# *The use of slow waves to design simple sound absorbing materials*

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in collaboration with

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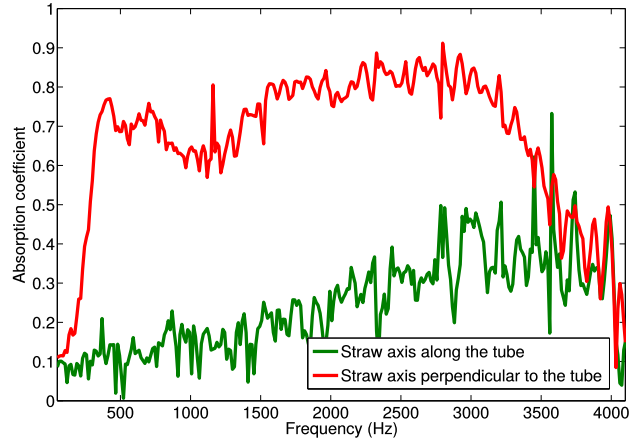
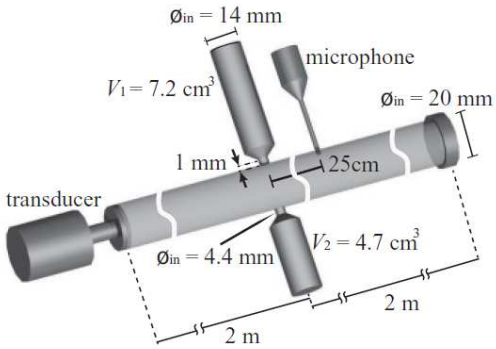
# Motivation

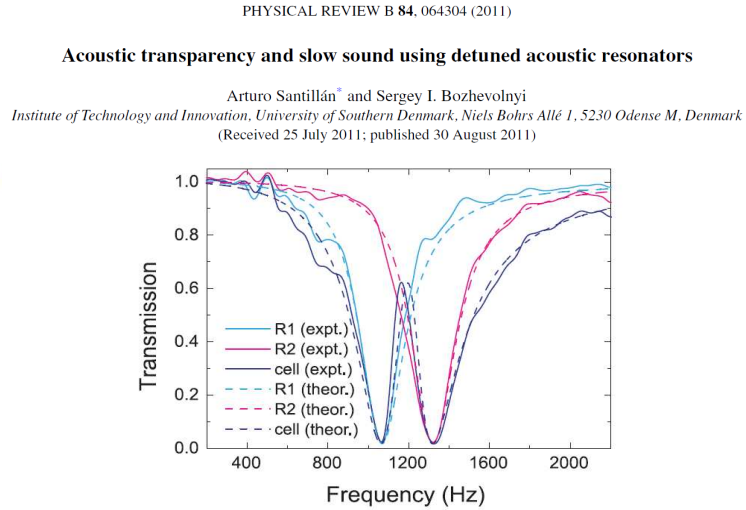
1. Straw as acoustic absorbers

42 mm thick sample




2. Evidence of slow sound in induced transparency band (narrow freq. band)

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**Acoustic transparency and slow sound using detuned acoustic resonators**

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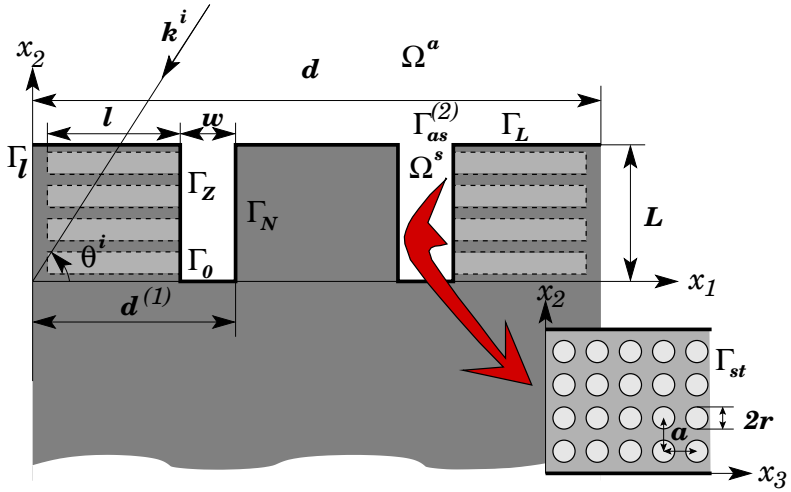
⇒ Use slow sound to absorb sound at low frequencies

# Description of the configuration

Propagation in the tube and in the narrow irregularities(Stinson, J.Acoust.Soc.Am., 1991)




Look for $p^g(\mathbf{x},\omega),\ \forall \mathbf{x}\in\Omega^g,\ s=a,s,t,$

$$\begin{cases} (\nabla+(k^g)^2)p^g(\mathbf{x})=0,\ \forall\mathbf{x}\in\Omega^s,\ g=a,s,t \\ k^a=\omega/c^a,\ k^g=\omega/c^g(\omega), g=s,t \\ + \\ p^a(\mathbf{x})-p^i(\mathbf{x})\sim \text{ outgoing waves; } |\mathbf{x}|\to\infty,\ x_2\geq L, \end{cases}$$

wherein $p^i(\mathbf{x})=A^i e^{\mathbf{i}k_1^i x_1-\mathbf{i}k_2^{ai}(x_2-L)}$

Boundary conditions: $$\begin{cases} [p(\mathbf{x})]=0,\ \forall\mathbf{x}\in\Gamma_{as}\ , \\ \left[(\rho(\mathbf{x}))^{-1}\partial_n p(\mathbf{x})\right]=0,\ \forall\mathbf{x}\in\Gamma_{as},\ \Gamma_N,\ \Gamma_L,\ \Gamma_0 \\ \mathbf{V}^s\cdot\mathbf{n}/p^s(\mathbf{x})=Z,\ \forall\mathbf{x}\in\Gamma_Z, \end{cases}$$

The field is quasi-periodic (*Floquet-Bloch condition*):

$$p(x_1+qd_1,x_2)=p(x_1,x_2)e^{\mathrm{i}k_1^i qd},\ \forall\mathbf{x}\in\mathbb{R}^2 \text{ and } \forall(q)\in\mathbb{Z}.$$

$\Rightarrow$ It is sufficient to determine the field in the unit cell $\mathcal{C}$.

# Solution of the problem

Domain decomposition method, field expressions: Bloch waves

$$p^a(\mathbf{x}) = \sum_{q\in\mathbb{Z}} \left[ A^i e^{-\mathrm{i}k^a_{2q}(x_2-L)}\delta_q + R_q e^{\mathrm{i}k^a_{2q}(x_2-L)} \right] e^{\mathrm{i}k^a_{1q}x_1} \text{ , with } k^a_{1q} = k^i_1 + \frac{2q\pi}{d}.$$

Modal representation in $\Omega^s$

$$p^{s(n)} = \sum_{m\in\mathbb{N}} A_n \cos\left(k^s_{1m}\left(x_1 - d^{(n)}\right)\right)\cos\left(k^s_{2m}x_2\right) \forall \mathbf{x} \in \Omega^{(n)} \ ,$$

wherein modes are bi-orthogonal(Lawrie, Q.J.I.Mech.Apl.Math., 1999) and satisfy

$$k^s_{1m}\tan\left(k^s_{1m}w\right) = \frac{-\mathrm{i}\omega\rho^s}{Z} \text{ with } Z = \frac{\mathrm{i}Z^t \operatorname{cotan}\left(k^t l\right)}{\phi^t}, \text{ and surface porosity } \phi^t = \frac{\pi r^2}{a^2},$$

Application of the B.C. on $\Gamma^{(n)}_{as}$ lead to

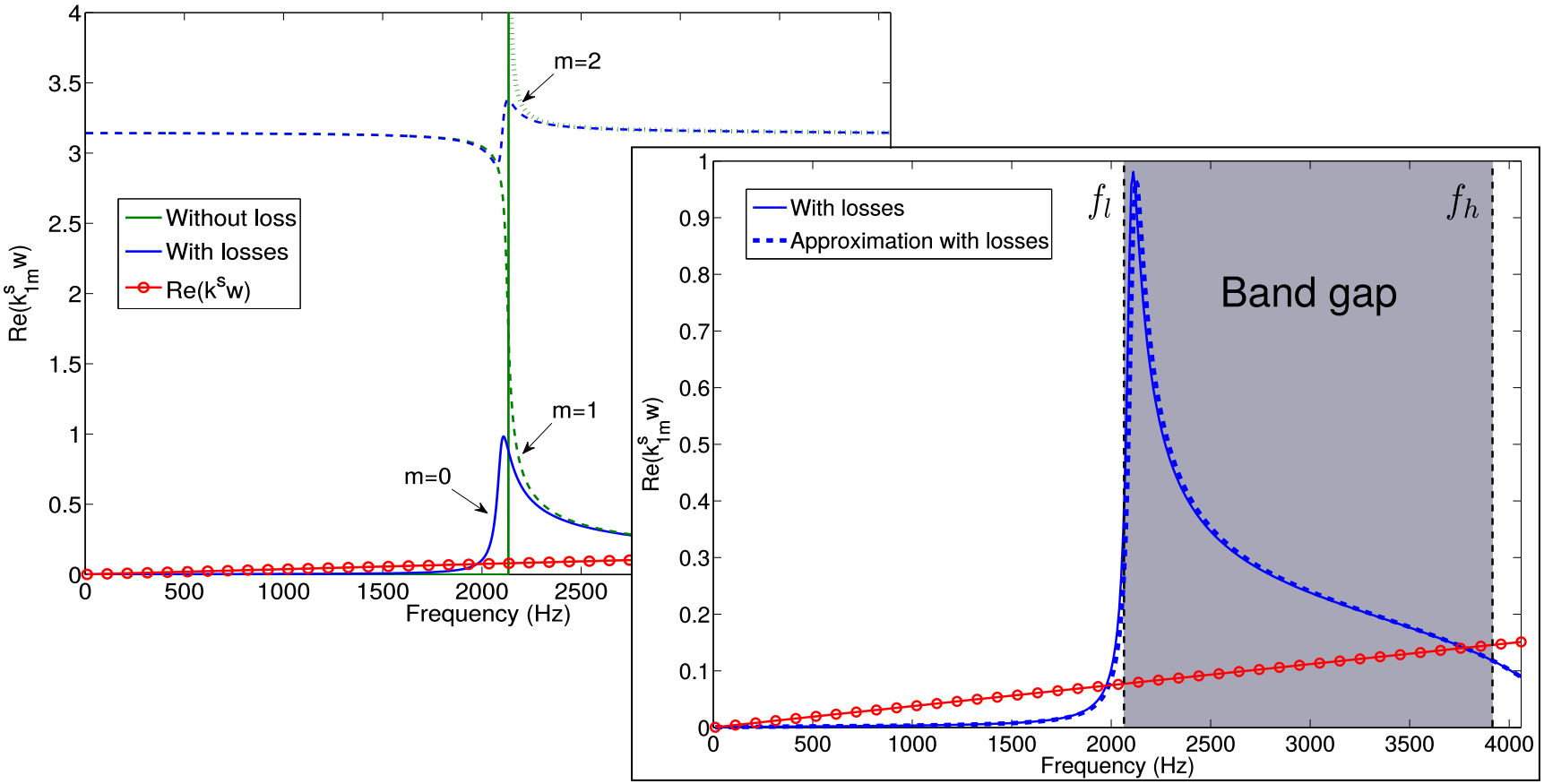
$$R_q - \frac{\mathrm{i}\rho^a}{k^a_{2q}} \sum_{Q\in\mathbb{Z}} R_Q \sum_{n\in\mathcal{N}} \sum_{m\in\mathbb{Z}} \frac{k^s_{2m}\phi^{s(n)}}{\rho^s N_m} \tan\left(k^s_{2m}L\right) I^{l/r(n)-}_{mq} I^{l/r(n)+}_{mQ} =$$

$$A^i\delta_q + A^i \sum_{n\in\mathcal{N}} \sum_{m\in\mathbb{Z}} \frac{\mathrm{i}\rho^a k^s_{2m}\phi^{s(n)}}{k^a_{2q}\rho^s N_m} \tan\left(k^s_{2m}L\right) I^{l/r(n)-}_{mq} I^{l/r(n)+}_{m0} \ ,$$

wherein $\phi^{s(n)} = w/d$ is the surface porosity of one slit, such that $\bigcup_{n\in\mathcal{N}} \phi^{s(n)} = \phi^s$

# Dispersion relation: $k^s_{1m}\tan\left(k^s_{1m}w\right) = -\mathrm{i}\omega\rho^s/Z$

$r = 2.5$ mm, $a = 7$ mm ($\phi^t \approx 0.4$), $l = 40$ mm, $w = 2$ mm

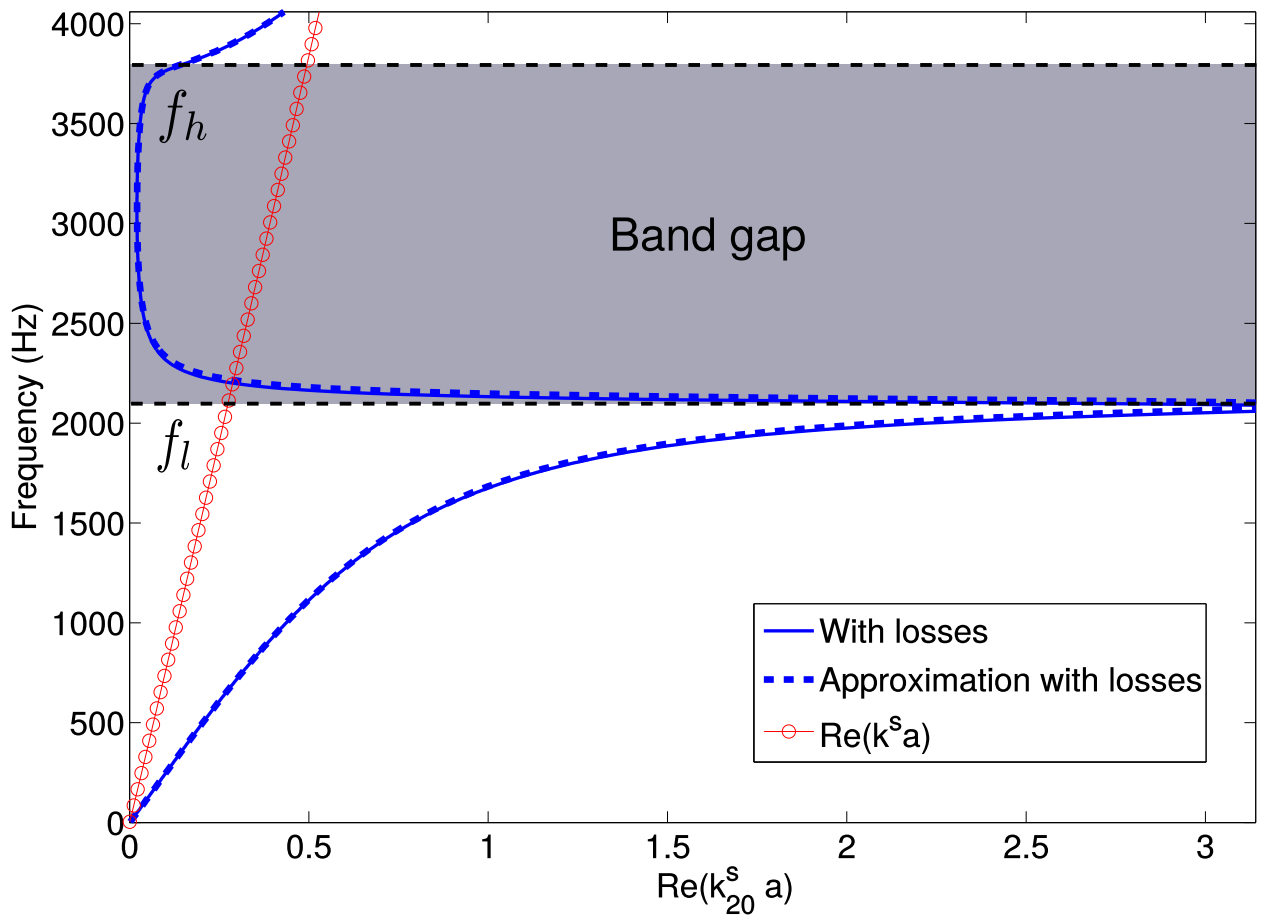



$k^s_{10}$ correctly approximated by $\tilde{k}^s_{10} = 1/w\sqrt{-\mathrm{i}\omega\rho^s w/Z}$

Lower and higher bounds of the BG: $f_l \approx c^t/4l$ and $f_h = 1/\left(2(l/c^t + wZ^t/c^s Z^s \phi^t)\right)$

# A more common representation

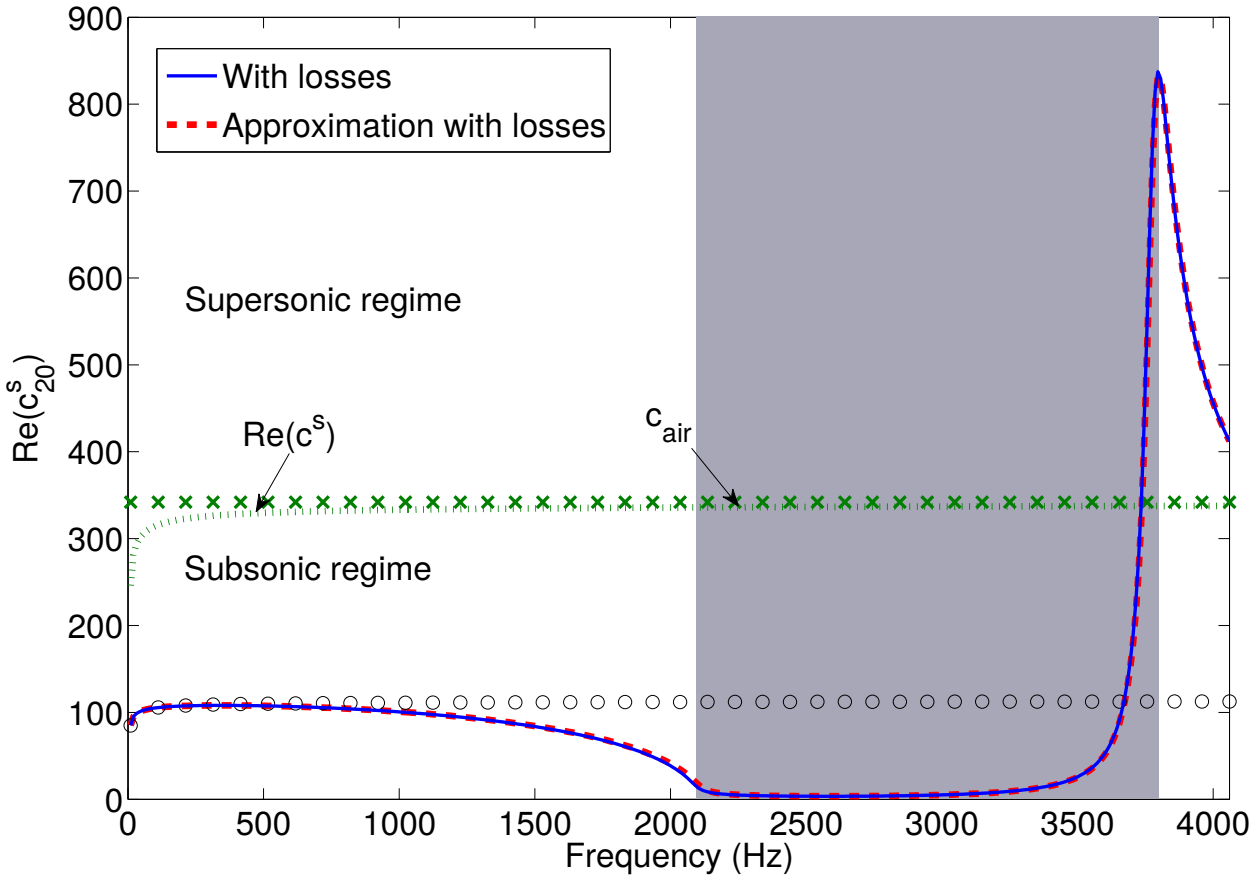
We look at Re $\left(k_{20}^s a\right)$, normalized wavenumber of the wave travelling in the slit




Below $f_l$, the slope of Re $\left(k_{20}^s a\right)$ is always smaller than the one of Re $\left(k^s a\right)$

# Evidence of slow sound

We look at Re $\left(c_{20}^s\right) = w/k_{20}^s = w/\sqrt{(k^s)^2 - (k_{10}^s)^2}$, speed of sound travelling in the slit




$\lim_{\omega \to 0} c_{20}^s(\omega) \approx c^s / \sqrt{1 + \dfrac{\phi^t l Z^s c^s}{w Z^t c^t}}$; without loss $\lim_{\omega \to 0} c_2^s \approx c^a / \sqrt{1 + \dfrac{\phi^t l}{w}}$

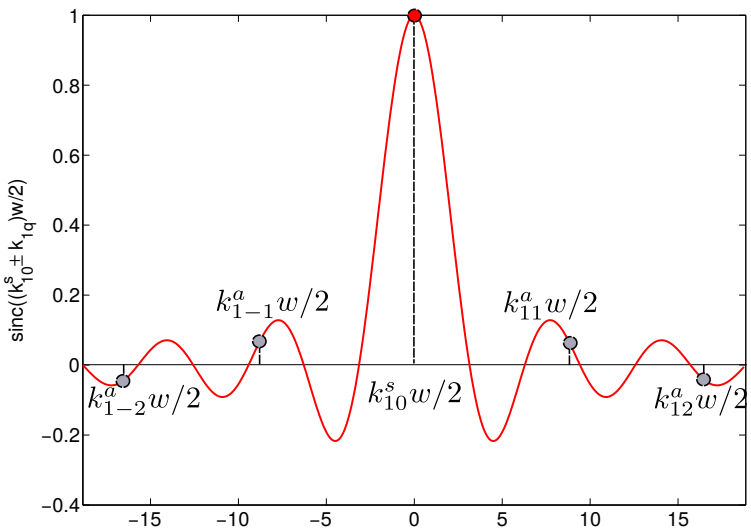
Sound speed & attenuation can be tuned by the ratio $\dfrac{\phi^t l}{w}$ and Im $\left(k_{20}^s(\omega)\right) >$ Im $\left(k^s\right)$

# Assumption to derive of the effective parameters

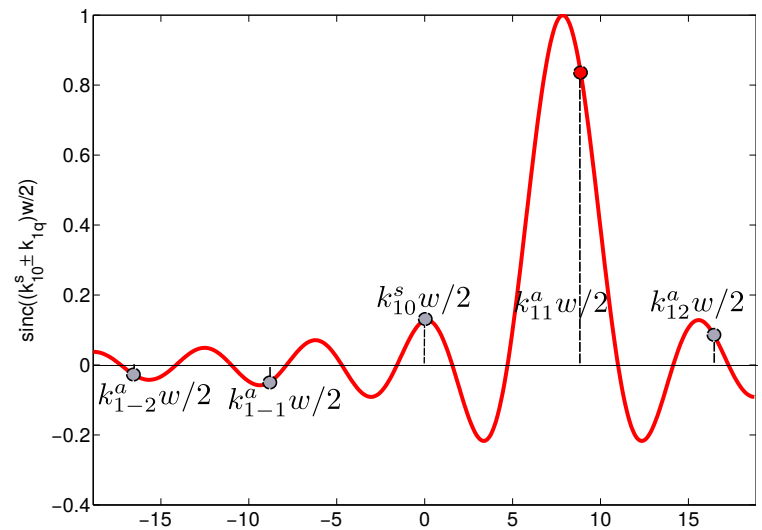
We assume $k_{10}^s w << 1$ so $N_0 \approx 1$.

$I_{0q}^{l/r(n)\pm}$ depends on sinc $\left((k_{10}^s \pm k_{1q}^a)w/2\right)$; at normal incidence $k_1^i = 0$, we have

Only 0-th order Bloch mode contribute




1-st order Bloch mode also contribute




$k_{10}^s$ being dispersive close to $f_l$, higher order Bloch modes contribute to the field

When $|\left(k_{10}^s \pm k_{11}^a\right)\frac{w}{2}| >> 1$, sinc $\left(\left(k_{10}^s \pm k_{10}^a\right)\dfrac{w}{2}\right) \approx 1$ and $I_{00}^{l/r(n)\pm} \approx 1$, we have

$$R_0 = A^i \frac{\mathrm{i}\omega\rho^s \cot\left(k_{20}^s L\right)/\phi^s k_{20}^s - Z_0/\sin\theta^i}{\mathrm{i}\omega\rho^s \cot\left(k_{20}^s L\right)/\phi^s k_{20}^s + Z_0/\sin\theta^i}.$$

# The effective parameters

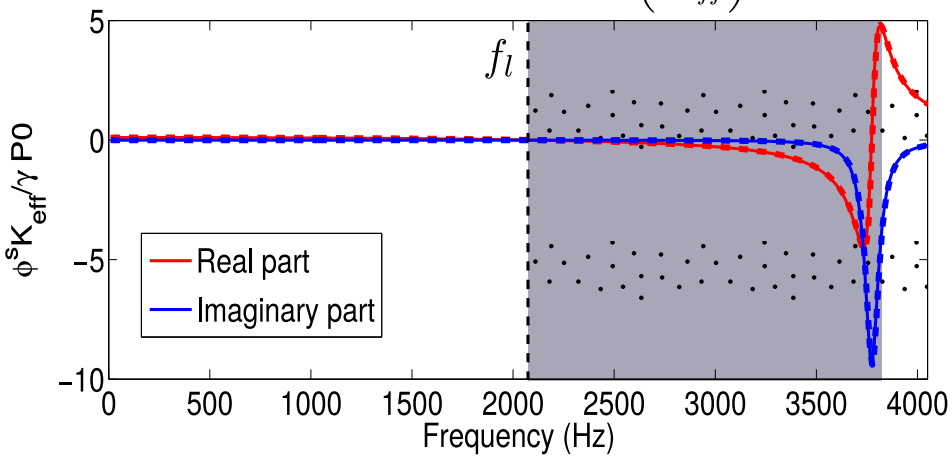
From which we can identify an effective anisotrop medium (locally reacting material)

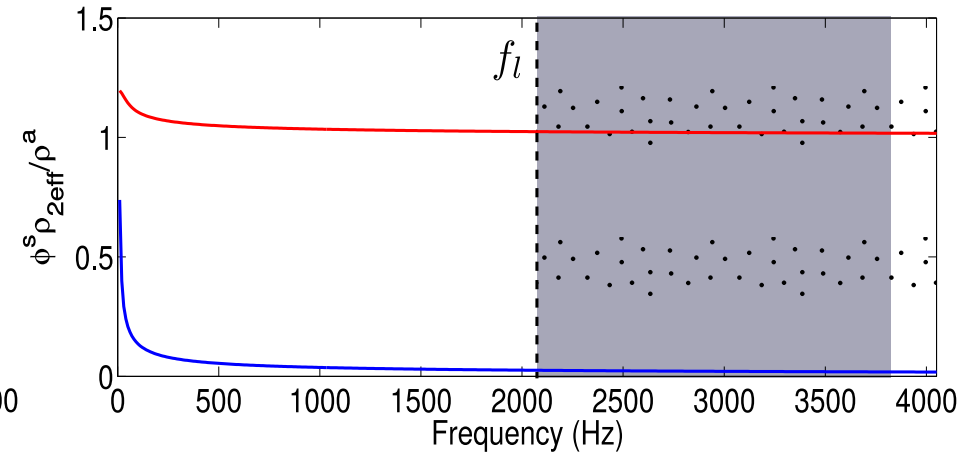
$$\bar{\bar{\rho}}_{eff} = \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{\rho^s}{\phi^s} \end{pmatrix} ,$$

$$K_{eff} = \frac{K^s}{\phi^s \left(1 + Z^s \phi^t \tan\left(k^t l\right) / Z^t k^s h\right)} .$$

$r = 2.5$ mm, $a = 7$ mm ($\phi^t \approx 0.4$), $N = 1$, $l = 40$ mm, $w = 2$ mm, $d = 42$ mm

$\mathrm{Re}\left(K^s_{eff}\right) < 0$ (Fang et al., Nature Mat., 2006)



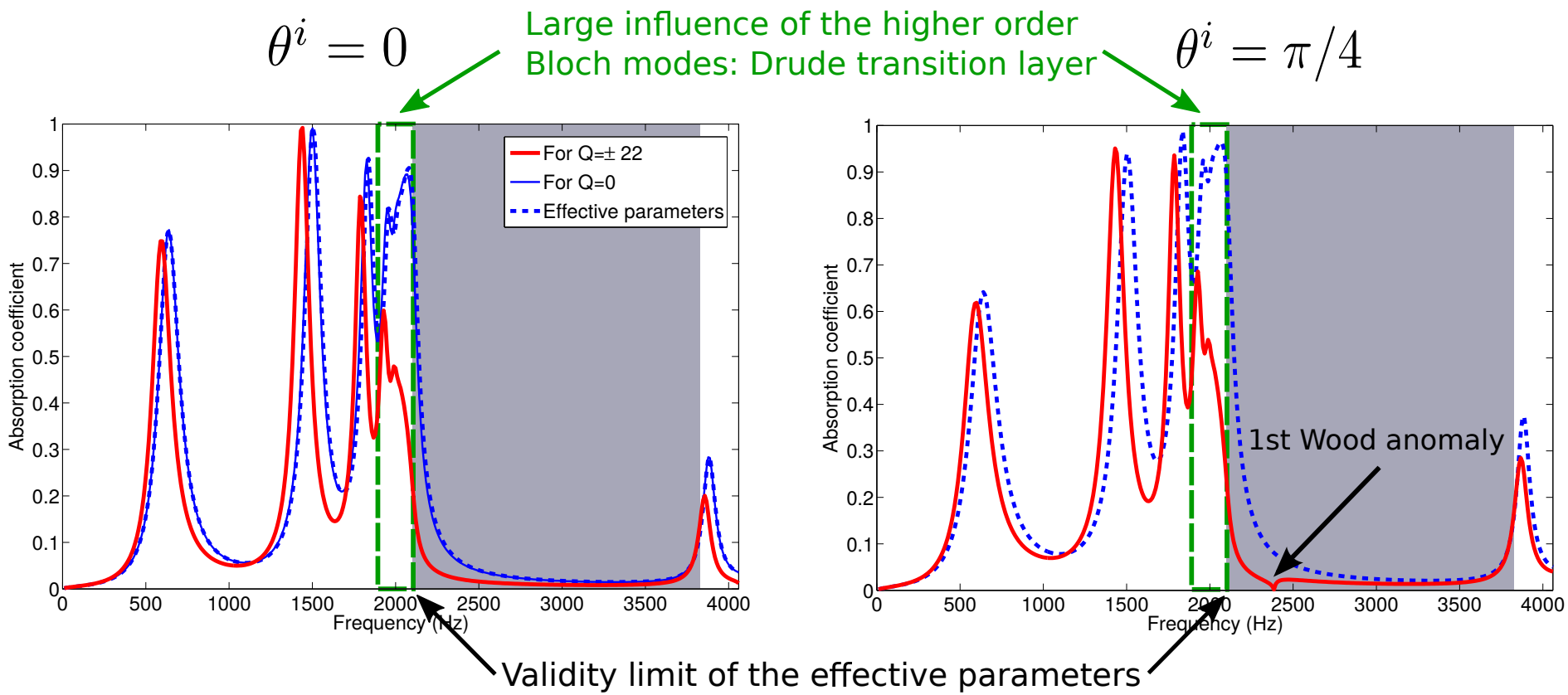




In the slit $\rho^s_{eff} = \rho_{2eff}\phi^s = \rho^s$ and $K^s_{eff} = K_{eff}\phi^s$

Only $K^s_{eff}$ is modified! Slow sound cannot be due to $\alpha_\infty$; only thermal effect

# The absorption curves

$L = 42$ mm, $N = 2$, $d = 84$ mm, $d^{(1)} = 42$ mm ($\Gamma_Z$, left) and $d^{(2)} = 42$ mm ($\Gamma_Z$, right)

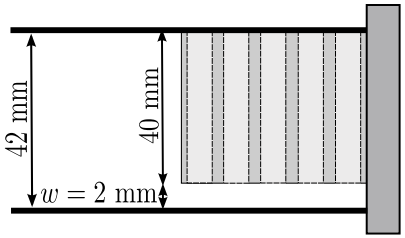


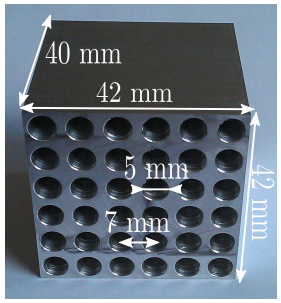

Effective parameters accurate below $f_l$: evidence of the Drude transition layers

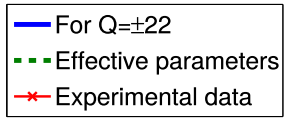
Large absorption peak at $590$ Hz: $\lambda^a \approx 14 \times L$

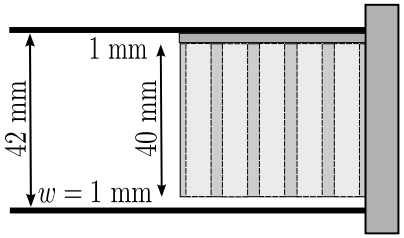
# Experimental validation

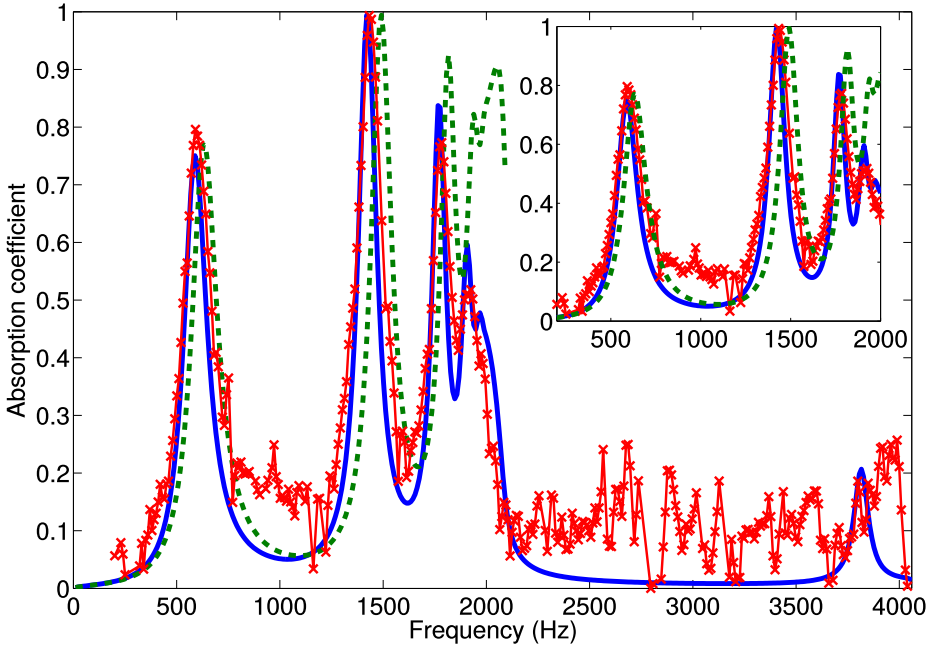
Square impedance tube to create the periodic pattern

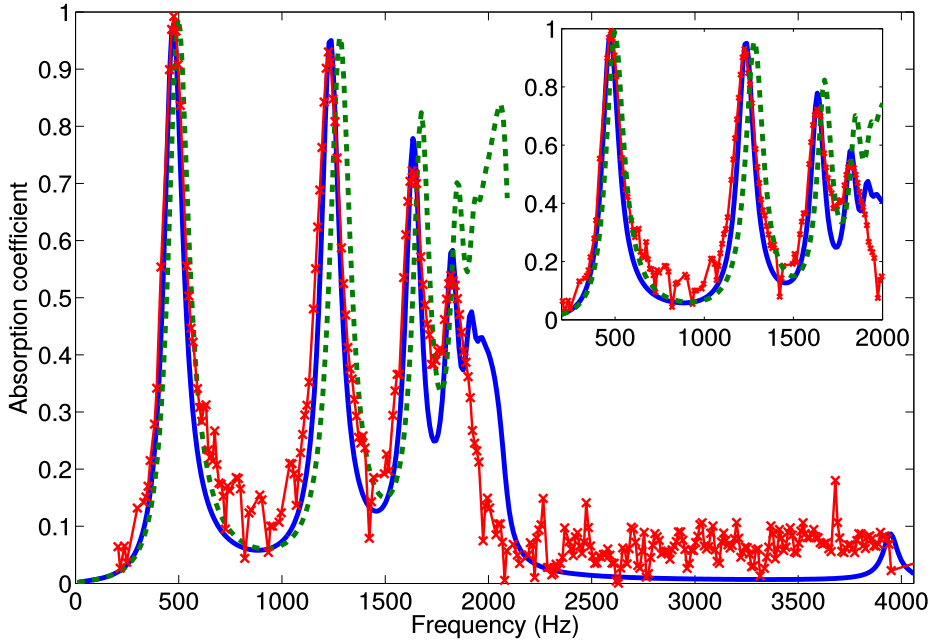



Linearity has been checked & validated (two different amplitudes)

# Conclusion & perspectives

- Slow sound enable to lower the quarter-wavelength resonance frequency

  $\Rightarrow$ Design a sound absorbing metamaterial

- Derivation of the effective properties

  $\Rightarrow$ Slow sound induced by a modification of $K_{eff}$; it is not due to $\alpha_\infty$

  $\Rightarrow$ Evidence of the Drude layer and validity limit

- Current work

  $\Rightarrow$ 3D rectangular pores with various types of resonators plugged on it

  $\Rightarrow$ Broaden the absorption frequency band by detuning resonators

- Prespectives

  $\Rightarrow$ Study wave propagation in waveguides with graded resonances

  $\Rightarrow$ More complex model for the straw (accounting for the irregularities?)

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# Thank you for your attention

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