



A Wave Based Method for the prediction and optimisation of absorption of twodimensional rigid-frame poroelastic periodic structures

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Introduction

Wave Based Method

WB UC model

Validation

Conclusion

- 1 Introduction
- 2 Wave Based Method
- 3 Wave Based Unit Cell model
- 4 Validation
- 5 Conclusion





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Foam-like materials:

- Absorption: low \leftrightarrow higher frequencies
- Quarter wavelength rule of thumb
- Bulky solutions

More efficient use of materials needed...

- Multilayering¹
- Heterogeneities:
 - Double porosity materials²
 - Periodic arrangements of surface irregularities³
 - Periodic arrangements of volume heterogeneities⁴

- [1] O. Tanneau et al. *J. Acoust. Soc Am.* **120** (2006).
 [2] C. Boutin et al. *Int. J. Solids Struct.* **5** (1998).
 [3] J.P. Groby et al. *J. Acoust. Soc Am.* **133** (2013).
 [4] J.P. Groby et al. *J. Acoust. Soc Am.* **130** (2011).



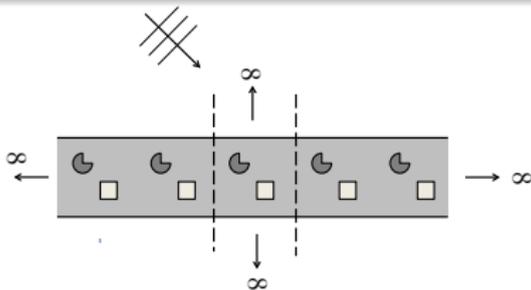


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Multipole Method

- Very efficient method
- Circular inclusions
- Bloch-Floquet conditions embedded

Finite Element method

- Any inclusion shape can be handled
- Fine discretisation needed, polynomial shape functions

Wave Based Method

- Basis functions are exact solutions of governing equations
- Meshless procedure
- Enhanced numerical convergence properties w.r.t. FEM
- Allows for optimisation studies

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General procedure
Unbounded problems
Multi-level approach

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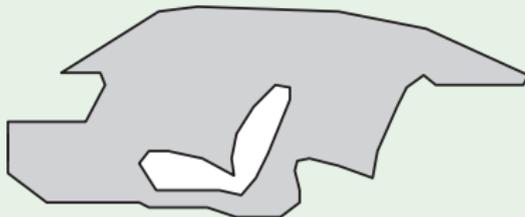
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2D acoustic problem

Problem domain



Helmholtz equation:

$$\nabla^2 p(\mathbf{r}) + k^2 p(\mathbf{r}) = 0$$

Boundary Conditions:

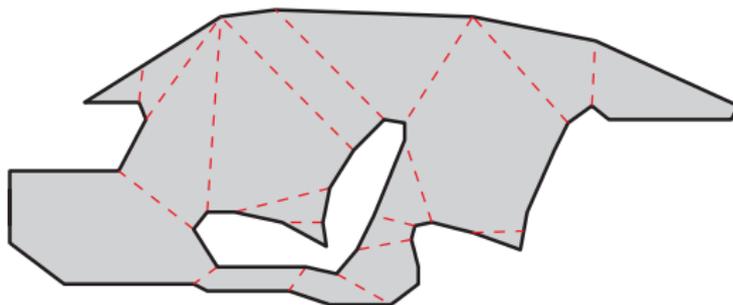
$$\mathbf{r} \in \Gamma_v : \frac{j}{\rho_0 \omega} \frac{\partial p(\mathbf{r})}{\partial n} = \bar{v}_n(\mathbf{r})$$

$$\mathbf{r} \in \Gamma_p : p(\mathbf{r}) = \bar{p}(\mathbf{r})$$

$$\mathbf{r} \in \Gamma_Z : \frac{j}{\rho_0 \omega} \frac{\partial p(\mathbf{r})}{\partial n} = \frac{p(\mathbf{r})}{Z_n(\mathbf{r})}$$

Four step procedure

1 Partitioning of the problem domain



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Four step procedure

- 1 Partitioning of the problem domain
- 2 Field variable expansion in each subdomain:

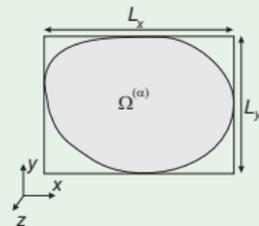
$$p^{(\alpha)}(\mathbf{r}) \simeq \sum_w p_w^{(\alpha)} \Phi_w^{(\alpha)}(\mathbf{r})$$

with:

$$\Phi_w^{(\alpha)}(\mathbf{r}(x, y)) = \begin{cases} \Phi_{w_r}^{(\alpha)}(\mathbf{r}(x, y)) = \cos(k_{xw_r}^{(\alpha)} x) e^{-jk_{yw_r}^{(\alpha)} y} \\ \Phi_{w_s}^{(\alpha)}(\mathbf{r}(x, y)) = e^{-jk_{xw_s}^{(\alpha)} x} \cos(k_{yw_s}^{(\alpha)} y) \end{cases}$$

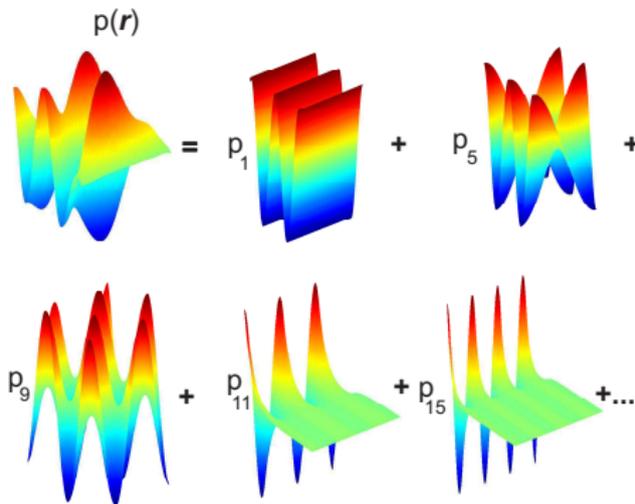
$$\begin{cases} (k_{xw_r}^{(\alpha)}, k_{yw_r}^{(\alpha)}) = \left(\frac{w_1^{(\alpha)} \pi}{L_x}, \pm \sqrt{k^2 - (k_{xw_r}^{(\alpha)})^2} \right) \\ (k_{xw_s}^{(\alpha)}, k_{yw_s}^{(\alpha)}) = \left(\pm \sqrt{k^2 - (k_{yw_s}^{(\alpha)})^2}, \frac{w_2^{(\alpha)} \pi}{L_y} \right) \end{cases}$$

$w_1, w_2 = 0, 1, 2, \dots$



Four step procedure

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Four step procedure

- 1 Partitioning of the problem domain
- 2 Field variable expansion in each subdomain:
- 3 Construction of the system of equations

$$\int_{v(\alpha)} \tilde{p}^{(\alpha)}(\mathbf{r}) R_v^{(\alpha)}(\mathbf{r}) + \int_{Z(\alpha)} \tilde{p}^{(\alpha)}(\mathbf{r}) R_Z^{(\alpha)}(\mathbf{r}) - \int_{p(\alpha)} \tilde{p}^{(\alpha)}(\mathbf{r}) \alpha R_p^{(\alpha)}(\mathbf{r}) d + \sum_{\beta=1, \beta \neq \alpha}^{N_\Omega} \int_{I(\alpha, \beta)} \tilde{p}^{(\alpha)}(\mathbf{r}) R_I^{(\alpha, \beta)}(\mathbf{r}) = 0.$$

Results in:

$$[A_{aa}] \{ \mathbf{p}_w \} = \{ \mathbf{b} \}.$$

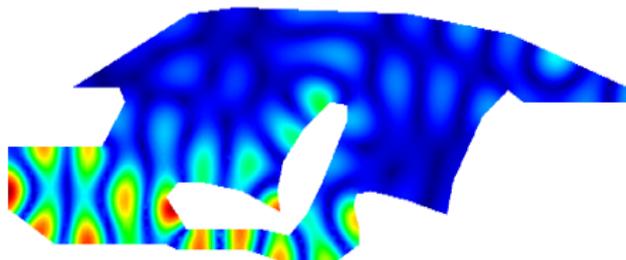


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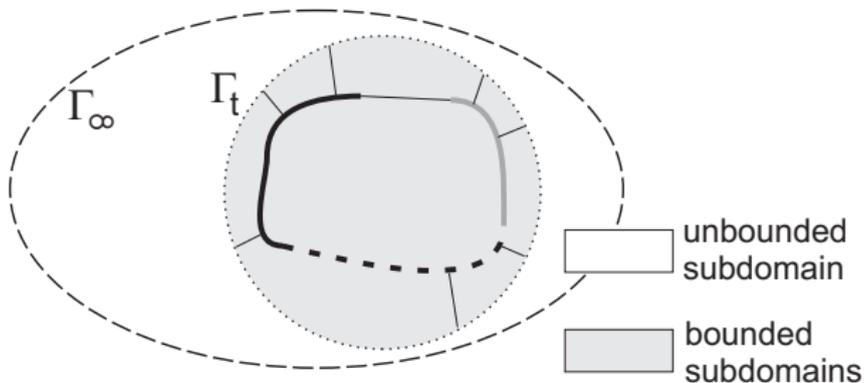
$$p^{(\alpha)}(\mathbf{r}) \simeq \sum_w p_w^{(\alpha)} \Phi_w^{(\alpha)}(\mathbf{r})$$

- 3 Construction of the system of equations
- 4 Postprocessing



The WBM for unbounded problems

Partitioning



- Truncation t : separate in bounded + unbounded region
- Bounded region: split in convex subdomains
→ convergence
- Unbounded region: modelled as a single subdomain
- Continuity conditions between subdomains
- Unbounded wave functions fulfill Helmholtz and Sommerfeld



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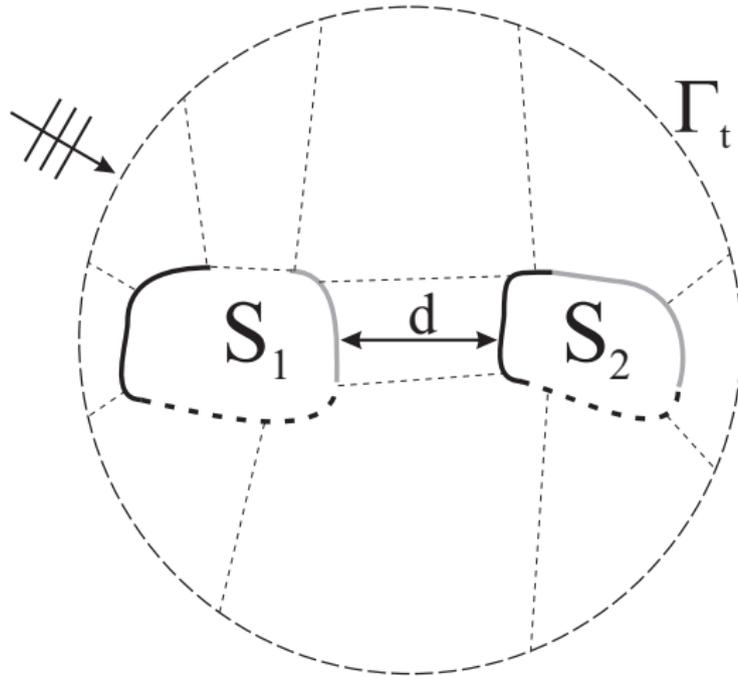
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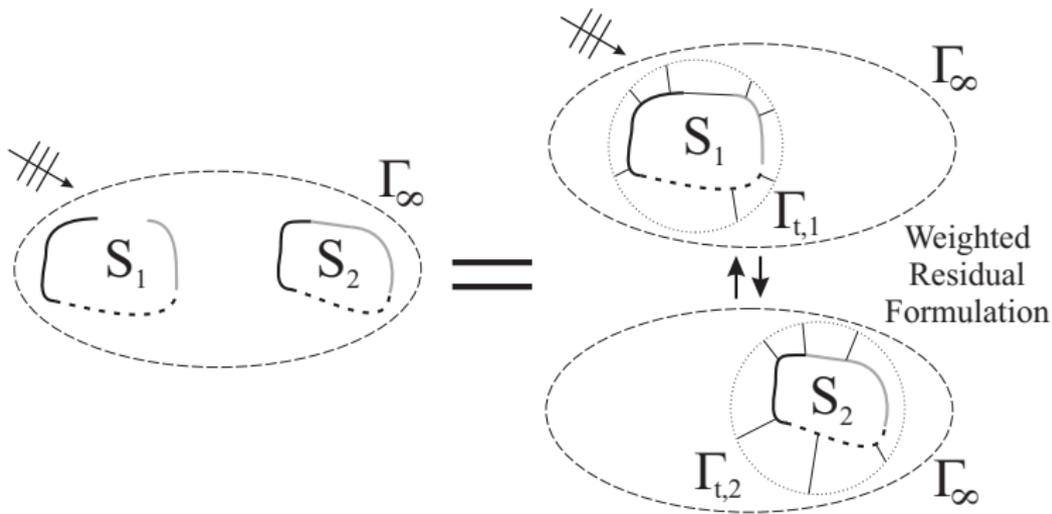
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Multi-level approach

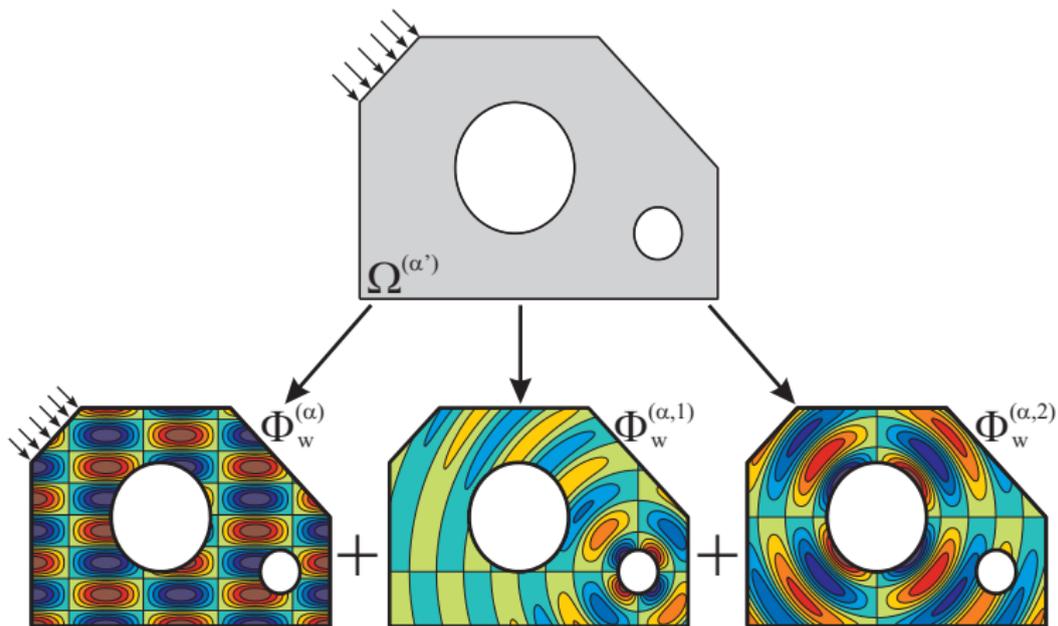
General concept





Multi-level approach

Bounded problems: general concept





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Wave Based Method

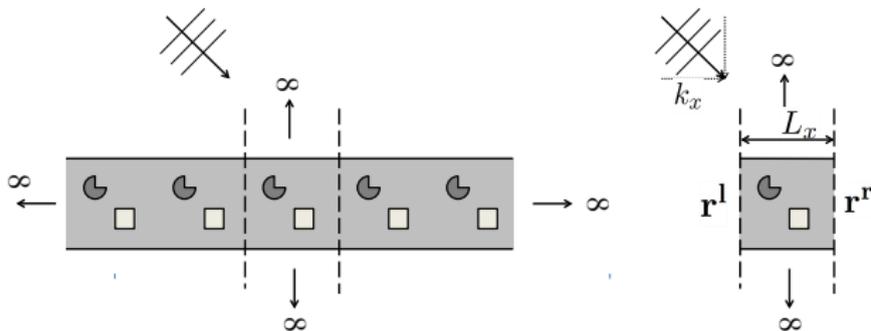
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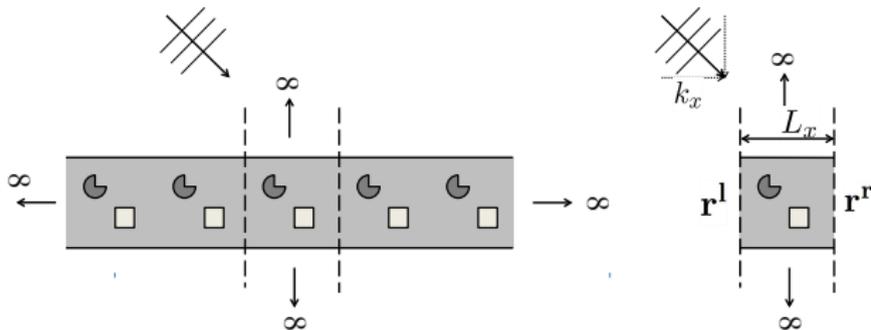


Periodicity in the x -direction:

- Due to plane wave nature
- Due to geometrical periodicity
- Shift N cells to the right of the considered cell:
 $\rightarrow p(x + NL_x, y) = p(x, y)e^{-jk_x NL_x}$

Needed adaptations:

- Embed BF conditions in the ML approach
- Define UB BF wave functions



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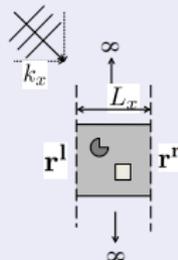
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Imposing BF conditions within the ML framework

Minimise following residuals:

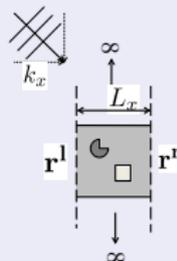
- $R_{BF_l} = p(\mathbf{r}^l) - p(\mathbf{r}^r) e^{jk_x L_x} = 0$
- $R_{BF_r} = v_n(\mathbf{r}^r) + v_n(\mathbf{r}^l) e^{-jk_x L_x} = 0$



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UB BF wave function definition

WFs should fulfill:

- Helmholtz
- Sommerfeld
- Periodic condition

$$\Phi_n^{(BF)}(x, y) = e^{-j(k_{BF,xn}x + k_{BF,yn}y)}$$

- $k_{BF,xn} = k_x + 2p\pi/L_x$
- $k_{BF,yn} = \pm \sqrt{k^2 - k_{BF,xn}^2}$
- $p \in \mathbb{Z}$
- $n = 1 : 2p_{max} + 1$
- sign of root \rightarrow outgoing waves

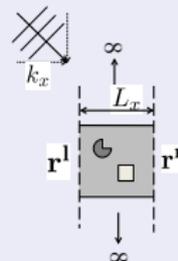




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Reflection, transmission and absorption coefficient

- Conservation of energy: $1 = \mathcal{A} + \mathcal{R} + \mathcal{T}$
- For \mathcal{R} and \mathcal{T} :
 - Only waves in y -direction carry energy
 - Amplitude defined by WF contribution factor
- \mathcal{A} calculated via conservation of energy
- Completely similar to work by Groby^[5]

[5] J.P. Groby et al. *J. Acoust. Soc Am.* **126** (2009).

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C-shaped inclusion

Optimisation study

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First validation steps:

- without inclusion → TMM
- with inclusions → Repetition of unit cells
- with inclusions → 1000 UCs in FEM

Validation cases available in literature:

- Absorption case: J.P. Groby et al. *J. Acoust. Soc Am.* **126** (2009).
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First optimisation of the geometry of the inclusion

- 'Openness' of a C-shaped inclusion



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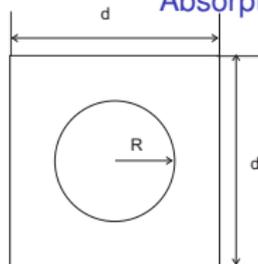
First optimisation of the geometry of the inclusion

- 'Openness' of a C-shaped inclusion

- Fireflex material
- Normal incidence
- $R=7.5\text{mm}$
- $d=2\text{cm}$

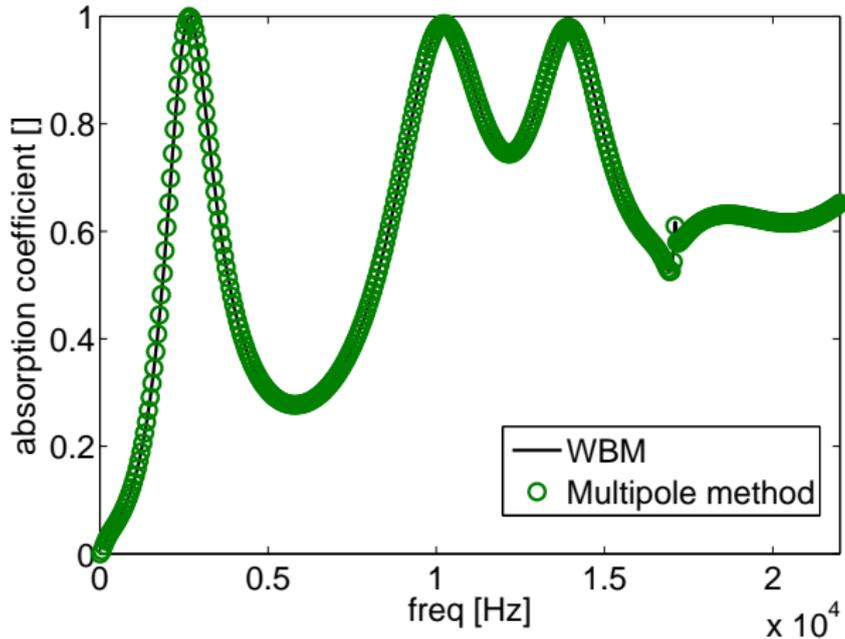
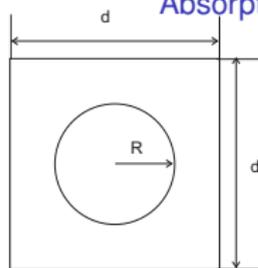
Validation

Absorption case





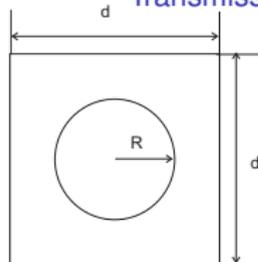
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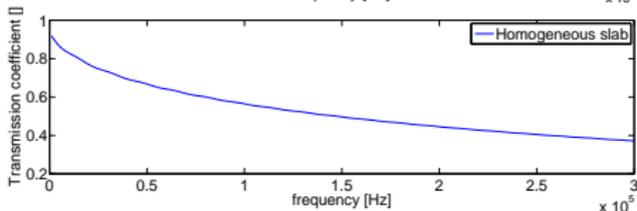
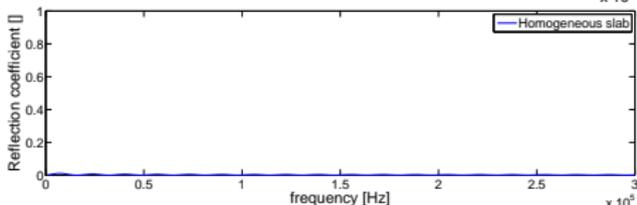
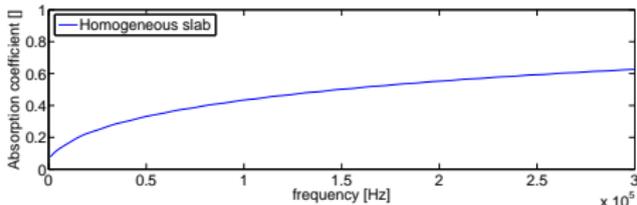
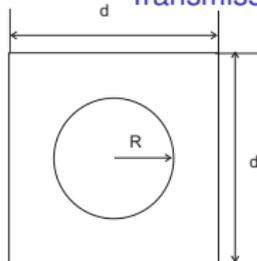
- Polyurethane foam
- Normal incidence
- $R=2.5\text{mm}$
- $d=1\text{cm}$

Validation

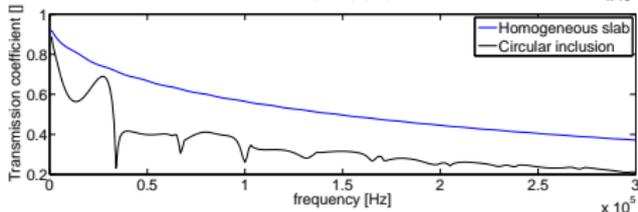
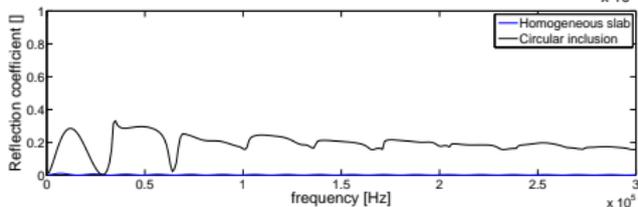
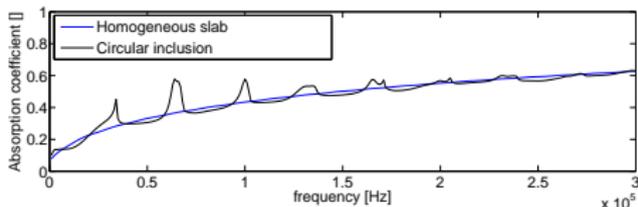
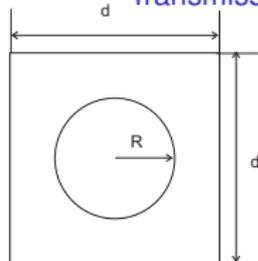
Transmission case



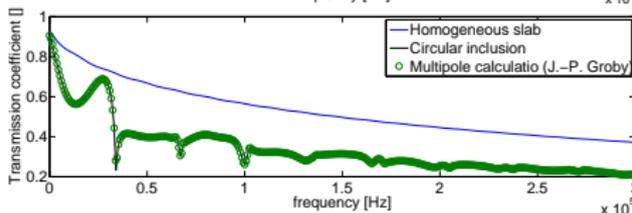
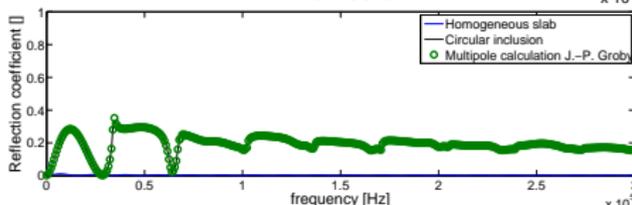
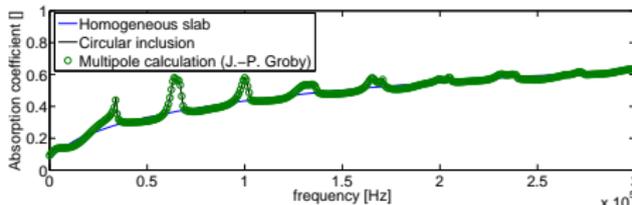
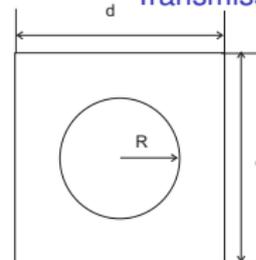
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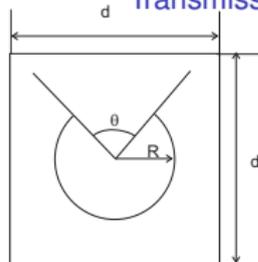


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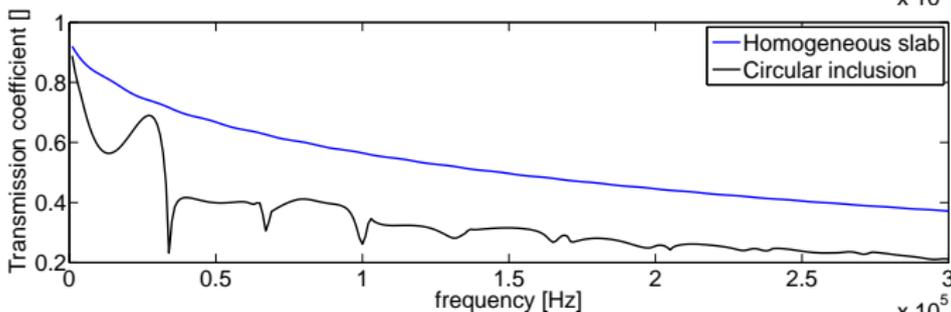
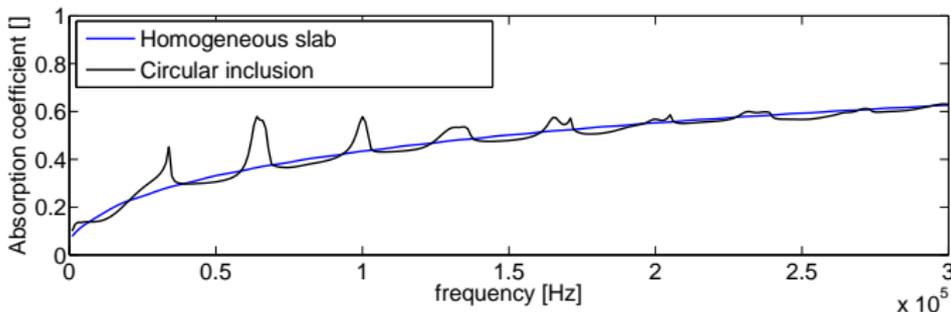
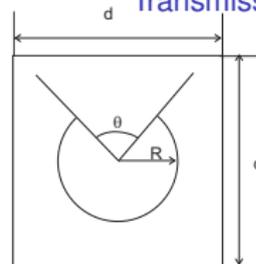
C-shaped inclusion:

- Same dimensions/material
- $\theta=45^\circ$



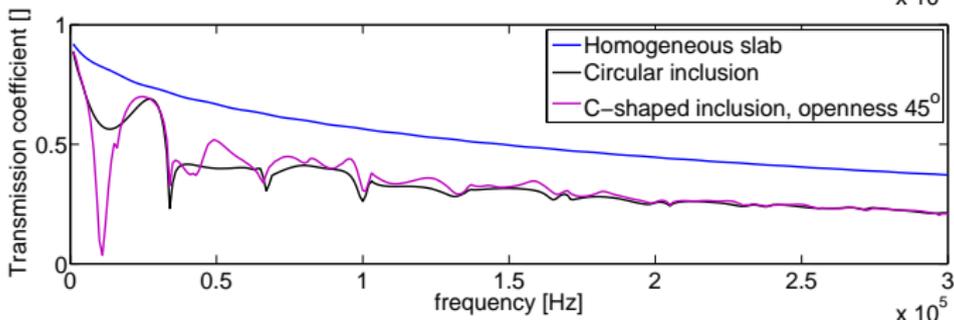
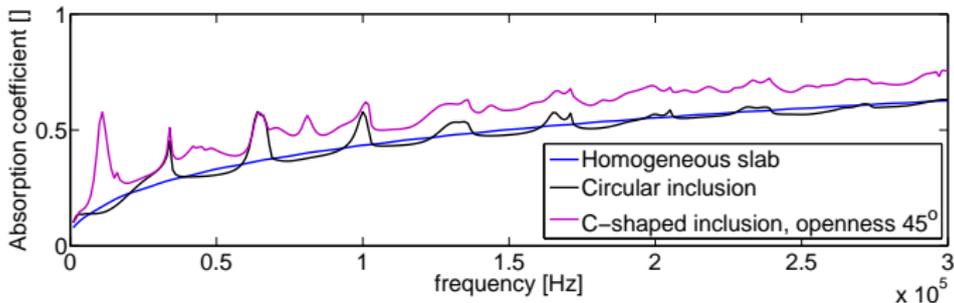
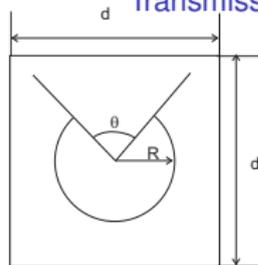
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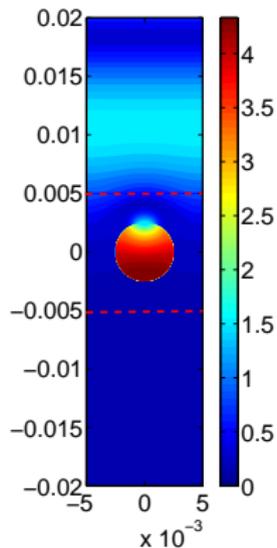
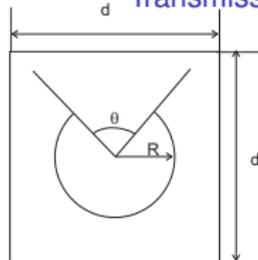


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WBM - advantages:

- Computationally efficient
- Subdomaining easy to parametrise
- Parts of the system matrices can be re-used

Optimisation:

- Minimise TC only using θ
- Target frequency 25kHz
- Genetic Algorithm
- Simulated Annealing

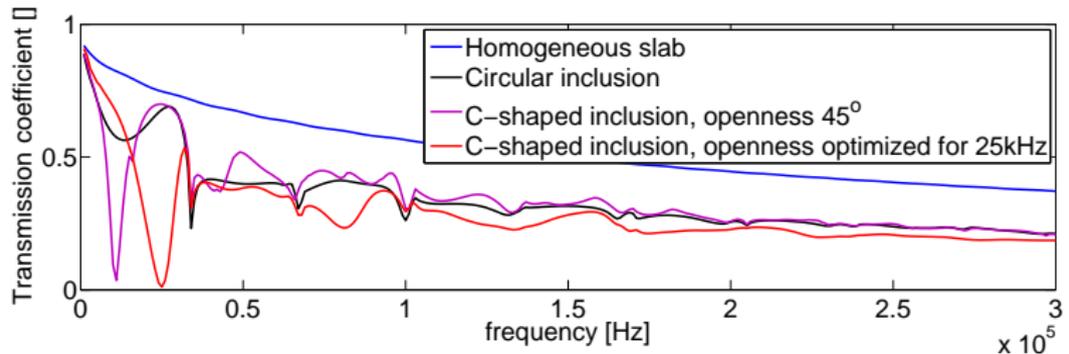


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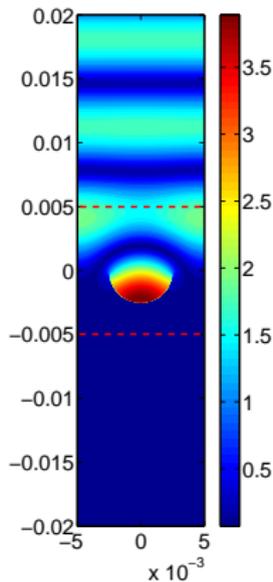


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Conclusions

Wave Based Method:

- + globally defined exact solutions = wave functions
- + low, mid (and high) frequency applications
- moderate geometrical complexity
- small, full, complex, and frequency dependent matrices

UC models with inclusions:

- BF conditions accounted for as boundary conditions
- Unbounded acoustic UC wave functions
- Allows for efficient optimisation studies

Future work:

- Orientation of C
- Combined optimisation
- Extend approach to full Biot

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Contact information:

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