Inhomogeneous media and metamaterials

ACOUSTIC METAPOROUS WITH PERIODIC INCLUSIONS: EXTENSION TO POROELASTIC MATERIALS

THOMAS WEISSER[1,A], J.-P. GROBY[1], O. DAZEL[1], F. GAUTIER[1]

S. FUTATSUGI[2], L. MONTEIRO[2]

[1]LAUM, Le Mans, France
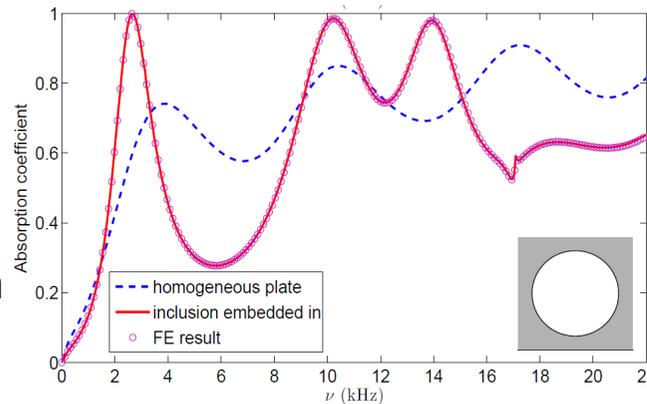
[A]Now in UHA, Mulhouse, France

[2]Embraer, Sao José dos Campos, Brasil

- ## Previous achievement with metaporous materials

Rigid inclusions (2D or 3D)

- Modified modes of the plate

- Trap mode
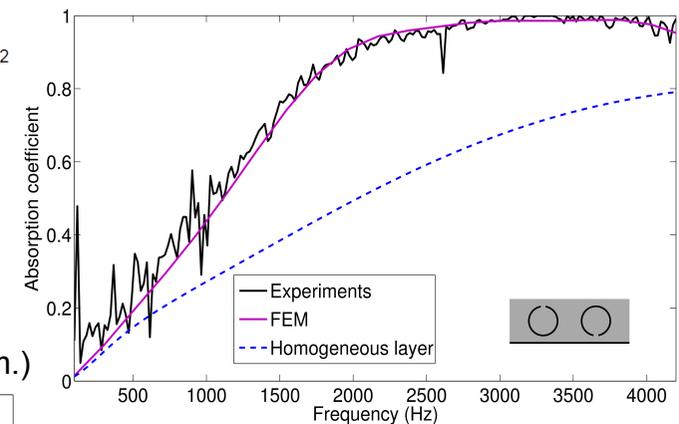
- Enhanced low frequency absorption

Split ring resonators

- Combine trap modes and inclusion resonances

- Study of the influence of geometry and orientation

- Large band absorption using "supercells"

Helmholtz resonators

- Absorption enhanced both in the viscous and inertial regimes

(Groby *et al*., J.Acoust.Soc.Am., 2011;

Nennig *et al*., J.Acoust.Soc.Am., 2012...)

(Lagarrigue *et al*., J.Acoust.Soc.Am., 2013)

(Groby *et al*., accepted to J.Acoust.Soc.Am.)
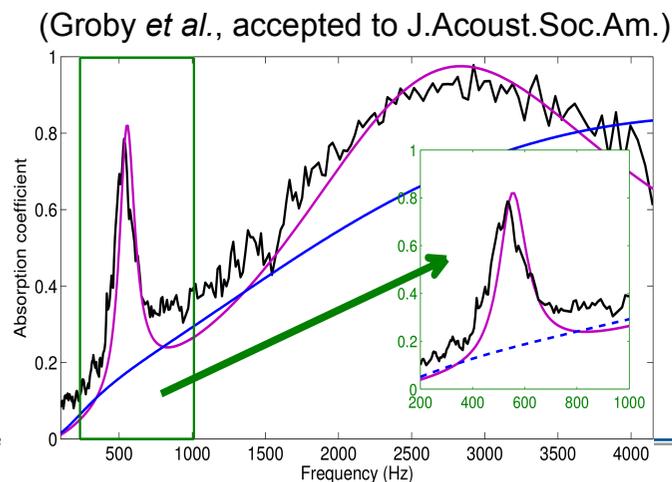
- ## General background
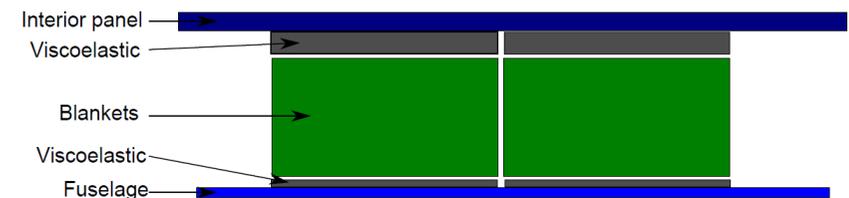
Acoustic isolation in aeronautic structures:

- Improvement and control of the acoustic comfort

- Broadband reduction (LF/MF) at audible frequencies

- Harsh weight and space minimization constraints

Classic technological solutions:

- Multi-layer structure

- Porous and viscoelastic materials

- Localized active devices
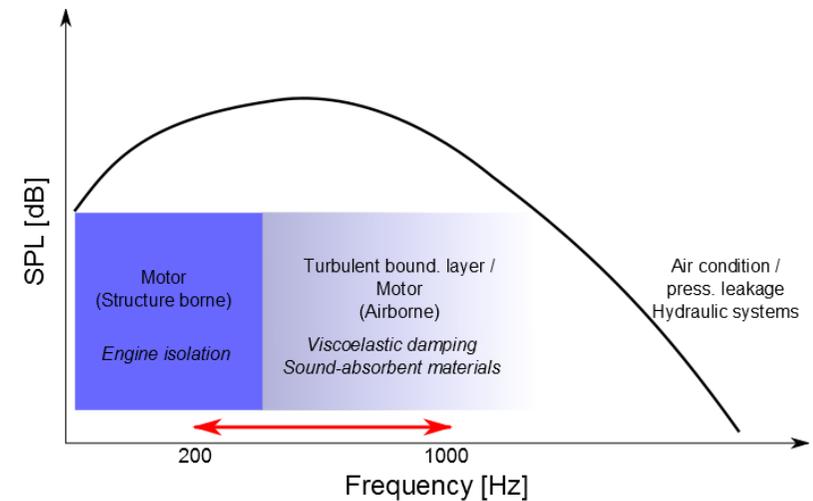
Typical cabin noise spectrum

Typical insulation system configuration

Goal: Extend the concept of metaporous materials to account for the skeleton motion

- ## Wave propagation in poro-elastic media

Biot's theory (1956):

- Constitutive relations:

$$\nabla \cdot \sigma^s = \rho_{11} \frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \rho_{12} \frac{\partial^2 \boldsymbol{U}}{\partial t^2}$$

$$\nabla \cdot \sigma^f = \rho_{12} \frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \rho_{22} \frac{\partial^2 \boldsymbol{U}}{\partial t^2}$$

- Expression of solid and fluid displacements using scalar and vector potentials:

$$\boldsymbol{u} = \nabla \varphi^s + \nabla \wedge \boldsymbol{\Psi}^s$$

$$\boldsymbol{U} = \nabla \varphi^f + \nabla \wedge \boldsymbol{\Psi}^f$$

⇨ 3 different types of propagating waves:

longitudinal: φ_P and φ_A ; shear: ψ_S

- Modeling of dissipation phenomena:

⇨ Johnson-Champoux-Allard model

⇨ modifies the effective density and bulk modulus:

$$\rho = \rho_0 \left[\alpha_\infty + \frac{\nu \phi}{i\omega q_0} G(\omega) \right]$$

$$K = \gamma P_0 / \left[\gamma - \frac{\gamma - 1}{1 + \frac{\nu' \phi}{i\omega q_0'} G'(\omega)} \right]$$

$$G_j(\omega) = \left[1 + \left(\frac{2\alpha_\infty q_0}{\phi \Lambda} \right)^2 \frac{i\omega}{\nu} \right]^{1/2}$$

$$G_j'(\omega) = \left[1 + \left(\frac{\Lambda'}{4} \right)^2 \frac{i\omega}{\nu'} \right]^{1/2}$$

- ## Nominal configuration

External pressure field:

$$p^{[0^+]} = e^{ik_1^i x_1} \left(e^{-ik_2^i(x_2-H)} + Re^{ik_2^i(x_2-H)} \right)$$

Internal wave potentials:

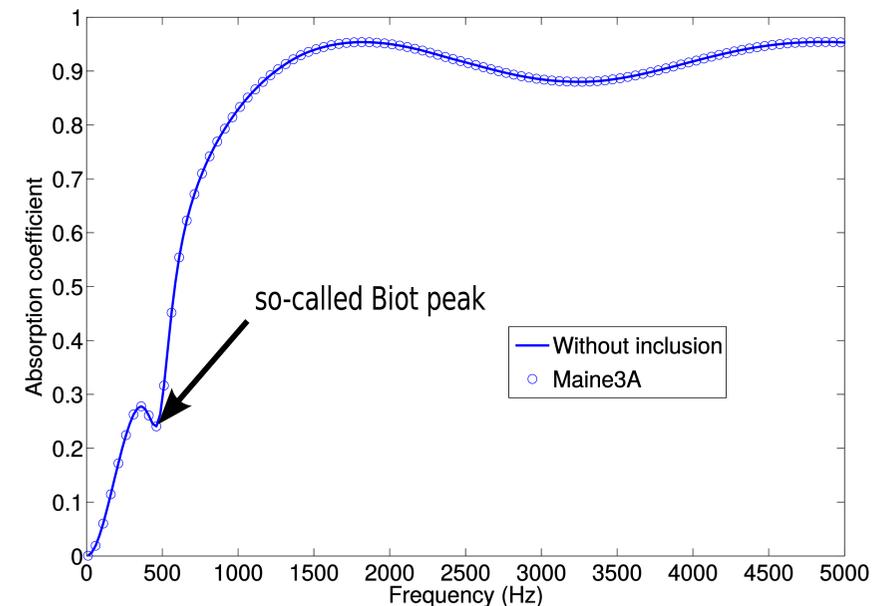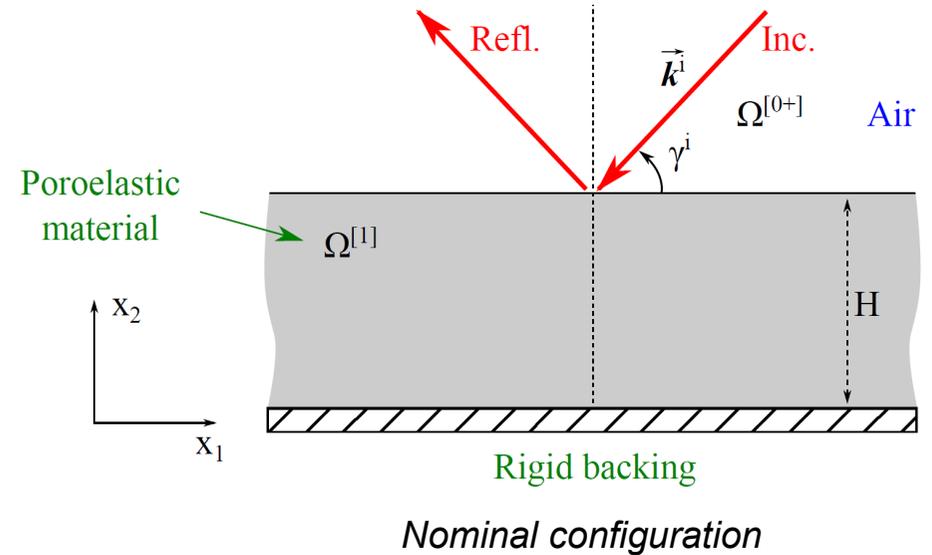$$\Xi_X^{[1]} = e^{ik_1^i x_1} \left(f_X^{[1]} e^{-ik_{X_2}^{[1]} x_2} + g_X^{[1]} e^{ik_{X_2}^{[1]} x_2} \right)$$

+ appropriate boundary conditions

Determination of the absorption coefficient:

Inverse computation of a 7x7 linear system : $\mathcal{M}\boldsymbol{f} = \boldsymbol{h}$

Application:

55 mm foam layer: melamine / "yellow" foam

Nominal configuration

Semi-analytical modeling

- ## Complete model assembly (1)

Bloch modal decomposition of the different fields:

Complete configuration

$$k_1^q = k_1^i + \frac{2p\pi}{d} \text{ and } k_2^{[j]q} = \sqrt{(k^{[j]})^2 - (k_1^q)^2} \quad , j = 0, 1$$
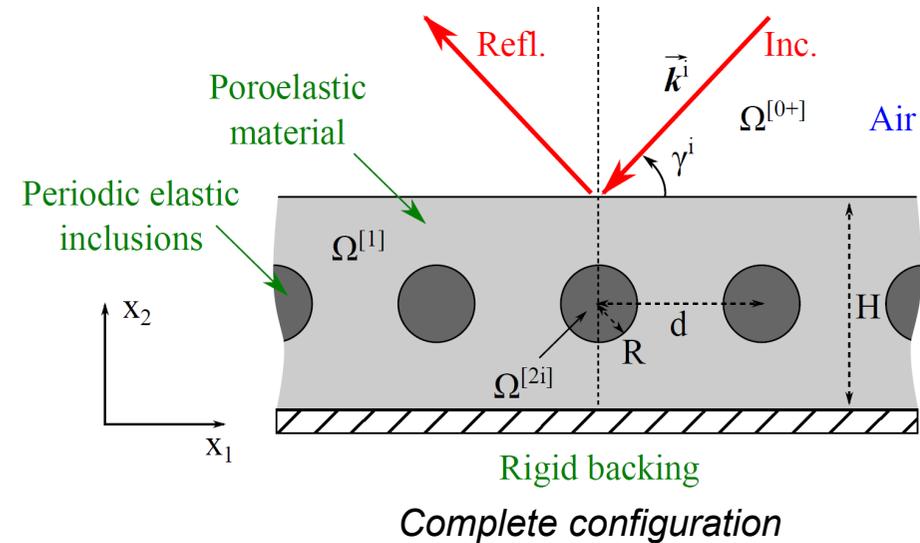
External pressure field:

$$p^{[0^+]}(\boldsymbol{x}) = \sum_{q \in \mathbb{Z}} e^{ik_1^q x_1} \left[A_q \, e^{-ik_2^{[0]q}(x_2 - H)} \delta_q + R_q \, e^{ik_2^{[0]q}(x_2 - H)} \right]$$

Potentials in the poroelastic layer:

Scattering coefficient

$$\Xi_X^{[2]}(\boldsymbol{x}) = \sum_{q \in \mathbb{Z}} e^{ik_1^q x_1} \left[f_X^{[2]q} \, e^{-ik_2^{[0]q} x_2} + g^{[2]q} \, e^{-ik_2^{[0]q} x_2} \right] + \sum_{q \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} K_{qn}^{X \pm} C_n^X \, e^{ik_1^q \left(x_1 - x_1^{(0)} \right) \pm ik_2^{[0]q} \left(x_2 - x_2^{(0)} \right)}$$

Reflection on the elastic plates   Scattered field expressed in Cartesian CS

- ## Complete model assembly (2)

Application of the B.C. on the air/material:

$$\mathcal{M}^q \boldsymbol{f}^q = \boldsymbol{h}^i A_q \delta_q + \sum_{n\in\mathbb{Z}} \boldsymbol{h}_{qn}^P C_n^P + \sum_{n\in\mathbb{Z}} \boldsymbol{h}_{qn}^A C_n^A + \sum_{n\in\mathbb{Z}} \boldsymbol{h}_{qn}^S C_n^S$$

« Identical » to the nominal system

Additional source terms involving scattering coefficients: numerical determination

Rewriting the potentials by replacing $\boldsymbol{f}^q$ by its semi-analytical expressions:

$$\Xi_X^{[2]}(\boldsymbol{x}) = \sum_{n\in\mathbb{Z}} C_n^X H_n^{(1)}(k_X r) e^{in\theta} + \sum_{n\in\mathbb{Z}} \mathcal{A}_n^X J_n(k_X r) e^{in\theta}$$

$$= \sum_{n\in\mathbb{Z}} C_n^X H_n^{(1)}(k_X r) e^{in\theta}$$

$$+ \sum_{n\in\mathbb{Z}}\sum_{m\in\mathbb{Z}} \left[\sigma_{mn}^X C_m^X + \sum_{q\in\mathbb{Z}} Q_{mnq}^{PX} C_m^P + \sum_{q\in\mathbb{Z}} Q_{mnq}^{AX} C_m^A + \sum_{q\in\mathbb{Z}} Q_{mnq}^{SX} C_m^S + S_n^X \right] J_n(k_x r) e^{in\theta}$$

« Schlömilch series »

- ## Complete model assembly (3)

Numerical determination of the scattering coefficients:

Using the multipole scattering approach:

$$C_n^X = T_n^{PX} \, \mathcal{A}_n^P + T_n^{AX} \, \mathcal{A}_n^A + T_n^{SX} \, \mathcal{A}_n^S$$

Applying the B.C. at the interface of the inclusion:

$$
\begin{bmatrix}
Id - \left[T^{PP}\left(\bar{\sigma}^P + \tilde{Q}^{PP}\right) + T^{AP}\tilde{Q}^{PA} + T^{SP}\tilde{Q}^{PS} \right] & -\left[T^{PP}\tilde{Q}^{AP} + T^{AP}\left(\bar{\sigma}^A + \tilde{Q}^{AA}\right) + T^{SP}\tilde{Q}^{AS} \right] & -\left[T^{PP}\tilde{Q}^{SP} + T^{AP}\tilde{Q}^{SA} + T^{SP}\left(\bar{\sigma}^S + \tilde{Q}^{SS}\right) \right] \\
-\left[T^{PA}\left(\bar{\sigma}^P + \tilde{Q}^{PP}\right) + T^{AA}\tilde{Q}^{PA} + T^{SA}\tilde{Q}^{PS} \right] & Id - \left[T^{PA}\tilde{Q}^{AP} + T^{AA}\left(\bar{\sigma}^A + \tilde{Q}^{AA}\right) + T^{SA}\tilde{Q}^{AS} \right] & -\left[T^{PA}\tilde{Q}^{SP} + T^{AA}\tilde{Q}^{SA} + T^{SA}\left(\bar{\sigma}^S + \tilde{Q}^{SS}\right) \right] \\
-\left[T^{PS}\left(\bar{\sigma}^P + \tilde{Q}^{PP}\right) + T^{AS}\tilde{Q}^{PA} + T^{SS}\tilde{Q}^{PS} \right] & -\left[T^{PS}\tilde{Q}^{AP} + T^{AS}\left(\bar{\sigma}^A + \tilde{Q}^{AA}\right) + T^{SS}\tilde{Q}^{AS} \right] & Id - \left[T^{PS}\tilde{Q}^{SP} + T^{AS}\tilde{Q}^{SA} + T^{SS}\left(\bar{\sigma}^S + \tilde{Q}^{SS}\right) \right]
\end{bmatrix}
\times
\begin{bmatrix} C^P \\ C^A \\ C^S \end{bmatrix}
=
\begin{bmatrix}
T^{PP}S^P + T^{AP}S^A + T^{SP}S^S \\
T^{PA}S^P + T^{AA}S^A + T^{SA}S^S \\
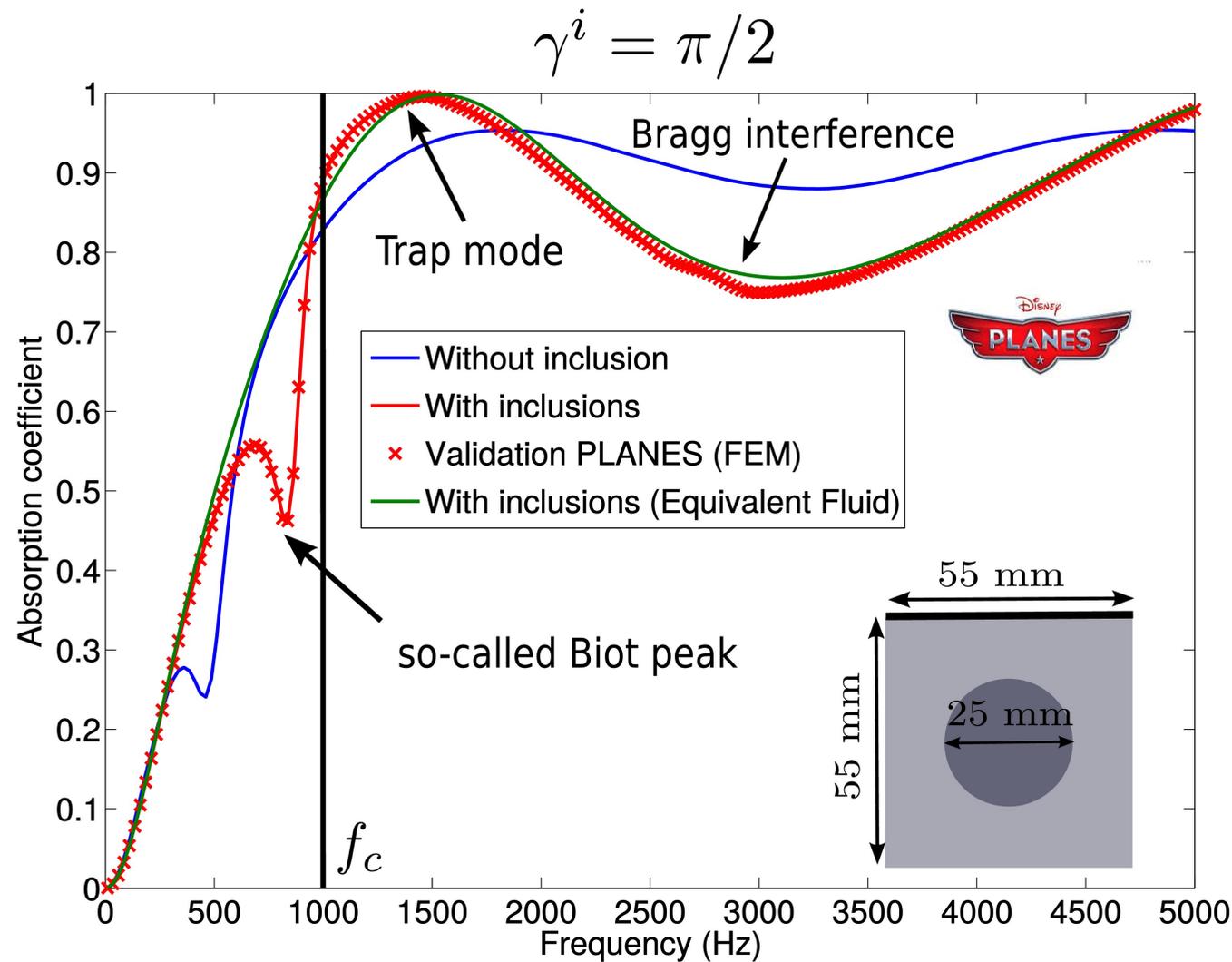T^{PS}S^P + T^{AS}S^A + T^{SS}S^S
\end{bmatrix}.
$$

Matrix system of size (3 x nbr of scattering modes) at each considered frequency.

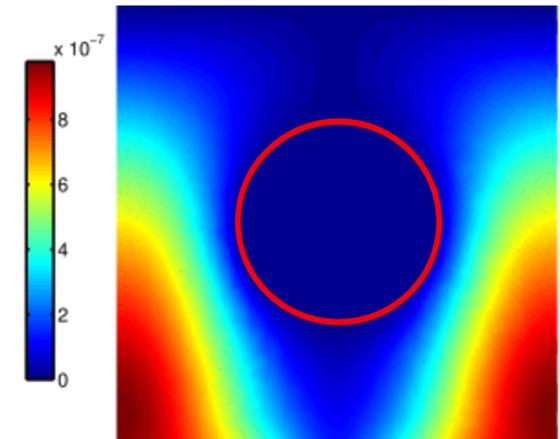**Reflection coefficients (inside f^q) are finally obtained by completely solving:**

$$\mathcal{M}^q \, f^q = h^i A_q \delta_q + \sum_{n\in\mathbb{Z}} h_{qn}^P \, C_n^P + \sum_{n\in\mathbb{Z}} h_{qn}^A \, C_n^A + \sum_{n\in\mathbb{Z}} h_{qn}^S \, C_n^S$$

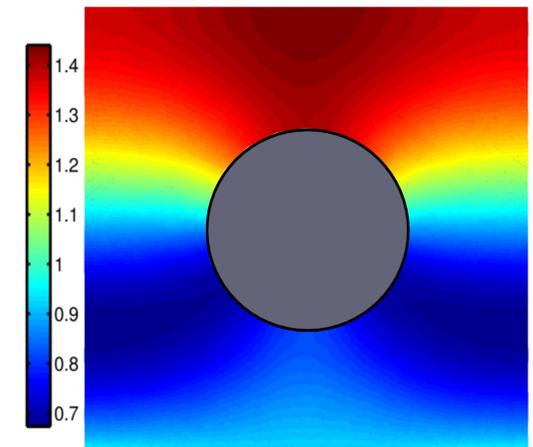- **Model cross validation (PVC inclusions)**

$$\gamma^i = \pi/2$$

Labels on the absorption-coefficient plot:
- Bragg interference
- Trap mode
- so-called Biot peak
- f_c

Legend:
- Without inclusion
- With inclusions
- × Validation PLANES (FEM)
- With inclusions (Equivalent Fluid)

Axes: Absorption coefficient vs Frequency (Hz)

Geometry inset: 55 mm × 55 mm, inclusion 25 mm

$|u^s|$ at $f = 850$ Hz

$|p|$ at $f = 1450$ Hz

- ## Parameter study: influence of filling fraction

$r = 12.5$ mm

thermal losses

viscous losses

so-called Biot peak

Without inclusion
r=2.5 mm
r=7.5 mm
r=12.5 mm
r=17.5 mm

55 mm

55 mm

$2r$

$r = 17.5$ mm

thermal losses

viscous losses

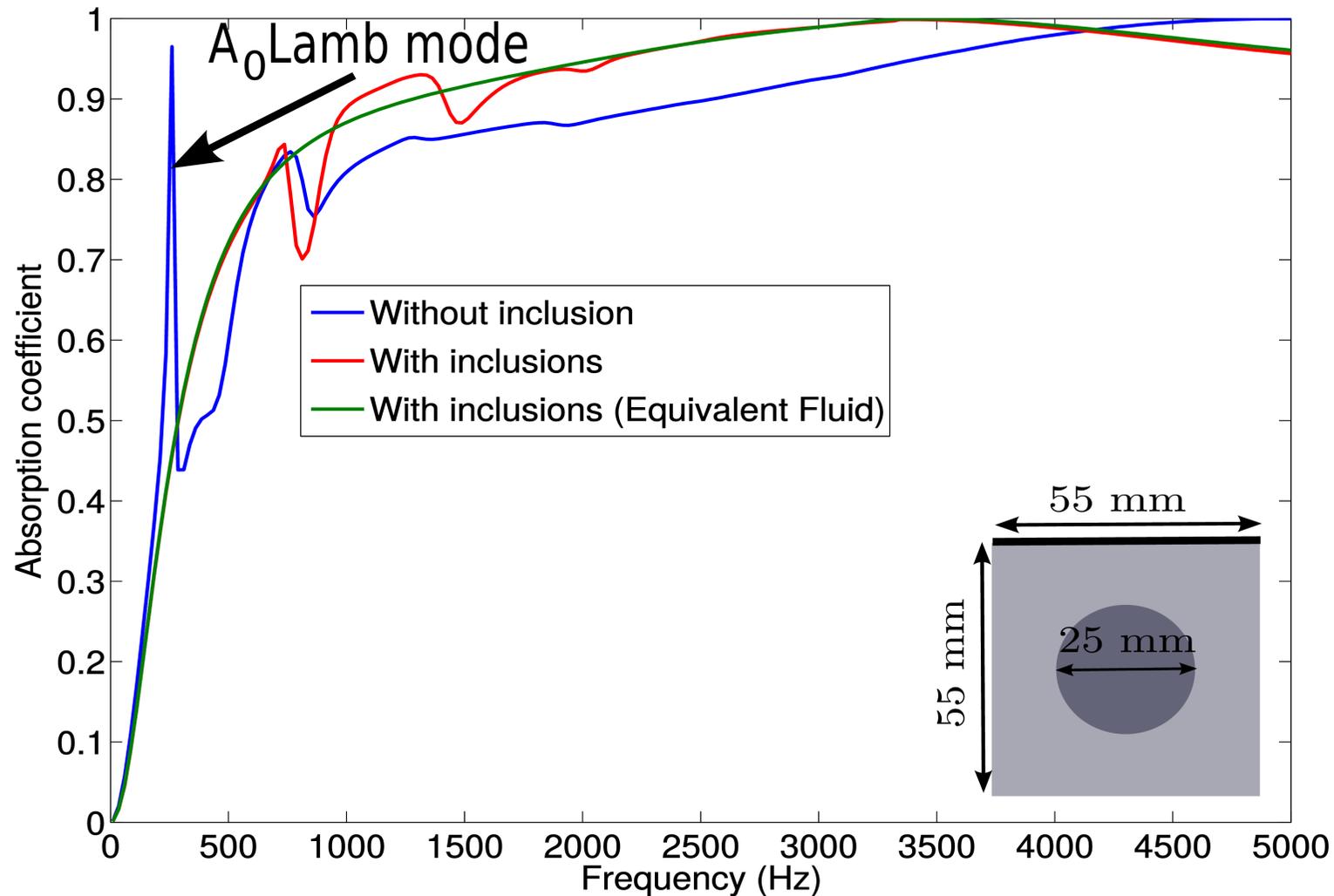LAUM   EMBRAER
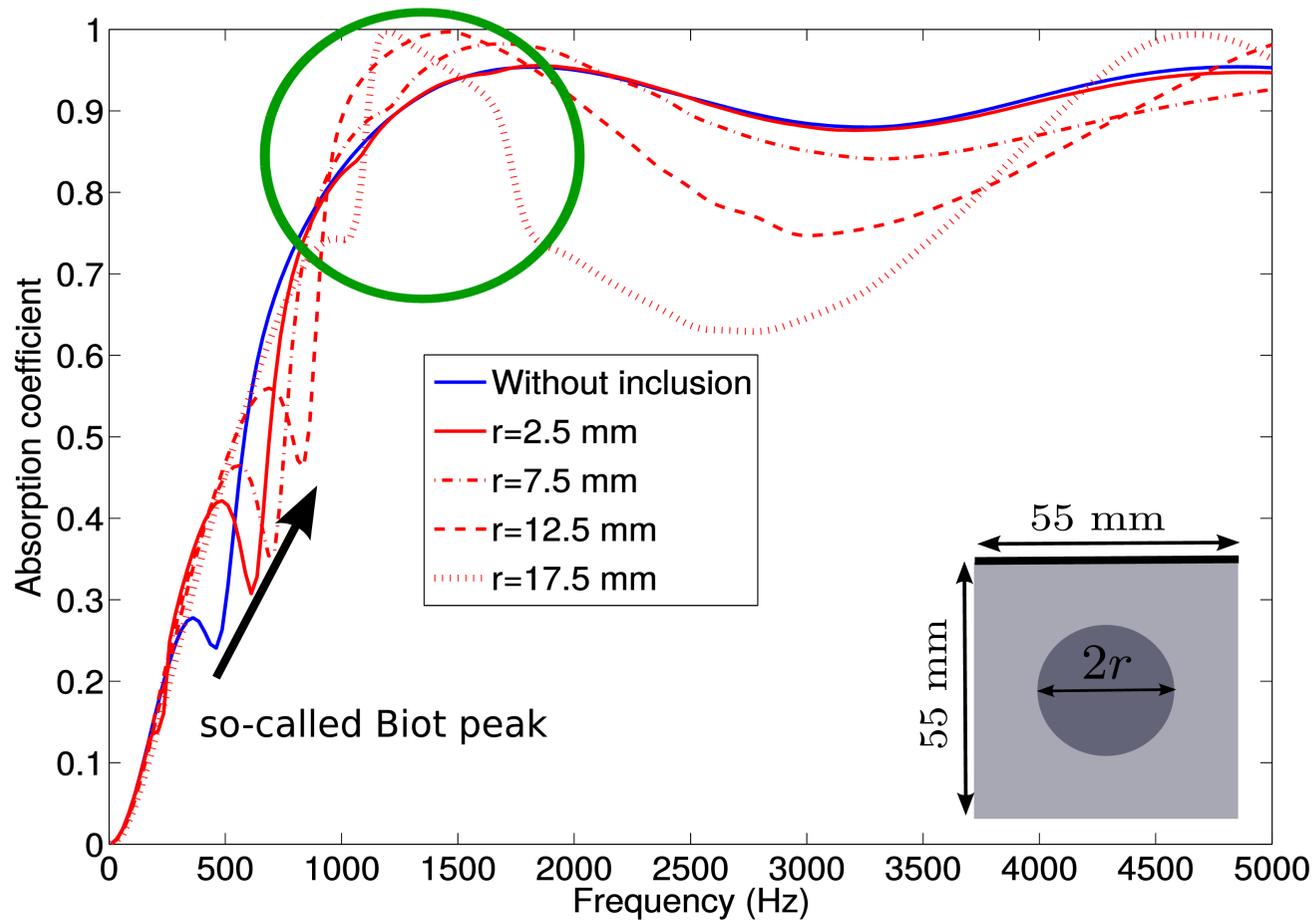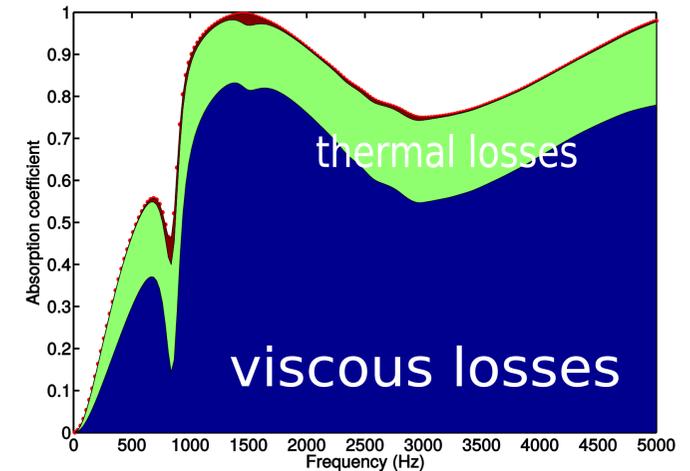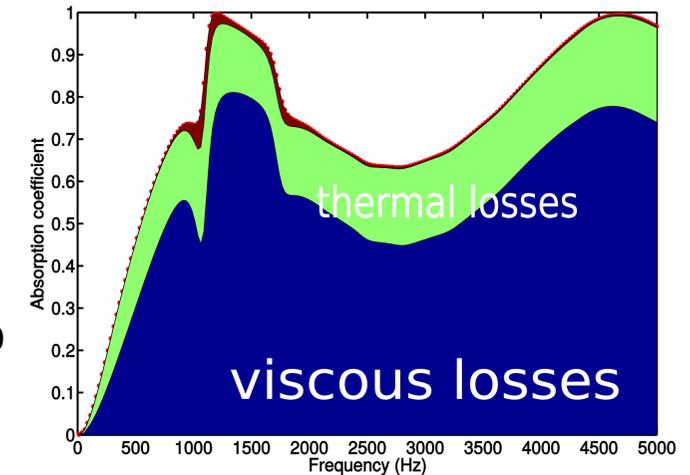
- Influence of the angle of incidence

$$\gamma^i = \pi/3$$

Conclusion and perspectives

Account for the skeleton motion of metaporous materials:

- Cross validation of the semi-analytical model with PLANES

- Inclusions tends to rigidify the structure
  - Shift of the so-called Biot peak to higher frequencies
  - Difficulty to excite Lamb mode

- Metaporous materials are more efficient to make a structure thinner, but...
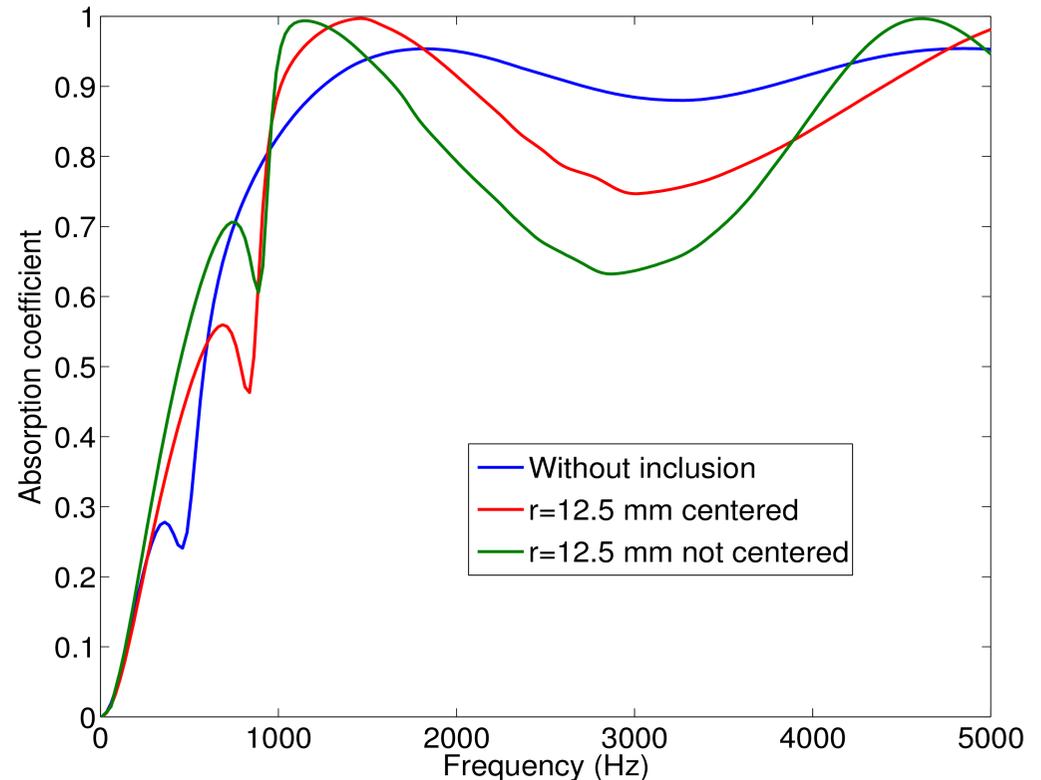
Further investigate the mode of the structure and the way to excite them

Investigate the inclusion mode in order to enhance absorption at low frequencies

Thank you for your attention!