

# Performance of a condensation procedure based on the projection on propagative waves

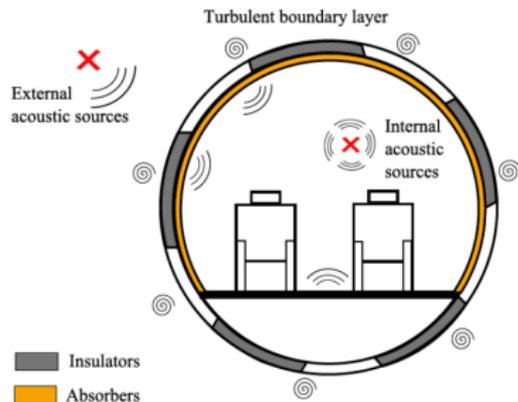
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# Dissipative interfaces in vibroacoustics



## 1 Context and problematic

Wave based condensation of the model

Performances and enhancements

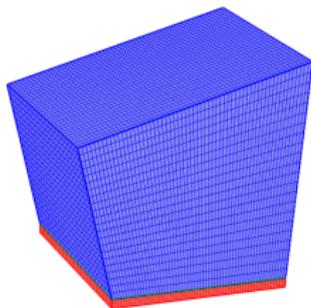
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## Observations

- ▶ Small wavelengths in the dissipative material
- ▶ Large energy dissipation
- Large numerical cost

⇒ How to solve for the problem with the lowest numerical cost as possible?

## Case of an absorption problem



Form of the system of equations:

$$\begin{bmatrix} \mathbf{D}_{AA}(\omega) & \mathbf{D}_{AP} \\ \mathbf{D}_{AP}^T & \mathbf{D}_{PP}(\omega) \end{bmatrix} \begin{pmatrix} \mathbf{p}_A(\omega) \\ \mathbf{q}_P(\omega) \end{pmatrix} = \begin{pmatrix} \mathbf{F}_A(\omega) \\ \mathbf{0} \end{pmatrix}$$

Drawbacks:

- ▶ Number of unknowns in the poroelastic domain
- ▶ Frequency dependence

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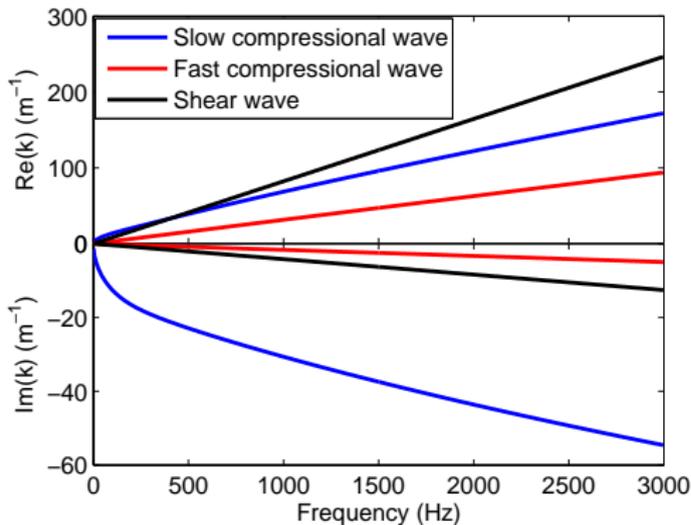
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## Biot's waves

- ▶ 3 waves : 2 compressional waves + 1 shear wave



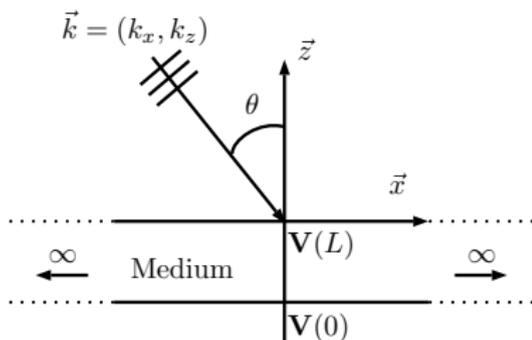
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- ▶ Plane wave acoustic excitation at incidence  $(k_x, k_y)$  on a flat, infinite structure



- ▶ Writing of the state vector as a function of the amplitudes of Biot's waves
- ▶ Computation of the transfer matrices in each medium
- ▶ Computation of the dynamic response of the material: e.g.  $Z_s, \alpha, TL$

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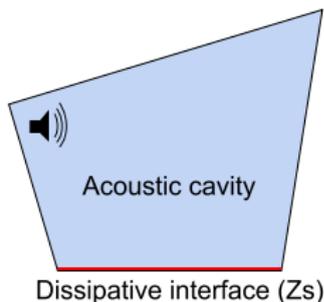
# Application to compute the forced response of the system

Representation of the interface by its impedance parameter:

$$Z_s = \frac{p_A}{\mathbf{v}_A \cdot \mathbf{n}_A}$$

Resulting system:

$$\left( \mathbf{D}_{AA}(\omega) - \underbrace{\frac{1}{i\omega Z_s(\omega)} \mathbf{C}}_{+\mathbf{D}_{\text{coupl}}(\omega)} \right) \mathbf{p}_A(\omega) = \mathbf{F}_A(\omega)$$



⇒ Elimination of all of poroelastic unknowns

Strong assumptions are used to approach the value of  $Z_s$ :  
arbitrary incidence angle, flat infinite structure.

⇒ Possibly large errors

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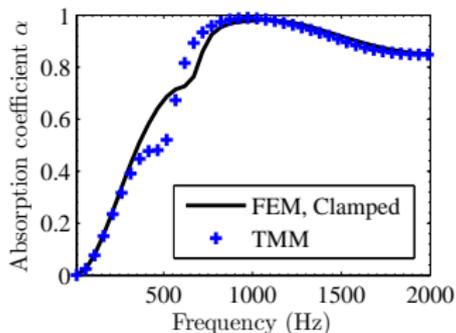
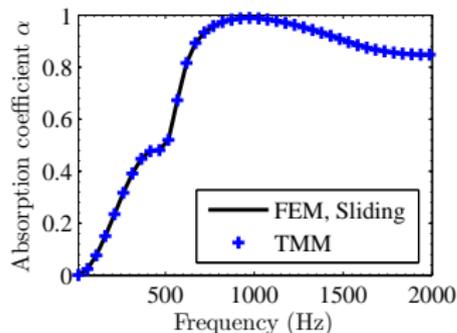
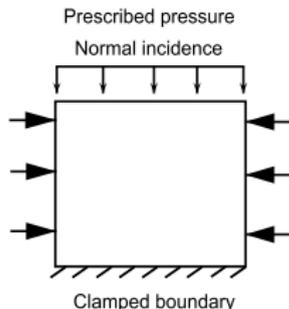
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# A limit of TMM

Section:  $10 \text{ cm} \times 8 \text{ cm}$   
Thickness:  $5 \text{ cm}$   
BC: Sliding or clamped boundary condition ( $\rightarrow$ )



Sliding boundary condition  
Clamped boundary condition  
 $\Rightarrow$  Influence of lateral boundary conditions

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Forced response on basis of progressive waves

Wave based condensation approach

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## Unidirectional wave propagation [Mea75,Mac05,Men05]

- ▶ Finite Element modeling of a substructure

$$\begin{bmatrix} \mathbf{D}_{LL}(\omega) & \mathbf{D}_{LR}(\omega) \\ \mathbf{D}_{RL}(\omega) & \mathbf{D}_{RR}(\omega) \end{bmatrix} \begin{pmatrix} \mathbf{q}_L \\ \mathbf{q}_R \end{pmatrix}(\omega) = \begin{pmatrix} \mathbf{F}_L \\ \mathbf{F}_R \end{pmatrix}(\omega)$$

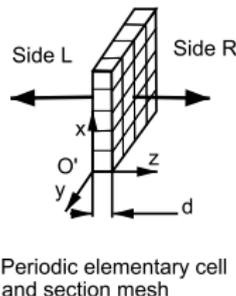
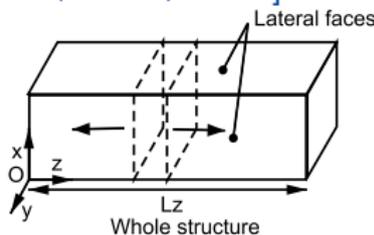
- ▶ Computation of the Transfer Matrix

$$\begin{pmatrix} \mathbf{q}_R \\ \mathbf{F}_R \end{pmatrix} = \mathbf{T}(d) \begin{pmatrix} \mathbf{q}_L \\ -\mathbf{F}_L \end{pmatrix}$$

- ▶ Wave modes are computed from the transfer matrix:

$$\text{Eigenvectors } \Phi = [\Phi_q^T, \Phi_F^T]^T$$

$$\text{Eigenvalues } \lambda = \exp(-ikd)$$



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# Forced response on progressive waves

- ▶ Start point : waves progressing in the material

$$\Phi_q^+, \Phi_q^-, \Phi_F^+, \Phi_F^-, \mu = \exp(-ik^+L)$$

- ▶ Boundary conditions

$$\Phi_q^+ Q^+(0) + \Phi_q^- Q^+(0) = \mathbf{q}_{\text{imp}}$$

$$\Phi_F^+ Q^+(L) + \Phi_F^- Q^+(L) = \mathbf{F}_{\text{imp}}$$

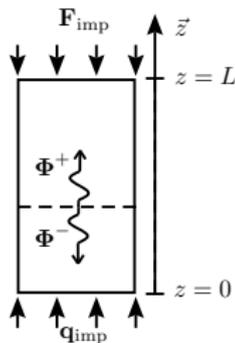
- ▶ Equations of propagation

$$Q^+(L) = \mu Q^+(0)$$

$$Q^-(0) = \mu Q^-(L)$$

- ▶ Solving for the amplitudes [Men10]

$$\underbrace{\begin{bmatrix} \mathbf{I} & (\Phi_q^+)^{-1} \Phi_q^- \mu \\ (\Phi_F^-)^{-1} \Phi_F^+ \mu & \mathbf{I} \end{bmatrix}}_{\mathbf{B}} \begin{pmatrix} Q^+(0) \\ Q^-(L) \end{pmatrix} = \underbrace{\begin{pmatrix} (\Phi_q^+)^{-1} \mathbf{q}_{\text{imp}} \\ (\Phi_F^-)^{-1} \mathbf{F}_{\text{imp}} \end{pmatrix}}_{\mathbf{F}_\Phi}$$



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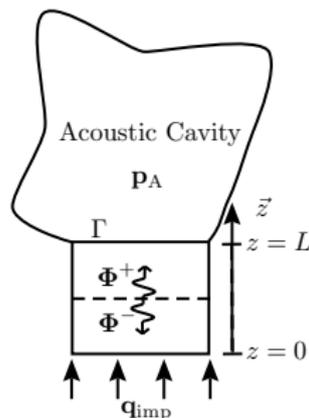
# Condensation of the dissipative interface

- ▶ Initial problem obtained with Finite Elements (UUt formulation)

$$\begin{bmatrix} \mathbf{D}_{AA}(\omega) & \mathbf{D}_{AP} \\ \mathbf{D}_{AP}^T & \mathbf{D}_{PP}(\omega) \end{bmatrix} \begin{pmatrix} \mathbf{p}_A(\omega) \\ \mathbf{q}_P(\omega) \end{pmatrix} = \begin{pmatrix} \mathbf{F}_A(\omega) \\ \mathbf{0} \end{pmatrix}$$

- ▶ Displacement on the interface

$$\mathbf{q}_\Gamma = \Phi \mathbf{Q}(\Gamma) = \Phi \mathbf{A} \mathbf{B}^{-1} \mathbf{C} \mathbf{p}_A$$



- ▶ Final system:

$$[\mathbf{D}_{AA}(\omega) + \mathbf{D}_{coupl}(\omega)] \mathbf{p}_A(\omega) = \mathbf{F}_A(\omega)$$

- ▶ Generalized impedance model
- ▶ Use of progressive waves
- ▶ Lateral effects, non-local reaction, no assumption on incidence

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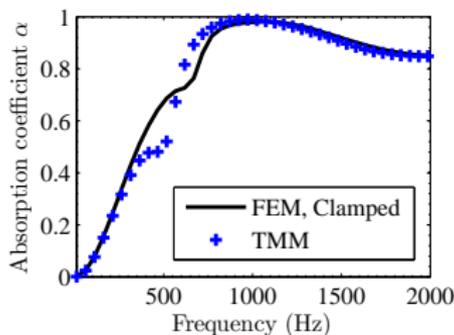
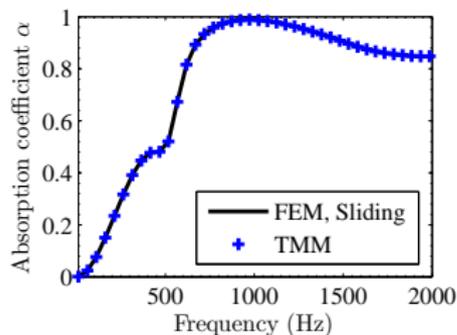
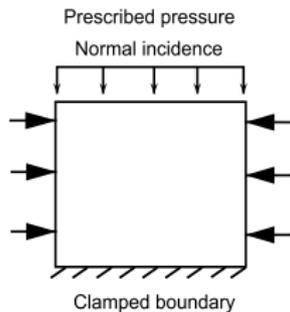
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# Validation: absorption case

Section: 10 cm  $\times$  8 cm  
Thickness: 5 cm  
BC: Sliding or clamped boundary condition ( $\rightarrow$ )



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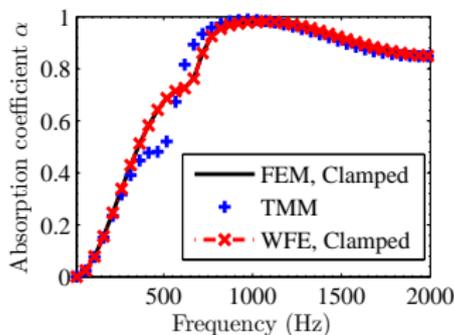
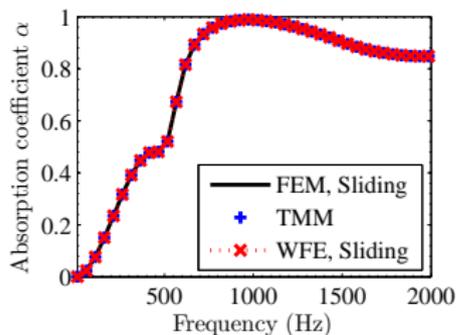
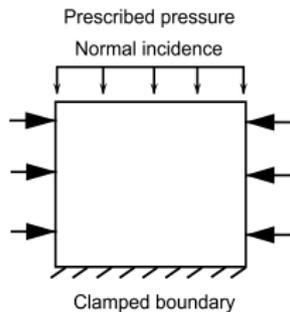
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# Validation: absorption case

Section:  $10 \text{ cm} \times 8 \text{ cm}$   
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$\Rightarrow$  Effects of lateral boundary conditions are predicted by the model

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# Key features of the method



- ▶ Algorithm of condensation (CBO):  
For all frequencies:
  - ▶ Step 1: Compute the wave modes
  - ▶ Step 2: Compute the coupling matrix
  - ▶ Step 3: Solve for the nodal pressures in the cavity

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For all frequencies:
  - ▶ Step 1: Compute the wave modes
  - ▶ Step 2: Compute the coupling matrix
  - ▶ Step 3: Solve for the nodal pressures in the cavity
- ▶ Main interests:
  - ▶ Resolution time does not depends on the thickness
  - ▶ Possible coupling with CMS to reduce Step 3
  - ▶ Can be used for any FEM model: Biot-Allard's model, equivalent fluid model

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  - ▶ Memory and cost to compute the waves (Step 1)
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  - ▶ Numerical errors

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- ▶ **Is the method efficient ?**

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- First observations

- Reduction of the number of contributing waves

- Results

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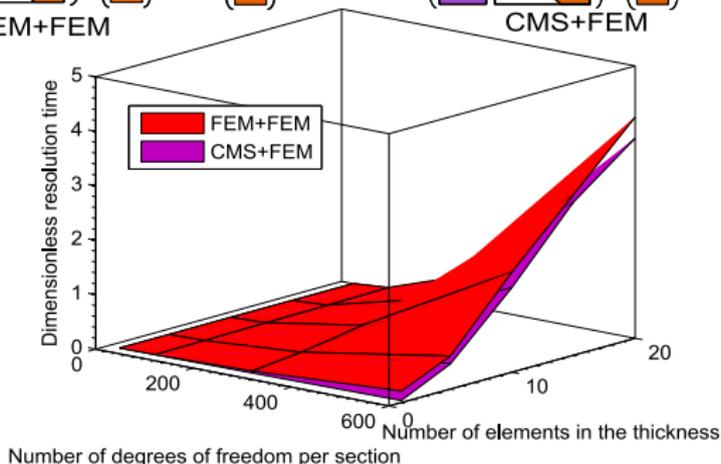
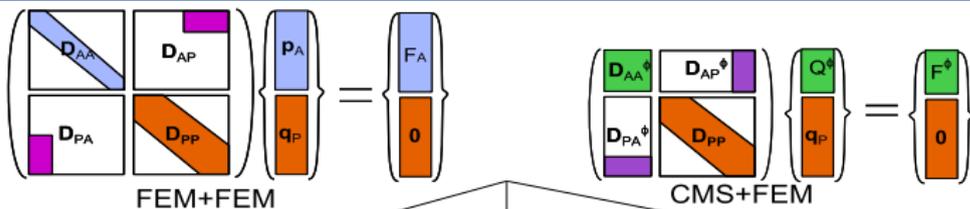
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# First performances observations



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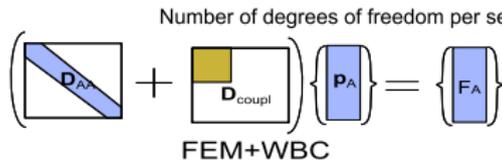
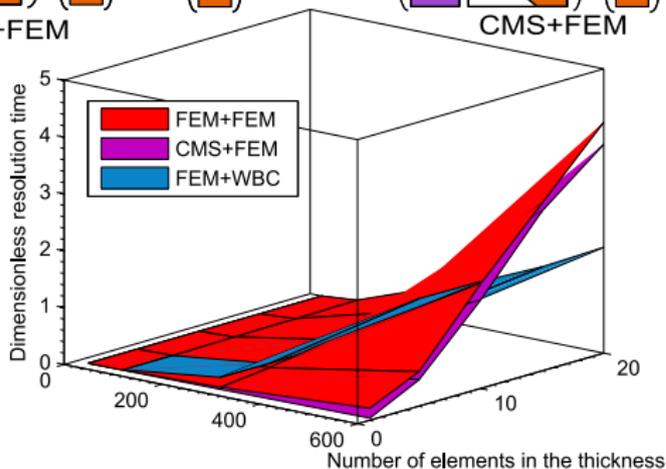
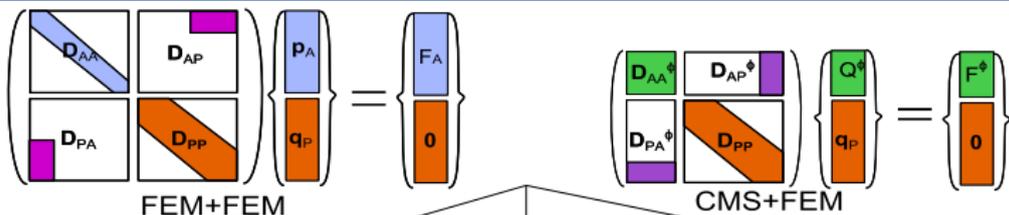
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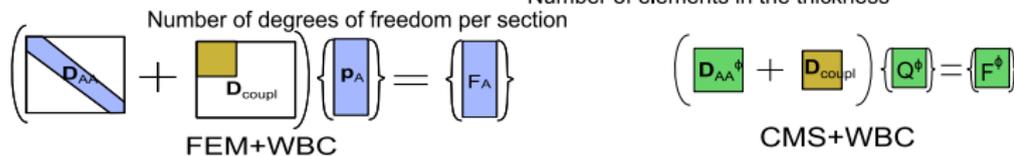
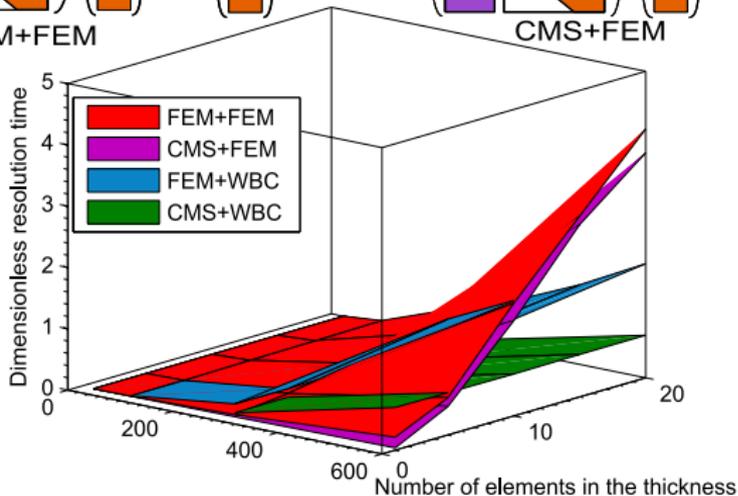
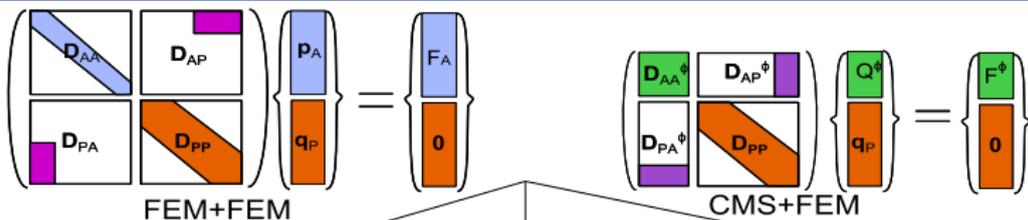
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# Key features of the method



- ▶ Efficient for thick samples with a small area only...

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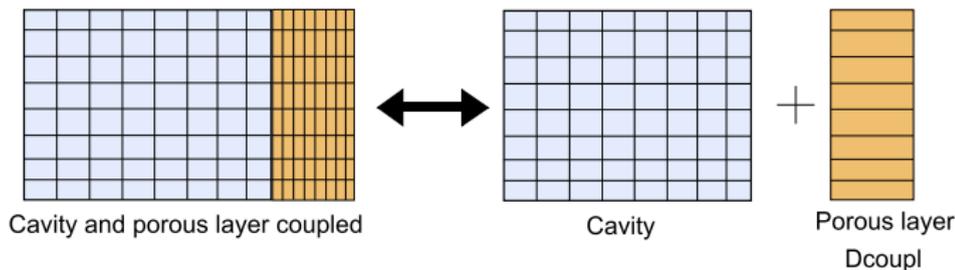
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# Key features of the method

- ▶ Efficient for thick samples with a small area only...
- ▶ Only one step of reduction: condensation in the thickness direction



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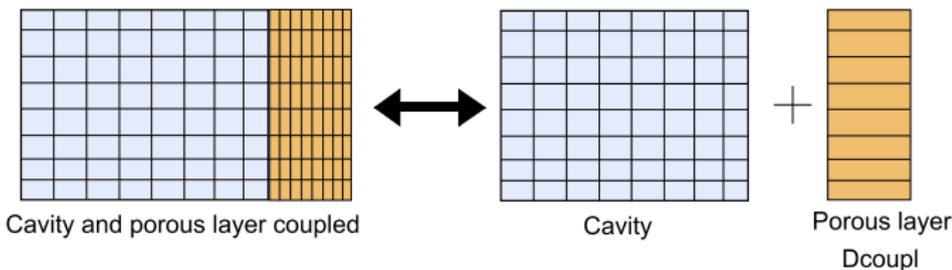
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- ▶ Efficient for thick samples with a small area only...
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- ▶ Other possible step:

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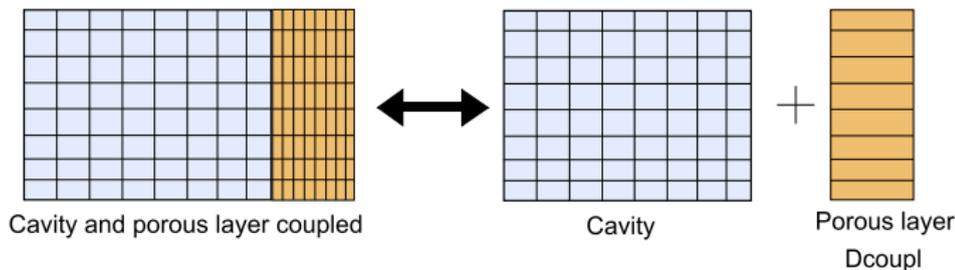
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- ▶ Other possible step:
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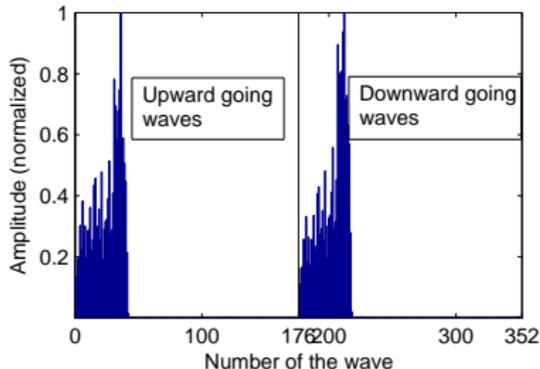
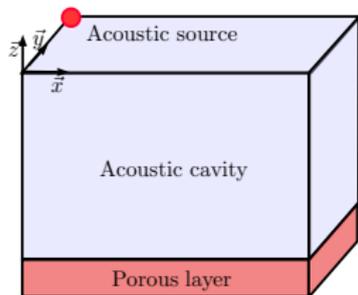
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# Reduction of the number of contributing waves

## Example of an absorption problem with an equivalent fluid (3D)



- Objective : robust and non expensive criterion of selection

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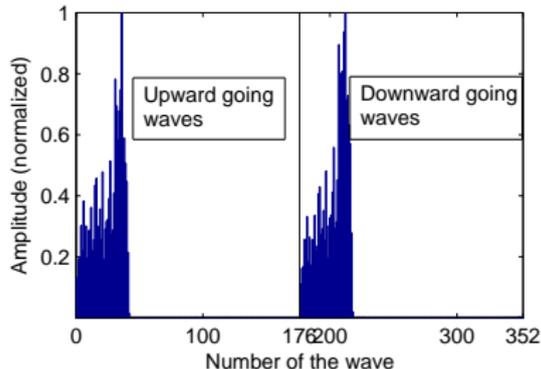
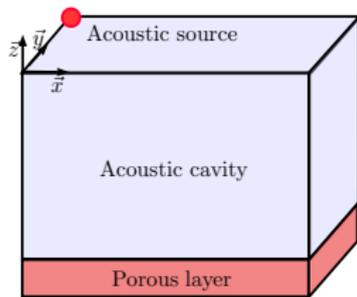
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## Example of an absorption problem with an equivalent fluid (3D)



- ▶ Objective : robust and non expensive criterion of selection
- ▶ Proposed approach: At  $f_{\max}$ ,  $|Q_{\text{sel}}| \leq \text{Crit} \times \max |Q|$

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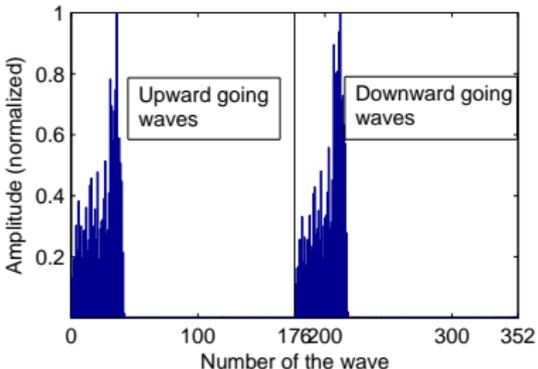
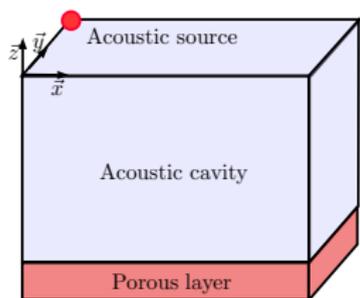
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- ▶ Objective : robust and non expensive criterion of selection
- ▶ Proposed approach: At  $f_{\max}$ ,  $|Q_{\text{sel}}| \leq \text{Crit} \times \max |Q|$
- ▶ Selection criterion? defined by the value of the error with respect to full basis:

$$\epsilon \text{dB} = |L_{p_{\text{ex}}} - L_{p_{\text{app}}}|$$

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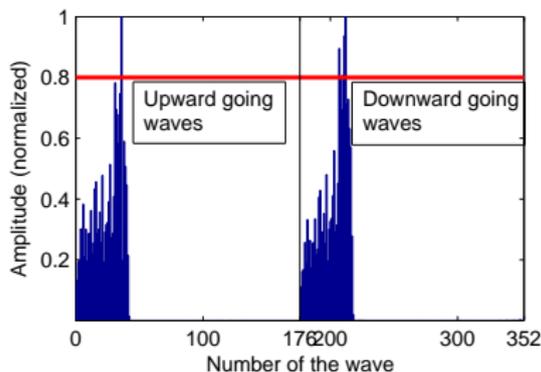
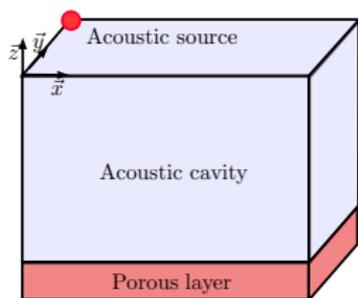
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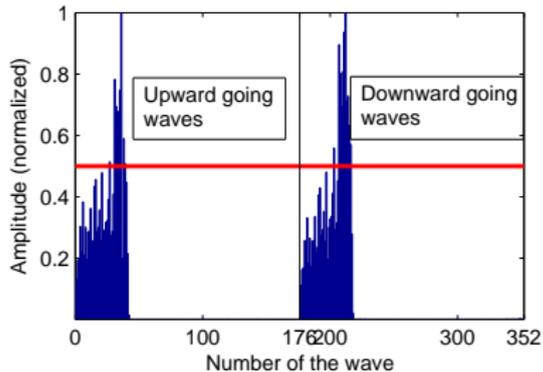
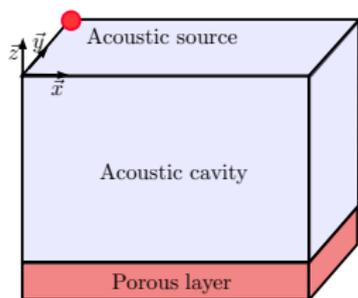
17 Reduction of the number of  
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# Reduction of the number of contributing waves

## Example of an absorption problem with an equivalent fluid (3D)



- ▶ Objective : robust and non expensive criterion of selection
- ▶ Proposed approach: At  $f_{\max}$ ,  $|Q_{\text{sel}}| \leq \text{Crit} \times \max |Q|$
- ▶ Selection criterion? defined by the value of the error with respect to full basis:

$$\epsilon_{\text{dB}} = |L_{p_{\text{ex}}} - L_{p_{\text{app}}}|$$

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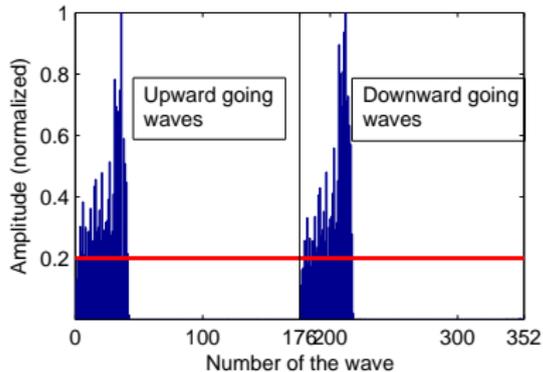
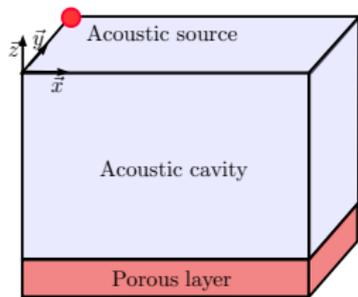
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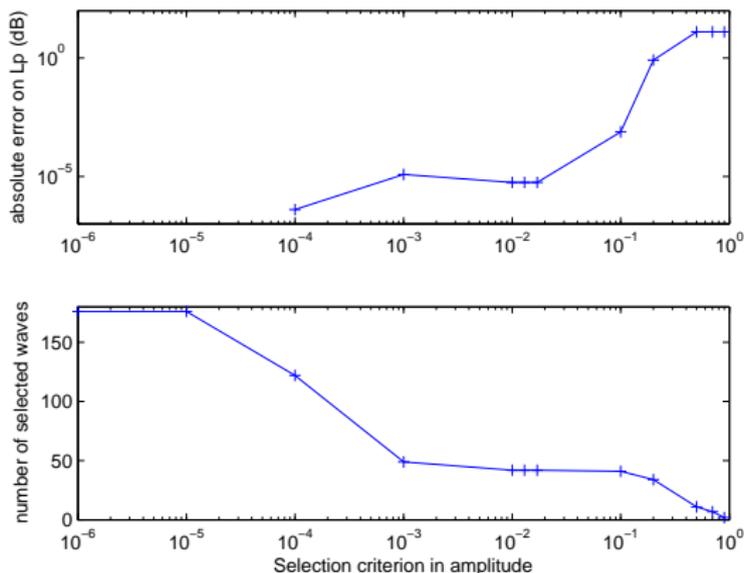
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# Reduction of the number of contributing waves

Evolution of the number of selected waves and of the error at the maximum frequency (2kHz)



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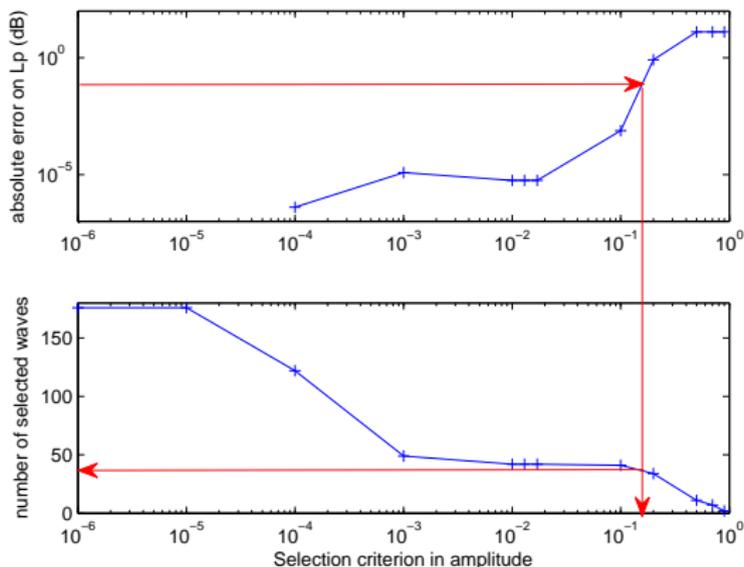
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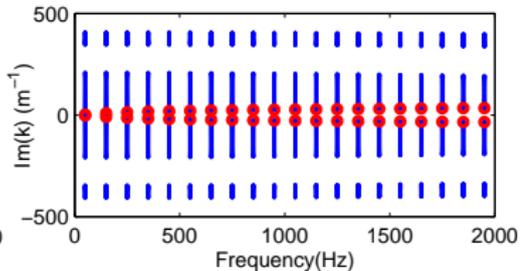
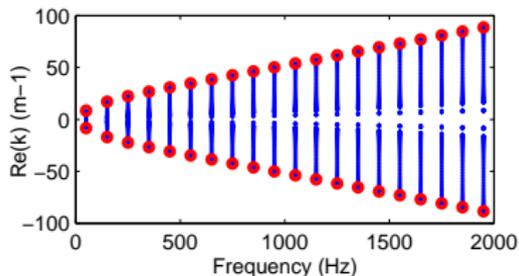
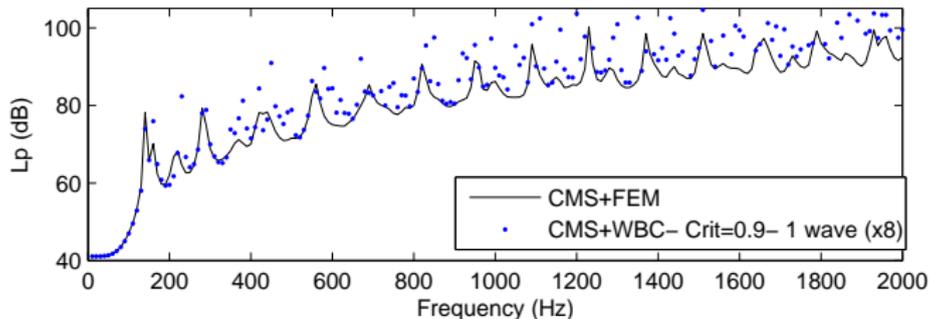
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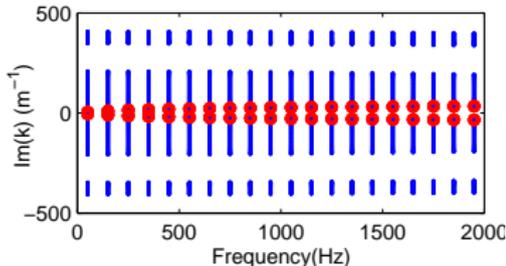
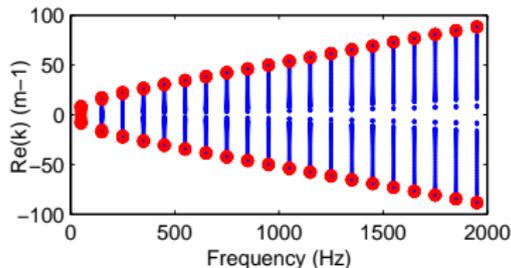
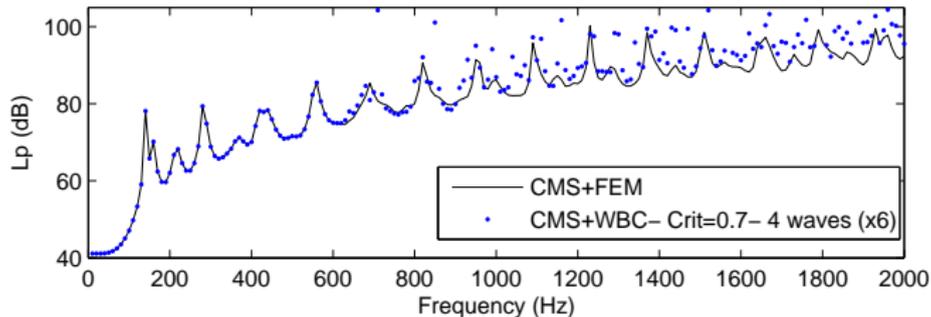
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→ The error decreases with the number of selected waves

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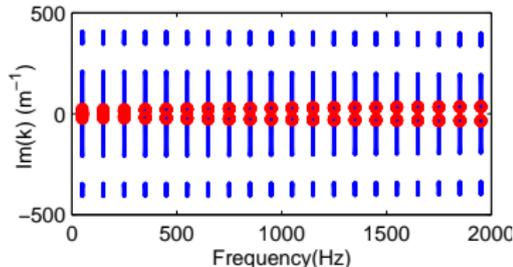
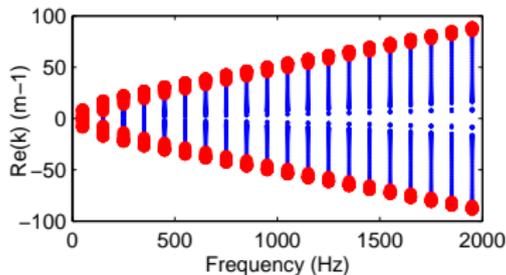
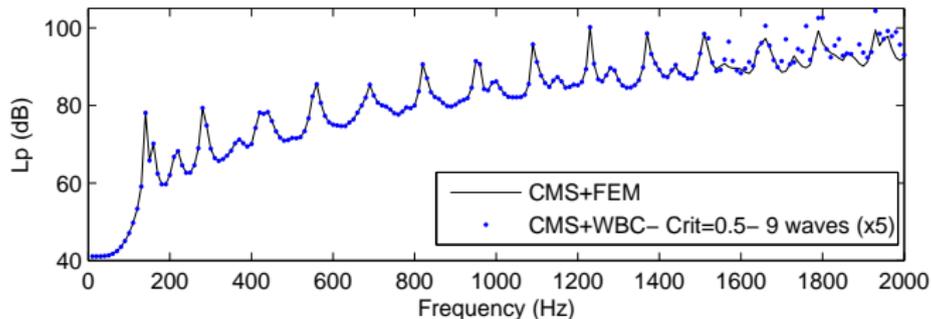
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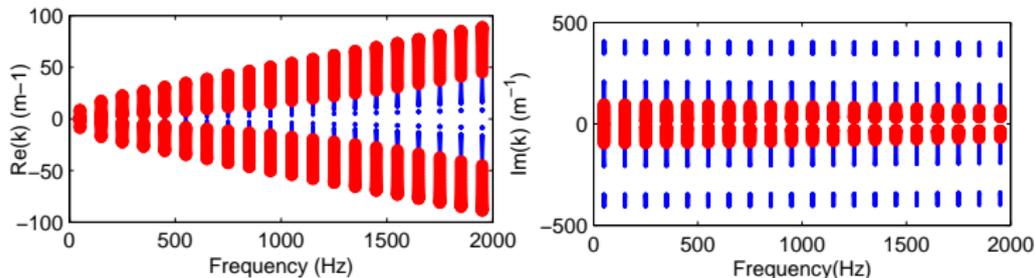
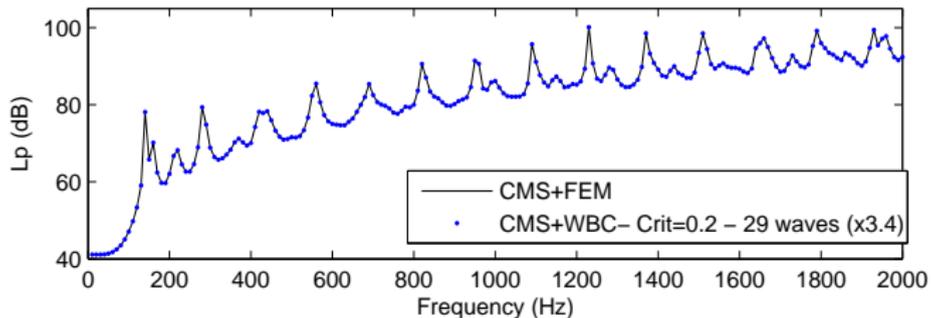
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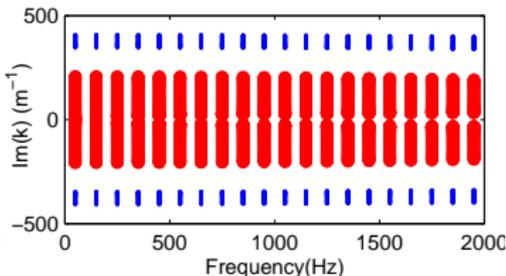
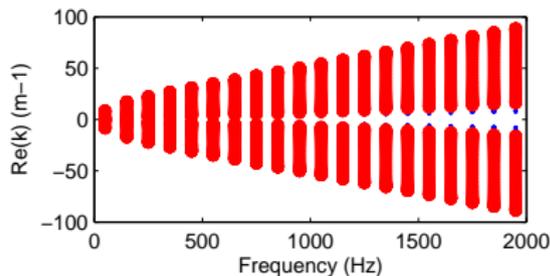
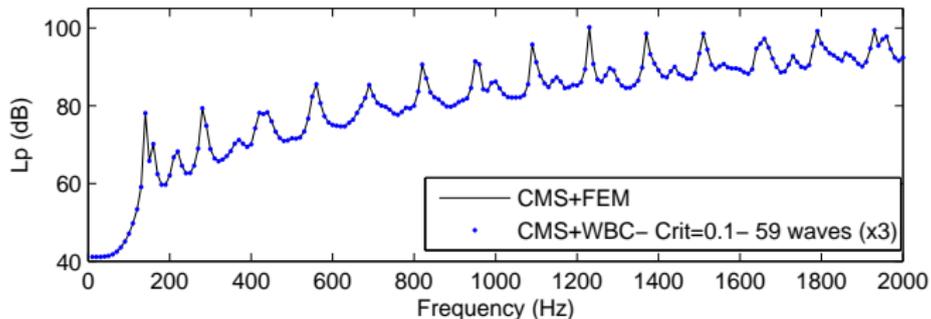
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→ But all of the waves are not needed.

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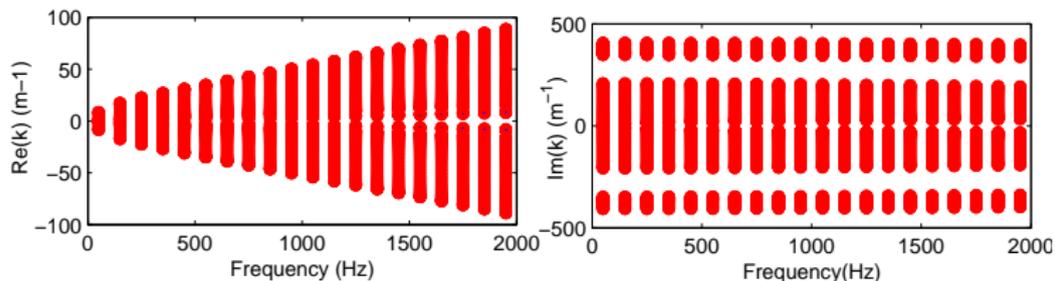
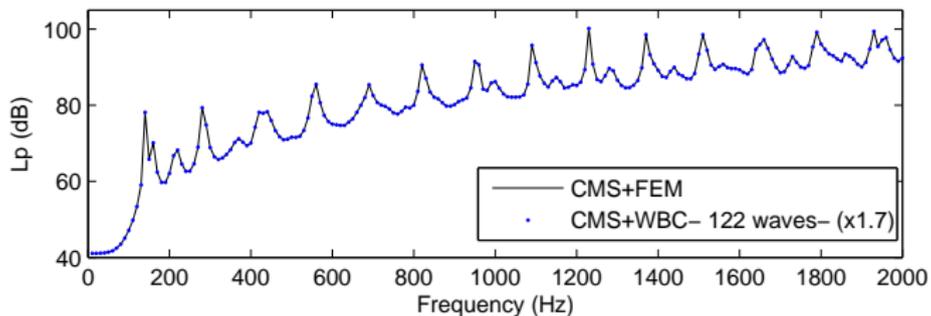
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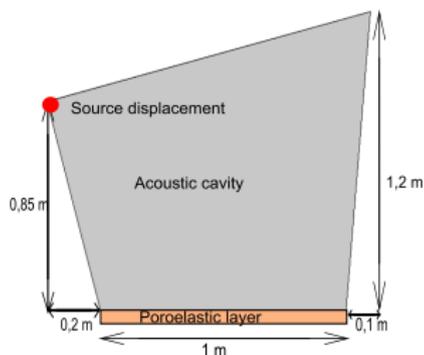
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## Poroelastic layer modeled by UUt formulation



- ▶ 5368 dofs for the cavity - 922 modes
- ▶ 8052 dofs for the poroelastic part
- ▶ Evaluation in 2000 frequencies
- ▶ Criterion for wave selection:  $\text{error} < 1\text{dB}$  at 1200Hz

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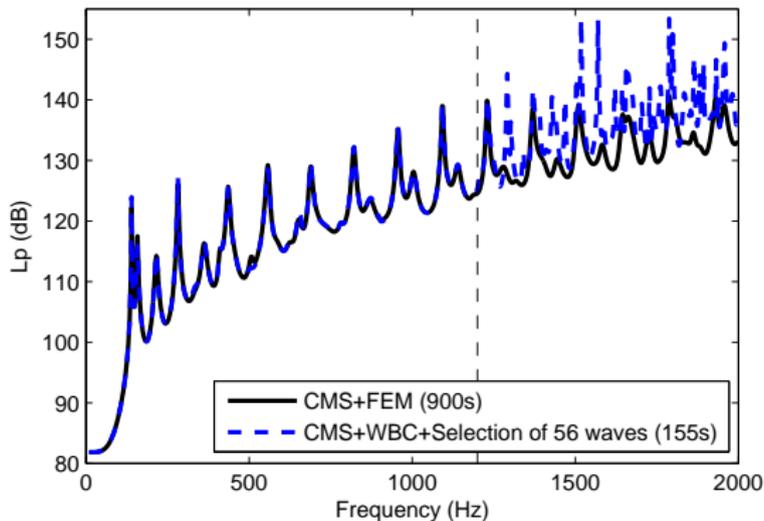
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⇒ Interesting reduction factors!

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# Conclusion

## Work realized

Development of a numerical model of dissipative interface

- ▶ Development of an original wave based condensation procedure for 3D acoustic/poroelastic systems
- ▶ Application to absorption and transmission problems, for single or two-layer packages

Proposals to enhance of the method

- ▶ Wave selection → interesting reduction factors

## Outlook

Application of the method to

- ▶ Acoustic cavity / Structure / Poroelastic coupled systems
- ▶ Multilayer packages
- ▶ Curved interfaces

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