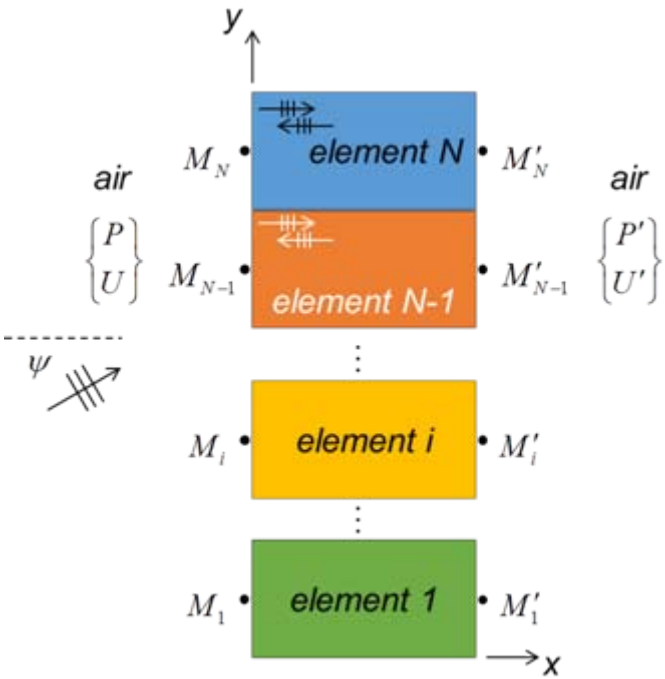
$\tilde{Z}_i$: Characteriztic impedance of medium i

$$\begin{pmatrix} p_i \\ v_i \end{pmatrix} = \begin{pmatrix} \cos \tilde{k}_i h_i & j\tilde{Z}_i \sin \tilde{k}_i h_i \\ j\dfrac{1}{\tilde{Z}_i} \sin \tilde{k}_i h_i & \cos \tilde{k}_i h_i \end{pmatrix} \begin{pmatrix} p_{i+1} \\ v_{i+1} \end{pmatrix} = \mathbf{T}_1 \begin{pmatrix} p_{i+1} \\ v_{i+1} \end{pmatrix}$$

$$\left.\begin{aligned} \begin{pmatrix} p_1 \\ v_1 \end{pmatrix} &= \mathbf{T}_1 \begin{pmatrix} p_2 \\ v_2 \end{pmatrix} \\ \begin{pmatrix} p_2 \\ v_2 \end{pmatrix} &= \mathbf{T}_2 \begin{pmatrix} p_3 \\ v_3 \end{pmatrix} \\ \begin{pmatrix} p_3 \\ v_3 \end{pmatrix} &= \mathbf{T}_3 \begin{pmatrix} p_4 \\ v_4 \end{pmatrix} \\ \begin{pmatrix} p_4 \\ v_4 \end{pmatrix} &= \mathbf{T}_4 \begin{pmatrix} p_5 \\ v_5 \end{pmatrix} \end{aligned}\right\} \begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \mathbf{T}_1 \mathbf{T}_2 \mathbf{T}_3 \mathbf{T}_4 \begin{pmatrix} p_5 \\ v_5 \end{pmatrix} = \mathbf{T}_G \begin{pmatrix} p_5 \\ v_5 \end{pmatrix}$$

→ Absorption Coefficient
Transmission Loss

Patchwork



key assumptions :

1)only plane waves propagate upstream and downstream the construction;

2)only normal incidence plane waves propagate in the construction;

3)no exchange exists between adjacent parallel elements;

4)the wavelength is much larger than the periodic elementary patchwork;

5)each element is represented by a 2x2 transfer matrix.

Transfer matrix of each element i **1**

$$\begin{Bmatrix} P_i \\ U_i \end{Bmatrix} = \begin{bmatrix} T_{i,11} & T_{i,12} \\ T_{i,21} & T_{i,22} \end{bmatrix} \begin{Bmatrix} P'_i \\ -U'_i \end{Bmatrix}$$

Admittance matrix of each element i **2**

$$\begin{Bmatrix} U_i \\ U'_i \end{Bmatrix} = \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$

These conditions are the same as the well known admittance sum method.

Connexion conditions **3**

$$\begin{Bmatrix} U \\ U' \end{Bmatrix} = \sum \begin{Bmatrix} r_i U_i \\ r_i U'_i \end{Bmatrix}$$

Continuity of upstream flow

Continuity of downstream flow

$$\begin{Bmatrix} P \\ P' \end{Bmatrix} = \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$

Uniformity of upstream pressure

Uniformity of downstream pressure for open-end cells

4

$$\begin{Bmatrix} r_i U_i \\ r_i U'_i \end{Bmatrix} = r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$

Admittance matrix for the parallel assembly **5**

$$\begin{Bmatrix} U \\ U' \end{Bmatrix} = \left(\sum r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \right) \begin{Bmatrix} P \\ P' \end{Bmatrix}$$

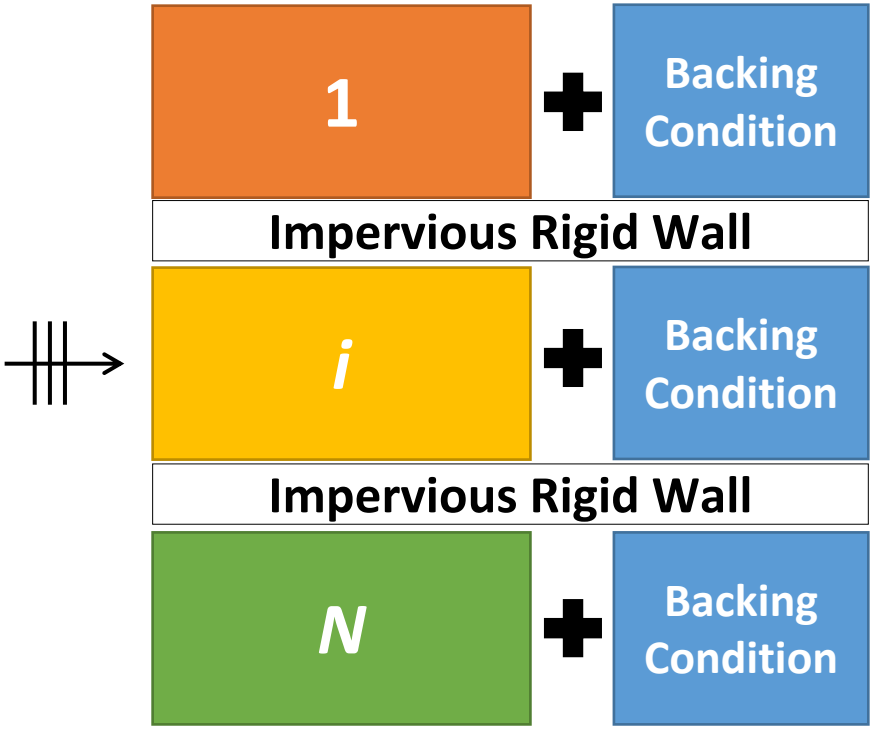
6 Transfer matrix for the parallel assembly with open-ended elements (*i*)

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = \mathbf{T_p} \begin{Bmatrix} P' \\ U' \end{Bmatrix} = \frac{-1}{\Sigma r_i y_{i,21}} \begin{pmatrix} \Sigma r_i y_{i,22} & -1 \\ \Sigma r_i y_{i,22} \Sigma r_i y_{i,11} - \Sigma r_i y_{i,12} \Sigma r_i y_{i,21} & -\Sigma r_i y_{i,11} \end{pmatrix} \begin{Bmatrix} P' \\ U' \end{Bmatrix}$$

6 Transfer matrix for the parallel assembly with open-ended elements (*j*) and close-ended elements (*k*)

$$\mathbf{T_p} = \frac{-1}{\Sigma r_j y_{j,21}} \begin{bmatrix} \Sigma r_j y_{j,22} & -1 \\ \Sigma r_j y_{j,22} \left(\Sigma r_i y_{i,11} - \Sigma r_k \dfrac{y_{k,12} y_{k,21}}{y_{k,22}} \right) - \Sigma r_j y_{j,12} \Sigma r_j y_{j,21} & \Sigma r_k \dfrac{y_{k,12} y_{k,21}}{y_{k,22}} - \Sigma r_i y_{i,11} \end{bmatrix}$$

Admittance Sum Method



How does PTMM compare to ASM ?

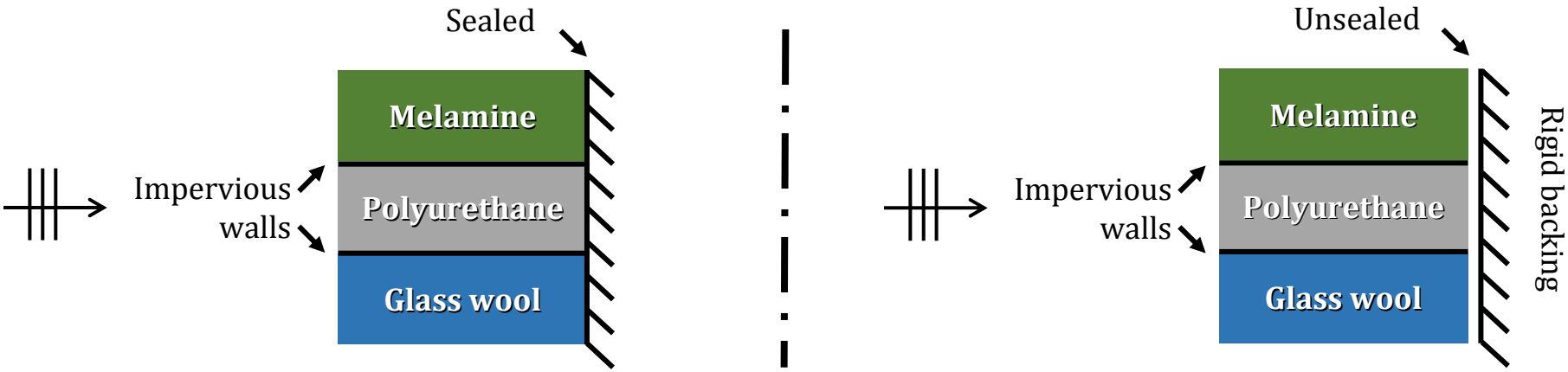
$$Y_G = \frac{1}{Z_s^G} = \sum_i r_i Y_i$$

C. Zwikker and C. Kosten, Sound Absorbing Materials (Elsevier, New York, 1949), p. 161.

Backing Condition	Close-ended	Anechoic	Air cavity of depth d
Admittance Y_i	$Y_i = \frac{T_{i,21}}{T_{i,11}}$	$Y_i = \frac{T_{i,21} + T_{i,22}/Z_0}{T_{i,11} + T_{i,12}/Z_0}$	$Y_i = \frac{T_{i,22} + jZ_0 \cot(k_0 d) T_{i,21}}{T_{i,12} + jZ_0 \cot(k_0 d) T_{i,11}}$

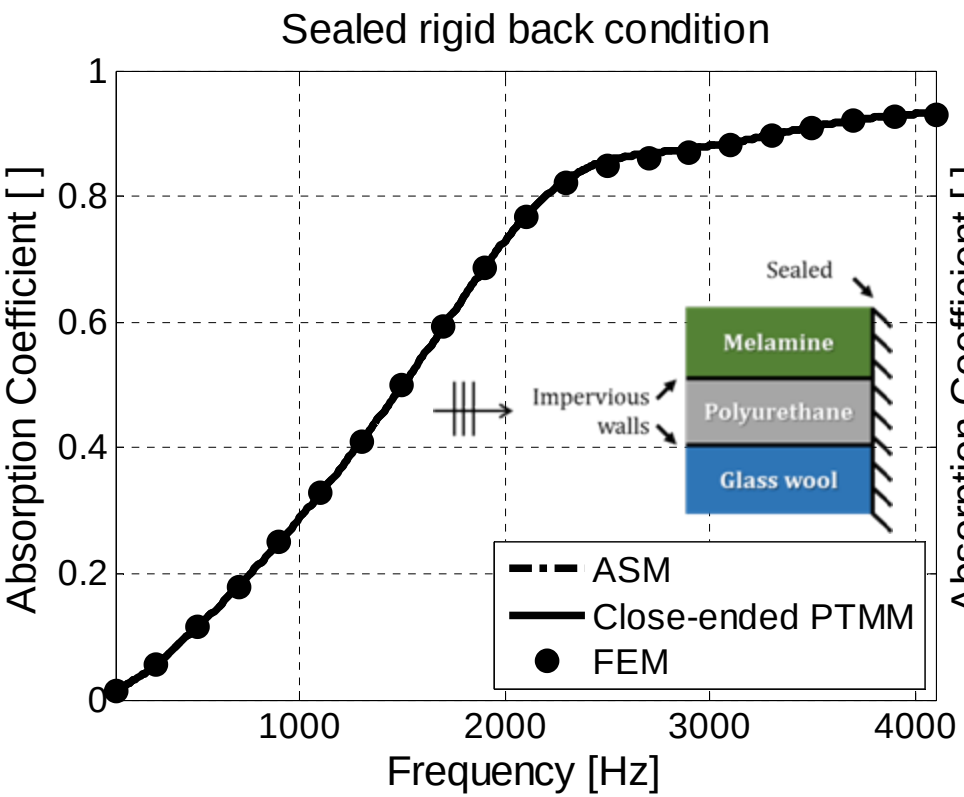
ASM

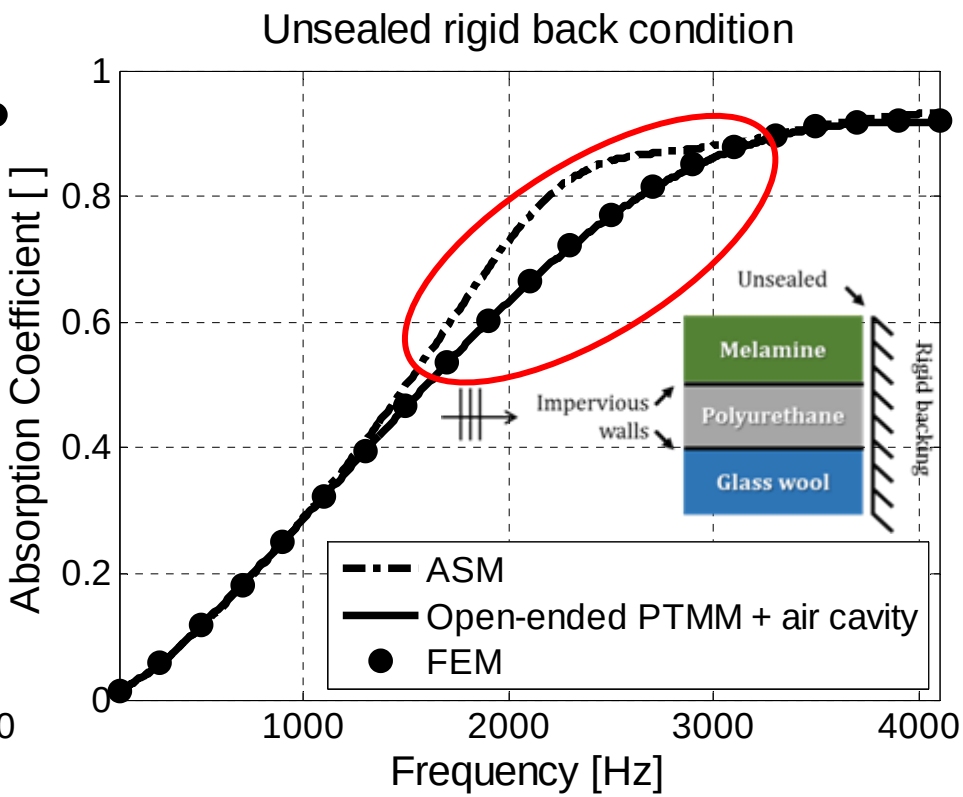
$$\alpha \Leftarrow Z_s \Leftarrow \begin{cases} \dfrac{1}{\dfrac{1/3}{Z_s^{Melamine}} + \dfrac{1/3}{Z_s^{Polyurethane}} + \dfrac{1/3}{Z_s^{Glasswool}}} & \text{for the sealed case} \\[2ex] \dfrac{1}{\dfrac{1/3}{Z_s^{Melamine+air}} + \dfrac{1/3}{Z_s^{Polyurethane+air}} + \dfrac{1/3}{Z_s^{Glasswool+air}}} & \text{for the unsealed case} \end{cases}$$



PTMM

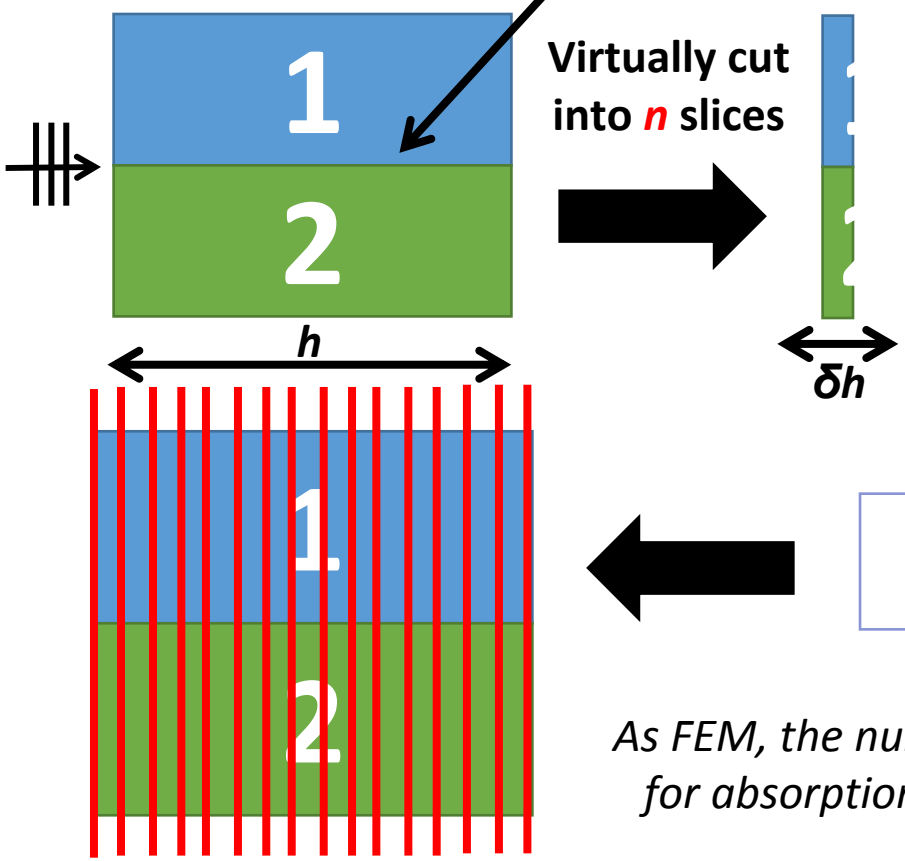
$$\alpha \Leftarrow Z_s \Leftarrow \begin{cases} TM_{PTMM}^{close-ended} & \text{for the sealed case} \\ TM_{PTMM}^{open-ended} TM_{air\ cavity} & \text{for the unsealed case} \end{cases}$$





a slight acoustical leak in the parallel construction can change significantly the acoustic response.

A means to avoid the no exchange assumption
No impervious interior wall is needed



Virtually cut into *n* slices

the slice *δh* is expressed by a parallel matrix *δT* by classical PTMM

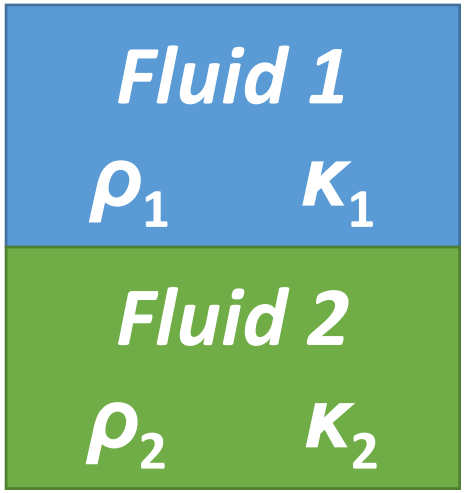
Back to the global patch :
$T = \delta T^n$

As FEM, the number of slices can be chosen when convergence for absorption coefficient and transmission loss is obtained.

Each red line corresponds to flow and pressure balances : the stack is homogenized.

Another method to homogenize …

Mixing Rules Method



$$\rho_{eq} = \frac{1}{\frac{r_1}{\rho_1} + \frac{r_2}{\rho_2}}$$

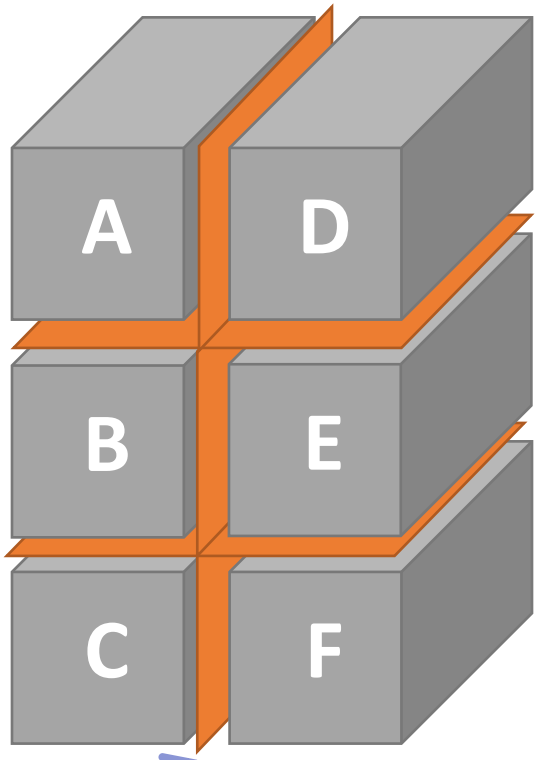
$$\kappa_{eq} = \frac{1}{\frac{r_1}{\kappa_1} + \frac{r_2}{\kappa_2}}$$

How does PTMM compare to MRM ?

generalized equation
with *i* materials

$$\rho_{eq} = \left(\sum_i \frac{r_i}{\rho_i} \right)^{-1}$$

$$\kappa_{eq} = \left(\sum_i \frac{r_i}{\kappa_i} \right)^{-1}$$



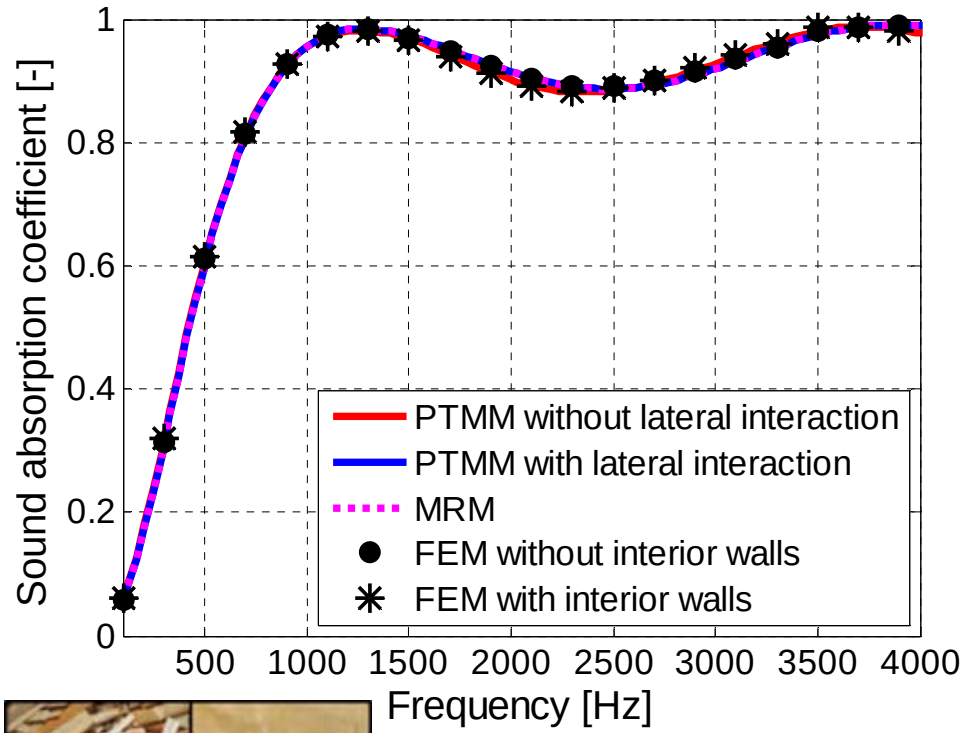
2 FEM solutions:
- With interior walls
- Without interior walls

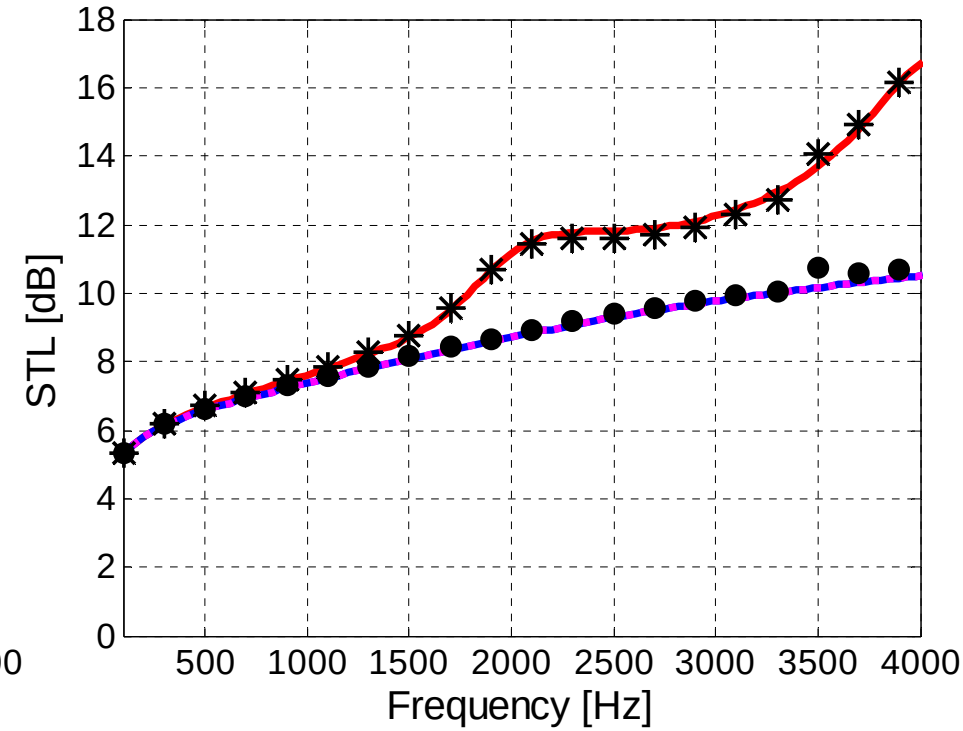
Compared with:
- PTMM
- MRM

With and without interior impervious walls

- PTMM without lateral interaction
- PTMM with lateral interaction

A: Switchgrass ; B: Glass wool ; C: Melamine foam ; D: Polyester fibre ; E: Polyurethane foam ; F: Rock wool



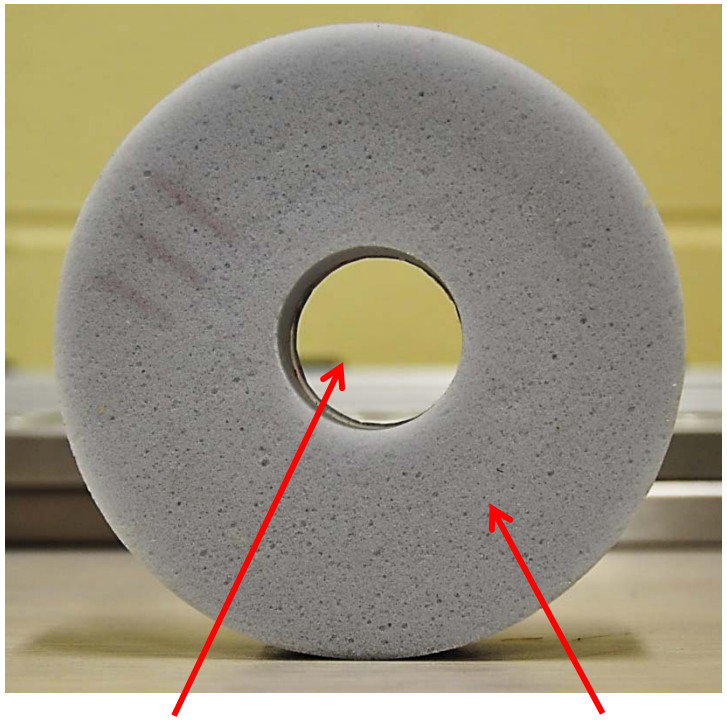


Be careful with the use of PTMM.

The question to ask:
Does it exist impervious walls between each element ?

A: Switchgrass ; B: Glass wool ; C: Melamine foam ; D: Polyester fibre ; E: Polyurethane foam ; F: Rock wool

A double porosity medium



Macro porosity (*p*) Micro porosity (*m*)

The acoustic model of this assembly depends on the contrast between the macroporous and the microporous phases.

For a low contrast ➔ no diffusion exists and MRM is used.

For a high contrast ➔ diffusion phenomenon appears and a modified version of MRM is used.

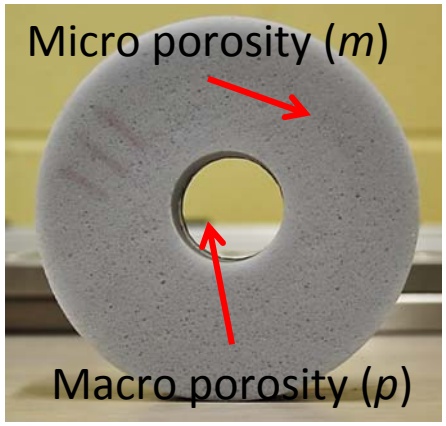
$$\kappa_{eq} = \left(\frac{r_p}{\kappa_p} + \frac{(1-r_p)F_d}{\kappa_m} \right)^{-1}$$

$$\rho_{eq} = \left(\frac{r_p}{\rho_p} + \frac{1-r_p}{\rho_m} \right)^{-1}$$

F_d is a function which depends on the form of the bilayer, the contact shape between the two phases and their porous properties.

X. Olny and C. Boutin, "*Acoustic wave propagation in double porosity media*," J. Acoust. Soc. Am. 114, July 2003, pp 73-89

How can diffusion phenomenon be taken into account with PTMM ?



$$\kappa_{eq} = \left(\frac{r_p}{\kappa_p} + \frac{(1-r_p)F_d}{\kappa_m} \right)^{-1}$$

$$\rho_{eq} = \left(\frac{r_p}{\rho_p} + \frac{1-r_p}{\rho_m} \right)^{-1}$$

$$\kappa_{m,eq} = \frac{\kappa_m}{F_d}$$

$$T_m = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) & j\sqrt{\rho_m\kappa_m}\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right)\Big/\sqrt{\rho_m\kappa_m} & \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) \end{bmatrix} \Rightarrow T_m^{diff} = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) & j\sqrt{\frac{\rho_m\kappa_m}{F_d}}\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right)\Big/\sqrt{\frac{\rho_m\kappa_m}{F_d}} & \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \end{bmatrix}$$

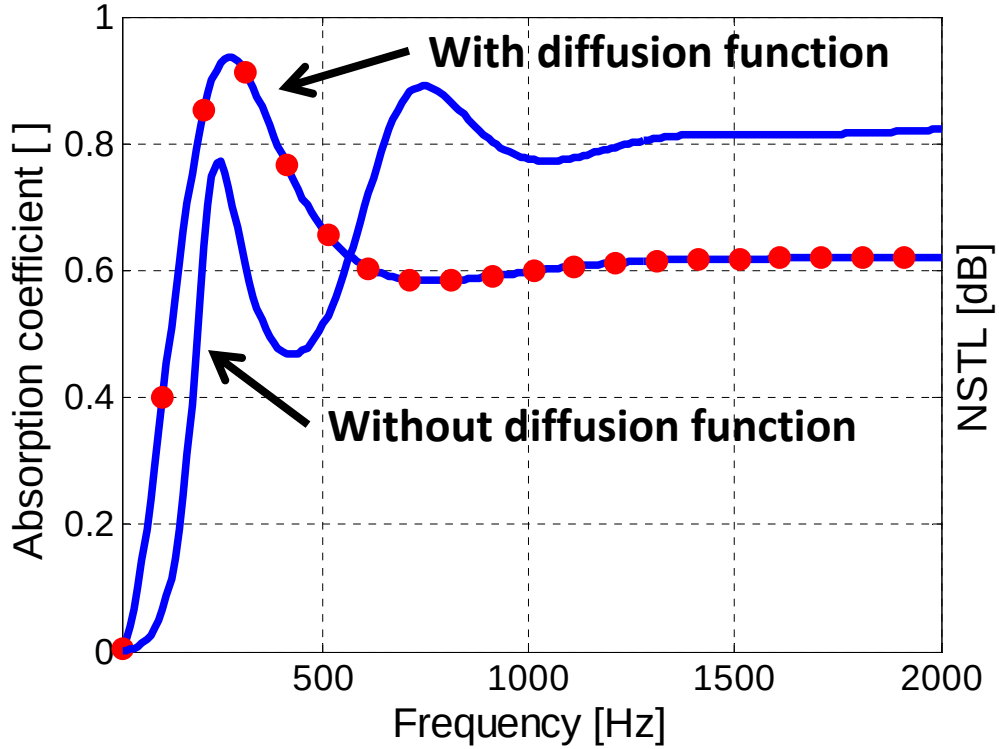
$$T_p = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) & j\sqrt{\rho_p\kappa_p}\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right)\Big/\sqrt{\rho_p\kappa_p} & \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \end{bmatrix} \Rightarrow$$

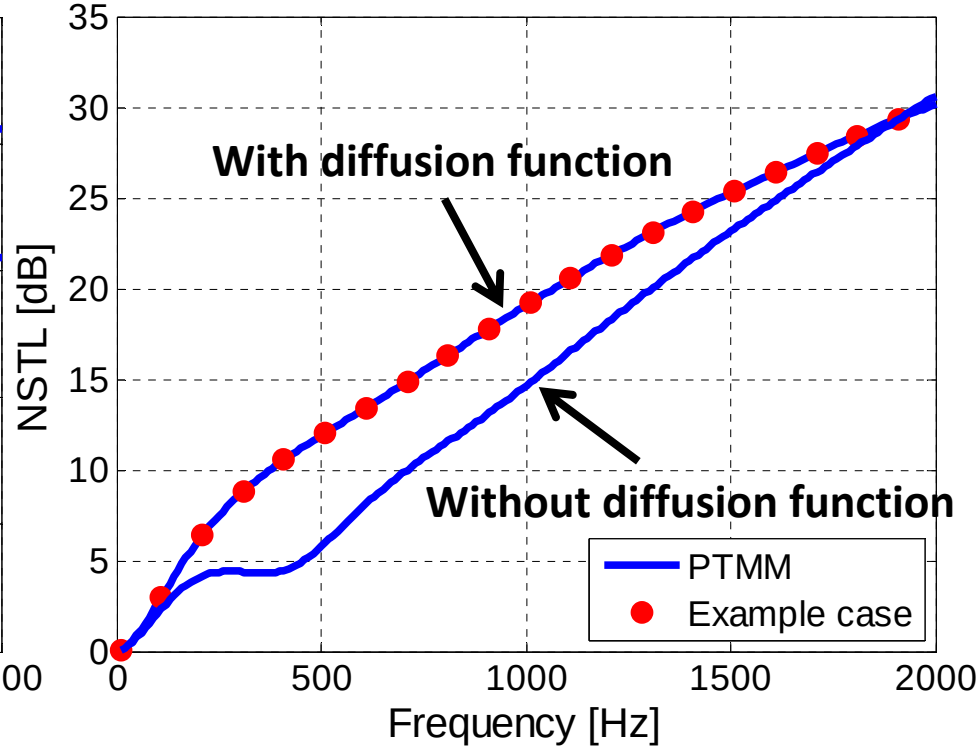
The macro porous transfer matrix is unchanged.

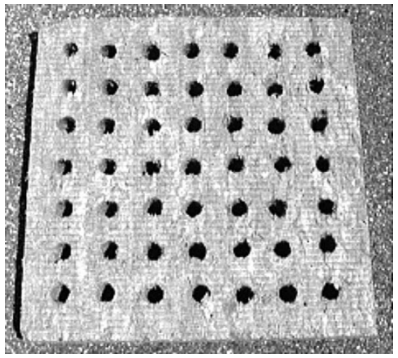
$$T_m^{diff} = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) & j\sqrt{\frac{\rho_m\kappa_m}{F_d}}\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right)\Big/\sqrt{\frac{\rho_m\kappa_m}{F_d}} & \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \end{bmatrix}$$

$$T_p = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) & j\sqrt{\rho_p\kappa_p}\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right)\Big/\sqrt{\rho_p\kappa_p} & \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \end{bmatrix}$$

Finally, both matrices are combined in parallel with PTMM.

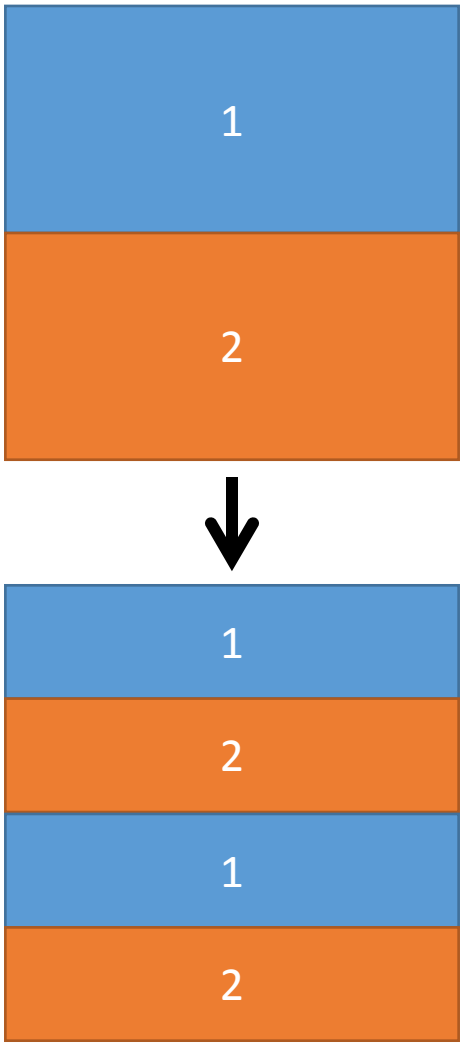


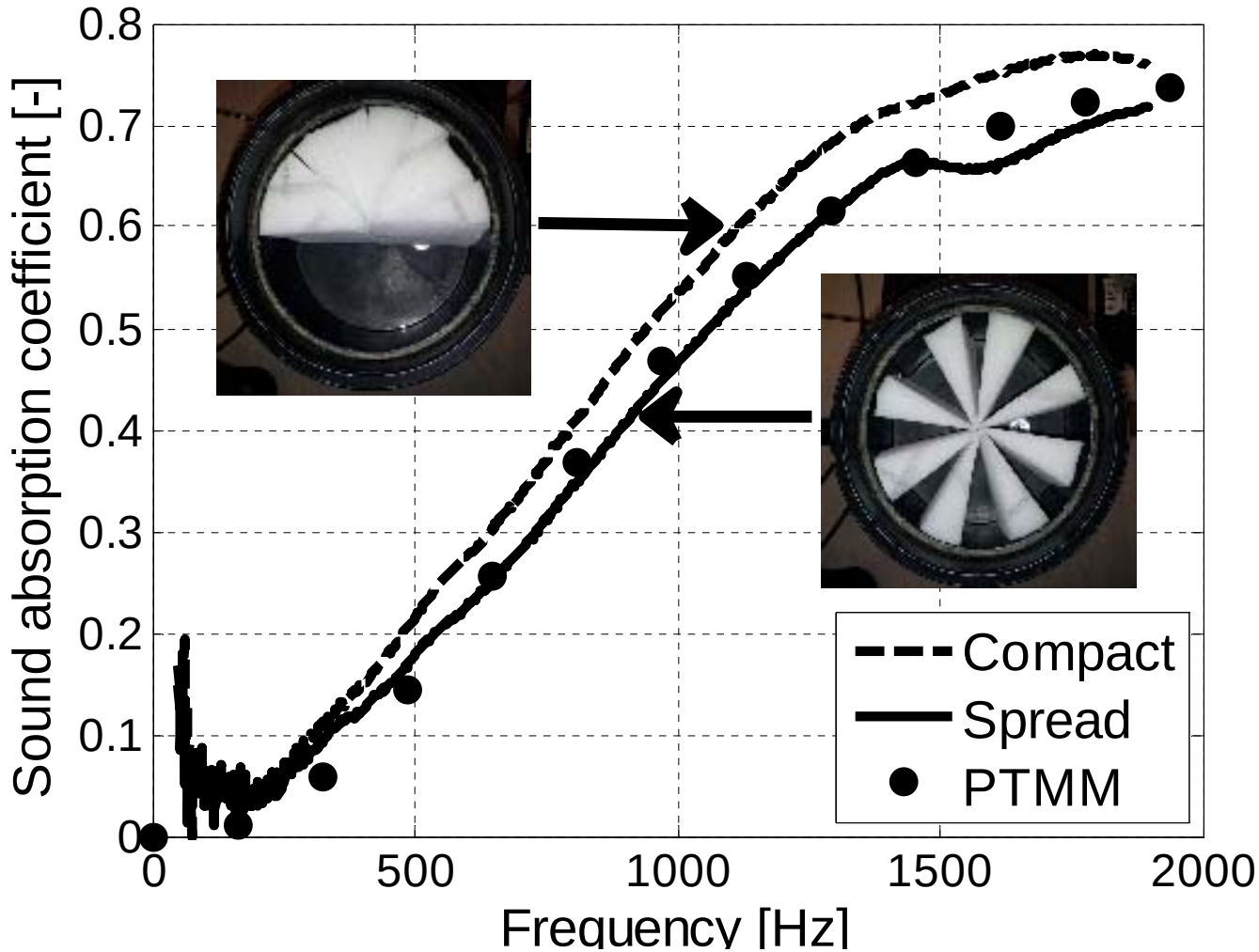


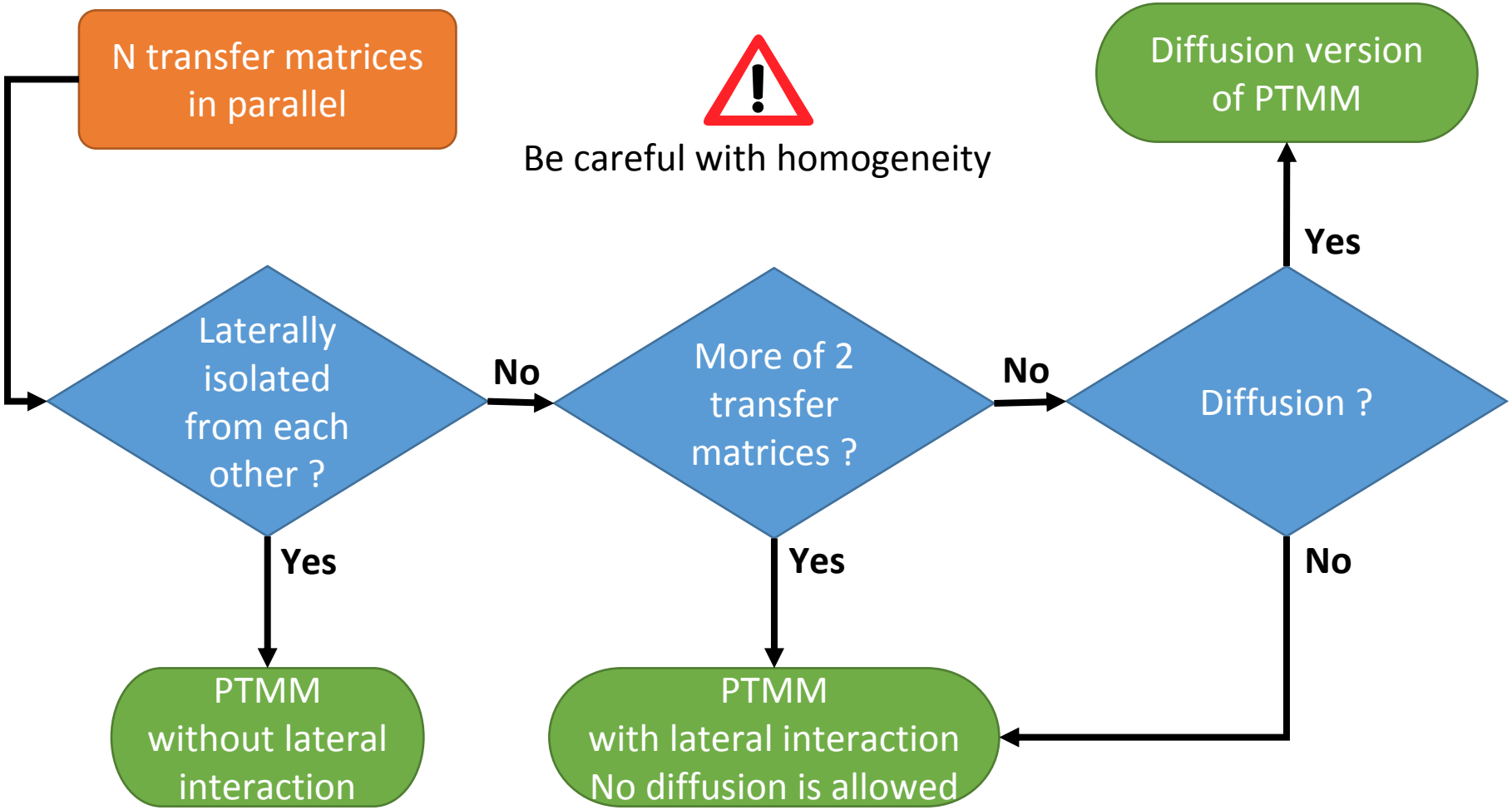


Example case extracts from a paper of F. Sgard, X. Olny, N. Atalla and F. Castel.
"On the use of perforations to improve the sound absorption of porous materials", Applied Acoustic (66) 2005, pp 625-651

Pressure uniformity






N transfer matrices in parallel
Be careful with homogeneity
Diffusion version of PTMM
Yes
Laterally isolated from each other ?
No
More of 2 transfer matrices ?
No
Diffusion ?
Yes
Yes
No
PTMM without lateral interaction
PTMM with lateral interaction No diffusion is allowed

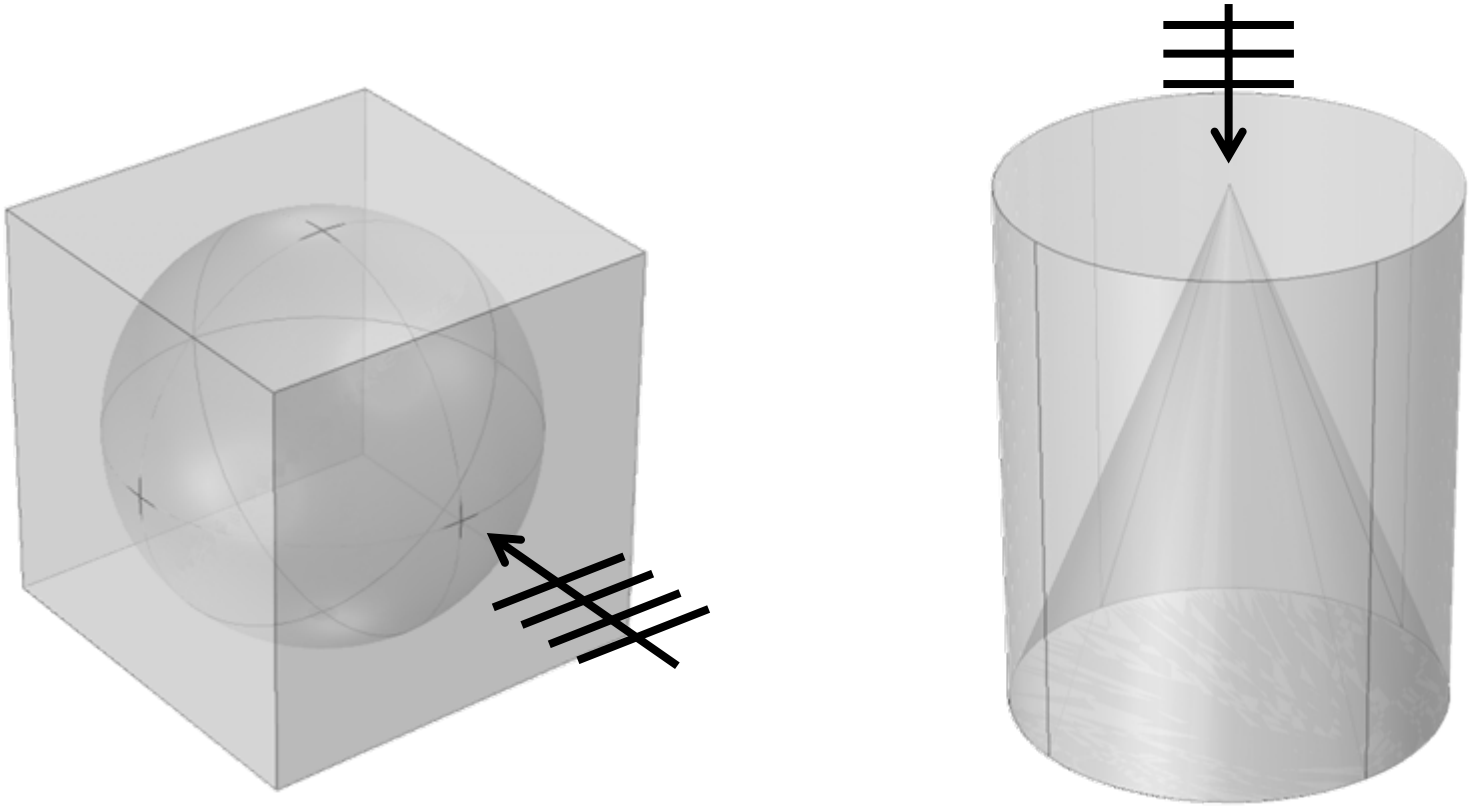
- Clarification on the use of PTMM has been given.
- The no exchange assumption can be avoid by using the discretized version of PTMM.
- Diffusion phenomenon can be taken into account by a modified transfer matrix and the discretized version of PTMM.
- The question of homogeneity has been raised:
  - To assure pressure uniformity as much as possible, the constituents have to be spread into the assembly.
- Theses discrepancies between experimental and theory could come from the uniform pressure assumption.
  - A new formulation without this assumption has to be written (Future Work).

Thank you!

Questions ?

If MRM and PTMM give the same results,
what are the advantages to use PTMM rather than MRM ?

Extra



PTMM is used when the distribution of each material is not constant along the thickness of the construction. As PTMM is based on a discretization, only a distribution profile of each material is needed.

pipe that branches into two pipes

Extra

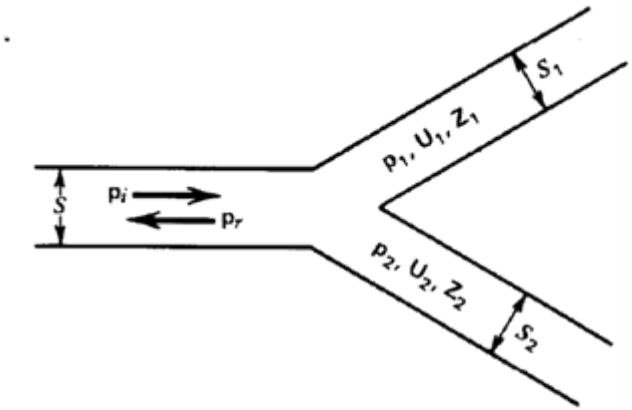


Figure 10.10.2 Conditions in the vicinity of a branch. The branches have cross-sectional areas S_1 and S_2 and input acoustic impedances $\mathbf{z}_1$ and $\mathbf{z}_2$.

As a second example, consider a pipe that branches into two pipes each with arbitrary input impedance, as indicated in Fig. 10.10.2. If the junction is at $x = 0$, the pressures in the three pipes very close to $x = 0$ are

$$\mathbf{p}_i = \mathbf{P}_i e^{j\omega t} \qquad \mathbf{p}_r = \mathbf{P}_r e^{j\omega t}$$
$$\mathbf{p}_1 = \mathbf{Z}_1 \mathbf{U}_1 e^{j\omega t} \qquad \mathbf{p}_2 = \mathbf{Z}_2 \mathbf{U}_2 e^{j\omega t} \tag{10.10.9}$$

where $\mathbf{Z}_1$, $\mathbf{Z}_2$, and $\mathbf{U}_1$, $\mathbf{U}_2$ are the input impedances and complex volume velocity amplitudes in the two branches. In the long wavelength approximation, continuity of pressure at the junction requires

$$\boxed{\mathbf{p}_i + \mathbf{p}_r = \mathbf{p}_1 = \mathbf{p}_2} \tag{10.10.10}$$

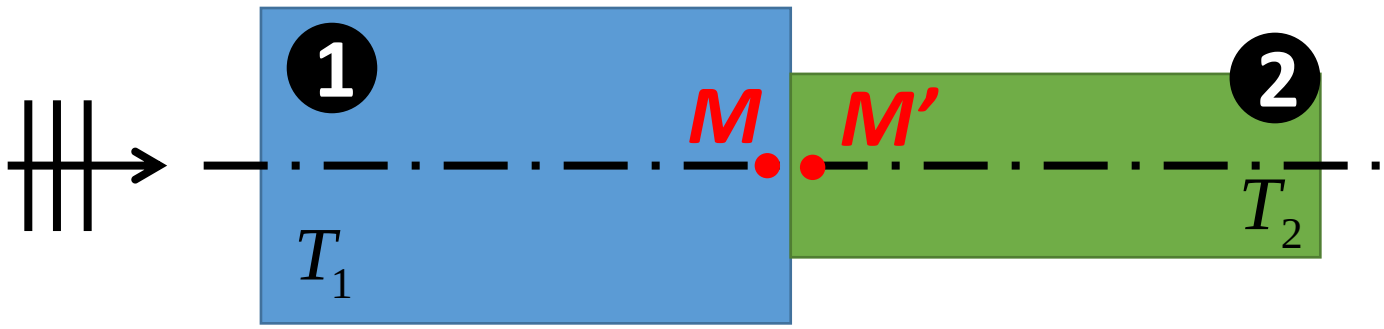
and continuity of volume velocity requires

$$\boxed{\mathbf{U}_i + \mathbf{U}_r = \mathbf{U}_1 + \mathbf{U}_2} \tag{10.10.11}$$

Lawrence E. Kinsler, Austin R. Frey, Alan B. Coppens, James V. Sanders
Fundamentals of Acoustics, 4th Edition, Wiley
December 1999, p. 290

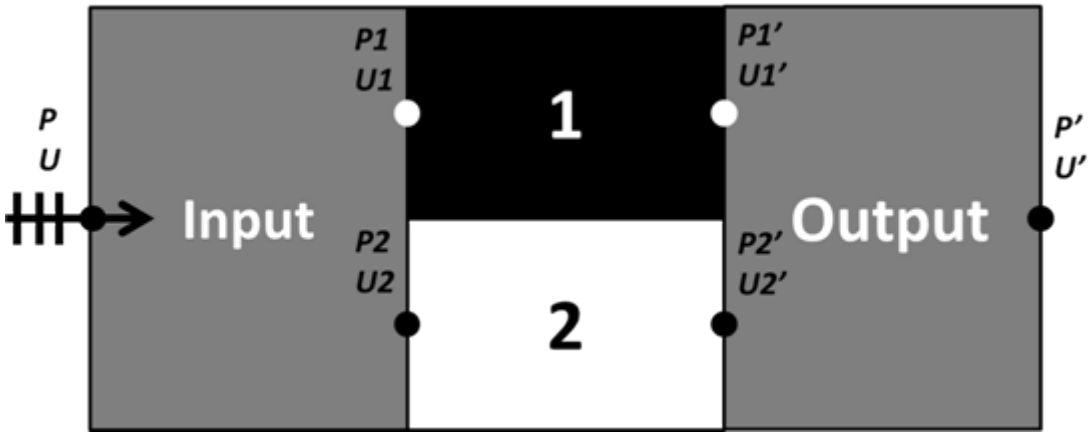
Cross Section Reduction – Expansion Chamber

Extra



$$\begin{pmatrix} P(M) \\ U(M) \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix} \begin{pmatrix} P(M') \\ U(M') \end{pmatrix}$$

Future Work



$$\begin{Bmatrix} P \\ U \end{Bmatrix} = T_{input} T_{assembly} T_{output} \begin{Bmatrix} P' \\ U' \end{Bmatrix}$$

No exchange assumption

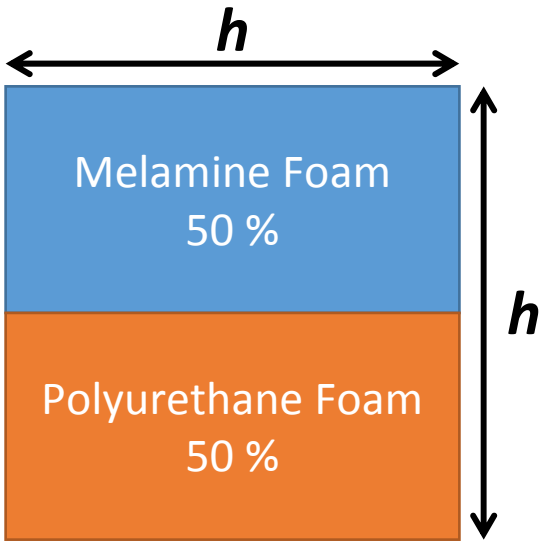
Transfer matrix of the first element

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = T_{input} \begin{Bmatrix} P_1 \\ U_1 \\ P_2 \\ U_2 \end{Bmatrix}$$

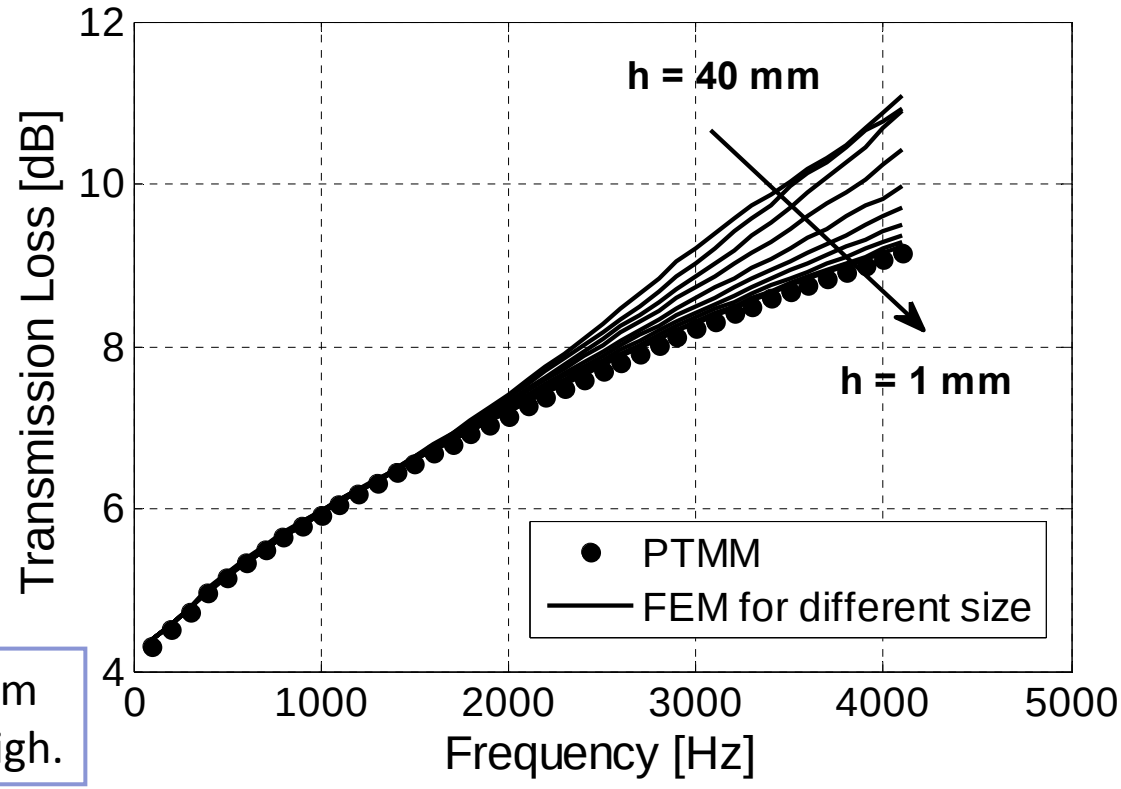
$$\begin{Bmatrix} P'_1 \\ U'_1 \\ P'_2 \\ U'_2 \end{Bmatrix} = T_{output} \begin{Bmatrix} P' \\ U' \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ U_1 \\ P_2 \\ U_2 \end{Bmatrix} = T_{assembly} \begin{Bmatrix} P'_1 \\ U'_1 \\ P'_2 \\ U'_2 \end{Bmatrix} \text{ with } T_{assembly} = \begin{bmatrix} T_{11,1} & T_{12,1} & 0 & 0 \\ T_{21,1} & T_{22,1} & 0 & 0 \\ 0 & 0 & T_{11,2} & T_{12,2} \\ 0 & 0 & T_{21,2} & T_{22,2} \end{bmatrix}$$

Effect of the lateral size *h*



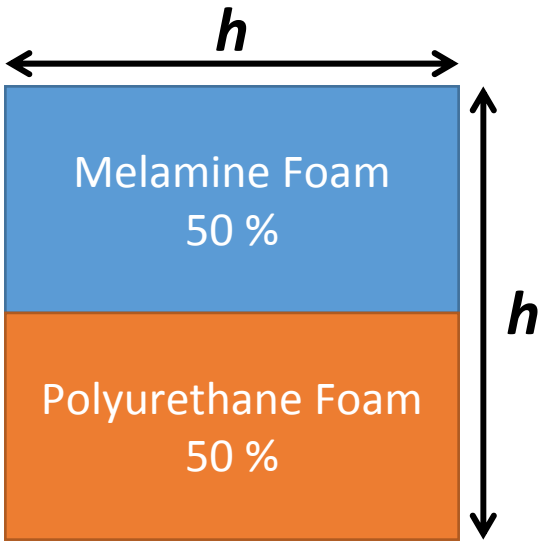
Thickness: 50 mm
FEM analysis



The contrast between melamine foam and polyurethane foam is relatively high.

	Porosity	Air resistivity [N.s.m^{-4}]	Tortuosity	Viscous length [µm]	Thermal length [µm]
Melamine foam	0.999	9724	1	110	122
Polyurethane					

Effect of the lateral size *h*



Thickness: 50 mm
FEM analysis

$$\varepsilon = \frac{\left| TL_{PTMM} - TL_{FEM} \right|}{TL_{FEM}} \cdot 100\%$$