Simulation of Sound and Vibrations in Porous Media using COMSOL Multiphysics

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Why numerical simulations?

- You can solve your equation analytically in an infinite medium / on the unit square, circle, etc...

$$U(r,t) = 1 - 2\sum_{n=1}^{\infty} \frac{J_0(\frac{\alpha_n r}{a})}{\alpha_n J_1(\alpha_n)} e^{-\alpha^2}$$

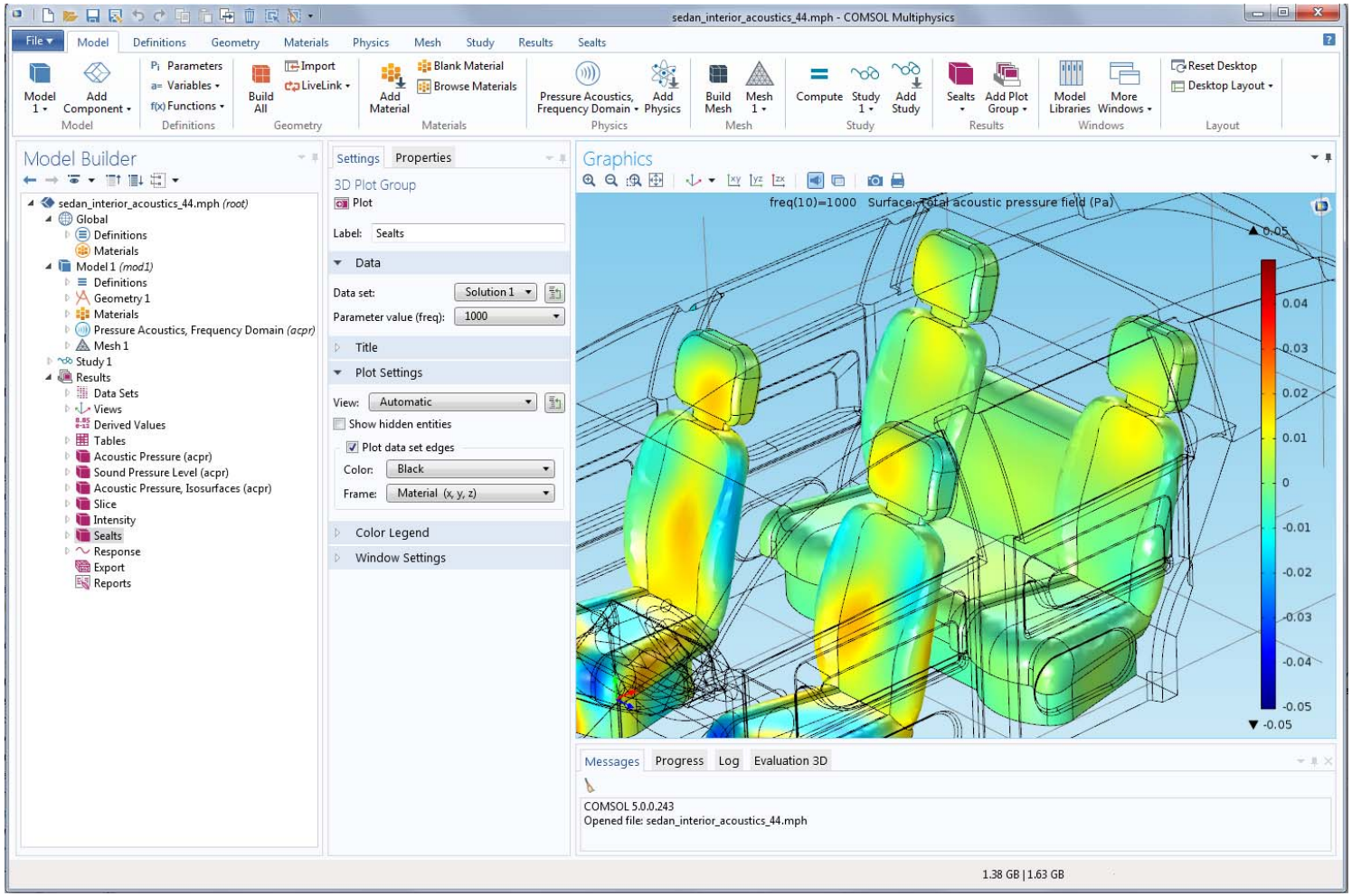
Why numerical simulations?

- But what about here?

$U(x, y, z, t) =$ I don't know!

Why COMSOL?

- Pressure Acoustics
- Poroacoustics (equivalent porous models)
- Poroeleastic Waves
- Your own arbitrary equation
  - Specify anything in terms of anything
  - Visualize anything, extract arbitrary results
  - Nonlocal model couplings


sedan_interior_acoustics_44.mph - COMSOL Multiphysics
File
Model
Definitions
Geometry
Materials
Physics
Mesh
Study
Results
Sealts
Model 1
Add Component
Parameters
Variables
Functions
Build All
Import
LiveLink
Add Material
Blank Material
Browse Materials
Pressure Acoustics, Frequency Domain
Add Physics
Build Mesh
Mesh 1
Compute
Study 1
Add Study
Sealts
Add Plot Group
Model Libraries
More Windows
Reset Desktop
Desktop Layout
Model Builder
sedan_interior_acoustics_44.mph (root)
Global
Definitions
Materials
Model 1 (mod1)
Definitions
Geometry 1
Materials
Pressure Acoustics, Frequency Domain (acpr)
Mesh 1
Study 1
Results
Data Sets
Views
Derived Values
Tables
Acoustic Pressure (acpr)
Sound Pressure Level (acpr)
Acoustic Pressure, Isosurfaces (acpr)
Slice
Intensity
Sealts
Response
Export
Reports
Settings
Properties
3D Plot Group
Plot
Label: Sealts
Data
Data set: Solution 1
Parameter value (freq): 1000
Title
Plot Settings
View: Automatic
Show hidden entities
Plot data set edges
Color: Black
Frame: Material (x, y, z)
Color Legend
Window Settings
Graphics
freq(10)=1000 Surface: Total acoustic pressure field (Pa)
▲ 0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
▼ -0.05
Messages
Progress
Log
Evaluation 3D
COMSOL 5.0.0.243
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1.38 GB | 1.63 GB

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Pressure Acoustics

- Acoustic wave equation valid for non-viscous fluids, solving for acoustic pressure *p*

$$\nabla \cdot -\frac{1}{\rho_c}\left(\nabla p_t - \mathbf{q}_d\right) - \frac{k_{eq}^2 p_t}{\rho_c} = Q_m$$

$$p_t = p + p_b$$

$$k_{eq}^2 = \left(\frac{\omega}{c_c}\right)^2$$

- Use different definitions of ρ_c, c_c to represent porous media

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Example: Delany-Bazley-Miki

$$\nabla \cdot -\frac{1}{\rho_c}\left(\nabla p_t - \mathbf{q}_d\right) - \frac{k_{eq}^2 p_t}{\rho_c} = Q_m$$

$$p_t = p + p_b$$

$$k_{eq}^2 = \left(\frac{\omega}{c_c}\right)^2$$

$$c_c = \frac{c}{\left(1 + C_1\left(\rho_f \frac{f}{R_f}\right)^{-C_2} - iC_3\left(\rho_f \frac{f}{R_f}\right)^{-C_4}\right)}$$

$$\rho_c = \frac{\rho_f c}{c_c}\left(1 + C_5\left(\rho_f \frac{f}{R_f}\right)^{-C_6} - iC_7\left(\rho_f \frac{f}{R_f}\right)^{-C_8}\right)$$

$$\epsilon_p \approx 1, \quad 10^3 < R_f < 50 \cdot 10^3 \, \mathrm{Pa \cdot s/m^2}$$

$$0.01 < \frac{\rho_f \cdot f}{R_f} < 1$$

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Other poroacoustics models

- Delany-Bazley-Miki
- Zwikker-Kosten
- Attenborough
- Wilson
- Johnson-Champoux-Allard
- Johnson-Champoux-Allard-Lafarge
- Johnson-Champoux-Allard-Pride-Lafarge

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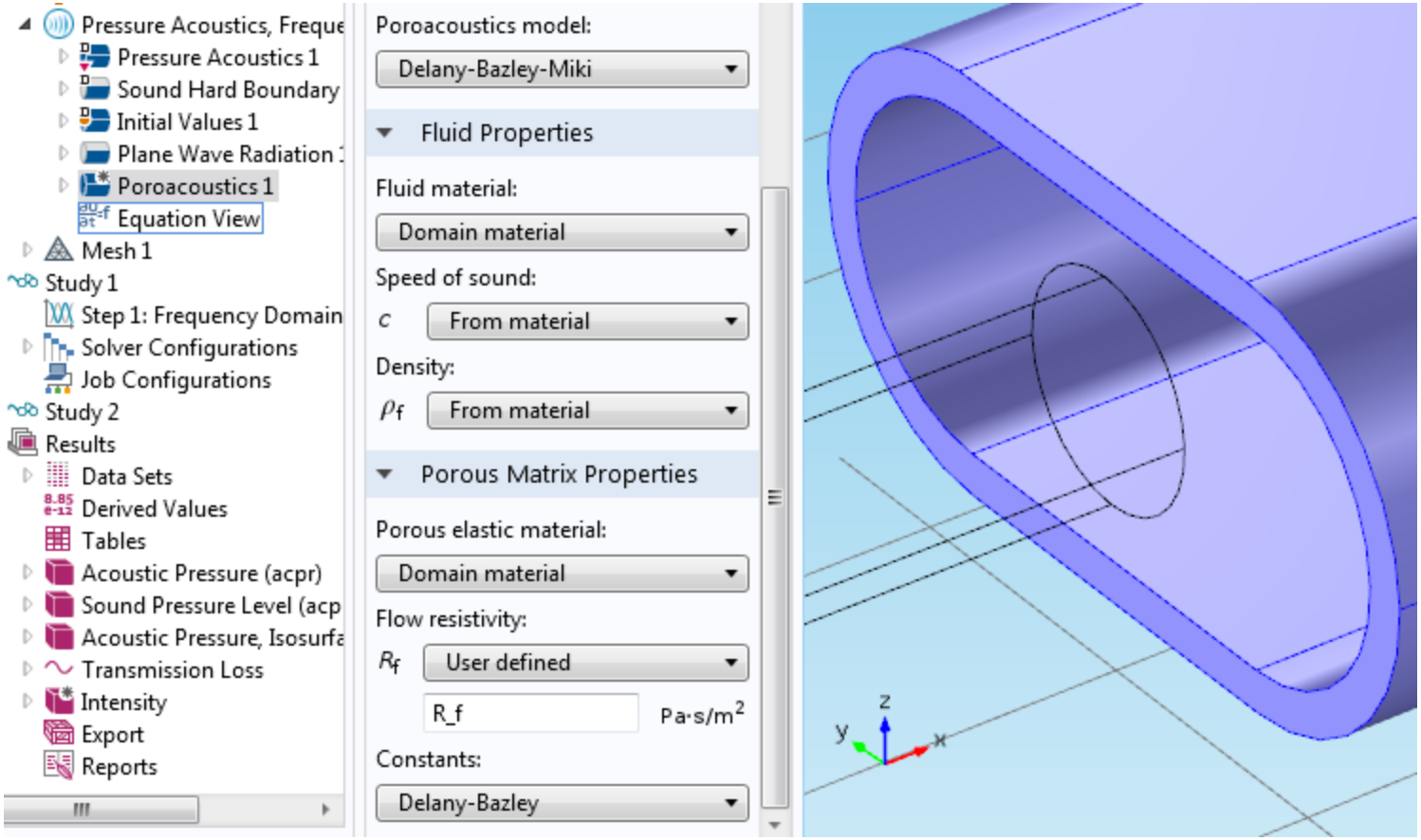
Poroacoustics in the GUI



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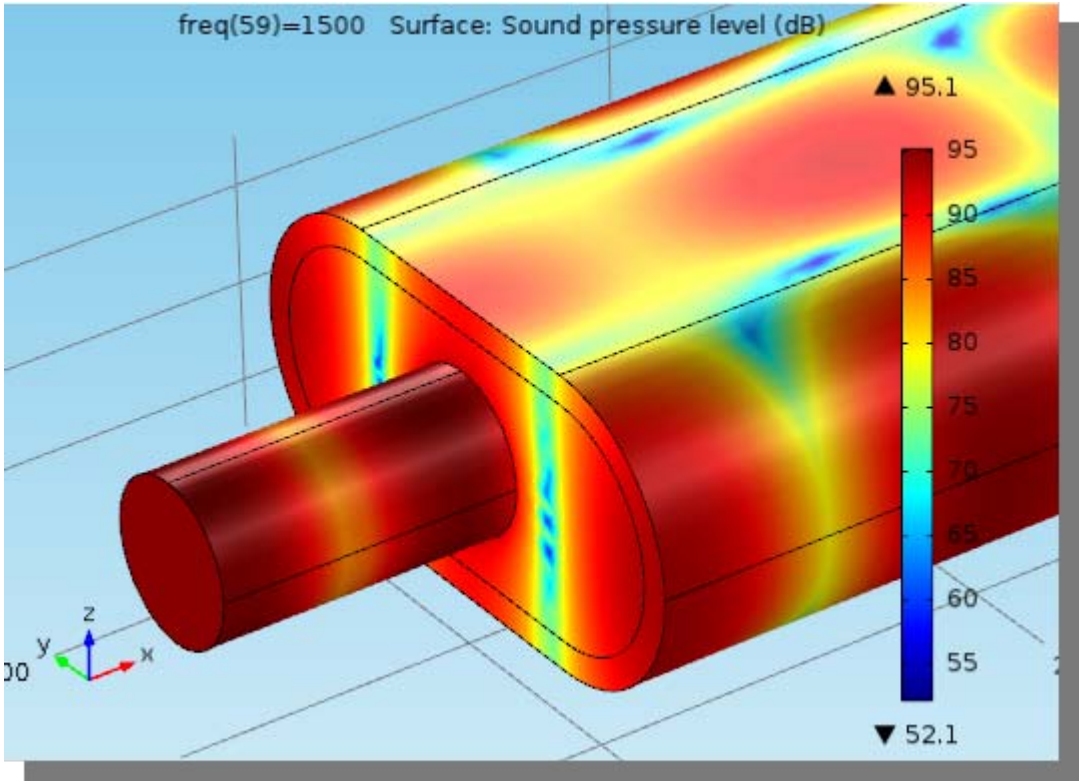
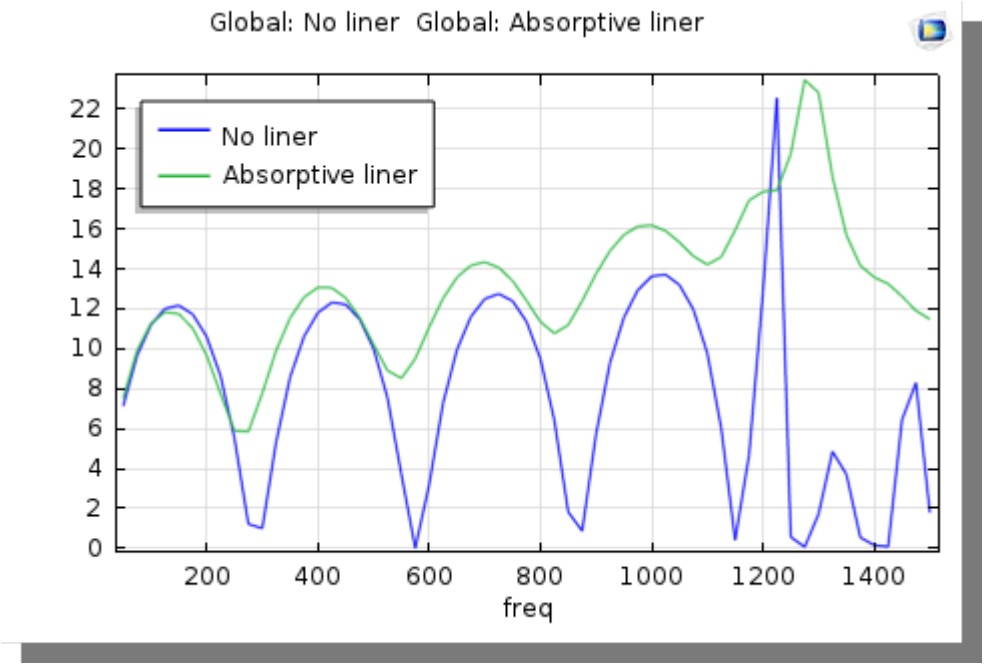
Results


freq(59)=1500 Surface: Sound pressure level (dB)
▲ 95.1
95
90
85
80
75
70
65
60
55
▼ 52.1
z
y
x

Results

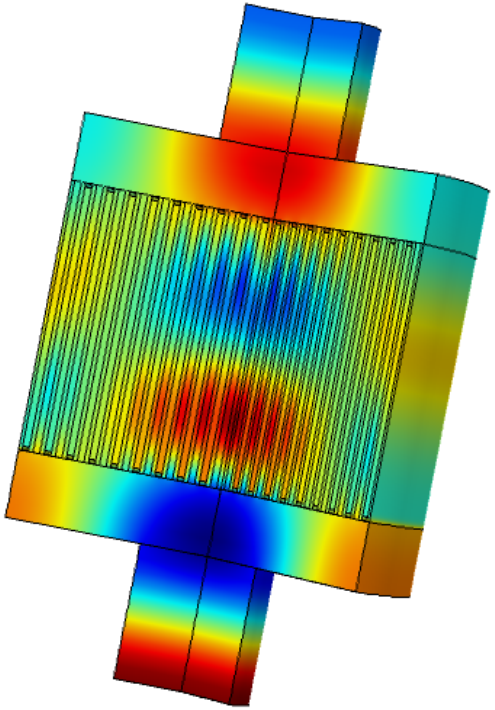

Global: No liner Global: Absorptive liner
No liner
Absorptive liner
22
20
18
16
14
12
10
8
6
4
2
0
200
400
600
800
1000
1200
1400
freq

Poroelastic Waves

- Solve both for deformations in solid phase and pressure in fluid phase
- Higher memory consumption
- Longer simulation time
- More material parameters needed
- But highly accurate results!

Poroelastic Waves

- Precise modeling of acoustics in porous materials
- Predefined multiphysics couplings:
  - Acoustic-Porous Boundary for coupling porous matrix displacement and pressure of saturating fluid to pressure in fluid.
  - Porous-Structure Boundary for coupling porous matrix displacement to solids
- Based on Biot theory
  - Fast and slow compressional waves
  - Shear waves



Acoustics of a Particulate-Filter-Like System model from the Model Library. Pressure distribution at f = 1700 Hz

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Poroelastic Waves Equation

- Material parameters:
  - Drained bulk modulus
  - Drained shear modulu
  - Drained density
  - Porosity
  - Permeability
  - Biot-Willis coefficient
  - Tortuosity factor

$$-\left(\rho_{av} - \frac{\rho_f^2}{\rho_c}\right)\omega^2\mathbf{u} - \nabla\cdot S = \mathbf{F}_V e^{i\phi} + \frac{\rho_f}{\rho_c}\nabla p$$

$$\rho_{av} = \rho_d + \epsilon_p\rho_f, \quad \rho_c = \frac{\tau_\infty \rho_f}{\epsilon_p} + \frac{\tilde{\mu}_f}{i\omega\kappa_p}, \quad \tilde{\mu}_f = \mu_f$$

$$S - S_0 = \mathbf{C} : (\epsilon - \epsilon_0 - \epsilon_{inel}) - \alpha_B p\mathbf{I}$$

$$\epsilon = \frac{1}{2}\left[(\nabla\mathbf{u})^T + \nabla\mathbf{u}\right]$$

$$\nabla\cdot -\frac{1}{\rho_c}\left(\nabla p - \omega^2\rho_f\mathbf{u}\right) - \frac{k_{eq}^2 p}{\rho_c} = \omega^2\alpha_B\nabla\cdot\mathbf{u}$$

$$k_{eq}^2 = \left(\frac{\omega^2}{M}\right)\rho_c, \quad \frac{1}{M} = \epsilon_p\chi_f + \frac{\alpha_B - \epsilon_p}{K}(1 - \alpha_B)$$

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Your Own Equations

- Set up arbitrary systems of PDEs
- Use COMSOL's geometry, mesh, solvers

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (-c\nabla \mathbf{u} - \alpha \mathbf{u} + \gamma) + \beta \cdot \nabla \mathbf{u} + a\mathbf{u} = f$$

$$\mathbf{u} = [u1, u2, u3]^T$$

$$\nabla = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}]$$

- Find others' work at www.comsol.com/papers

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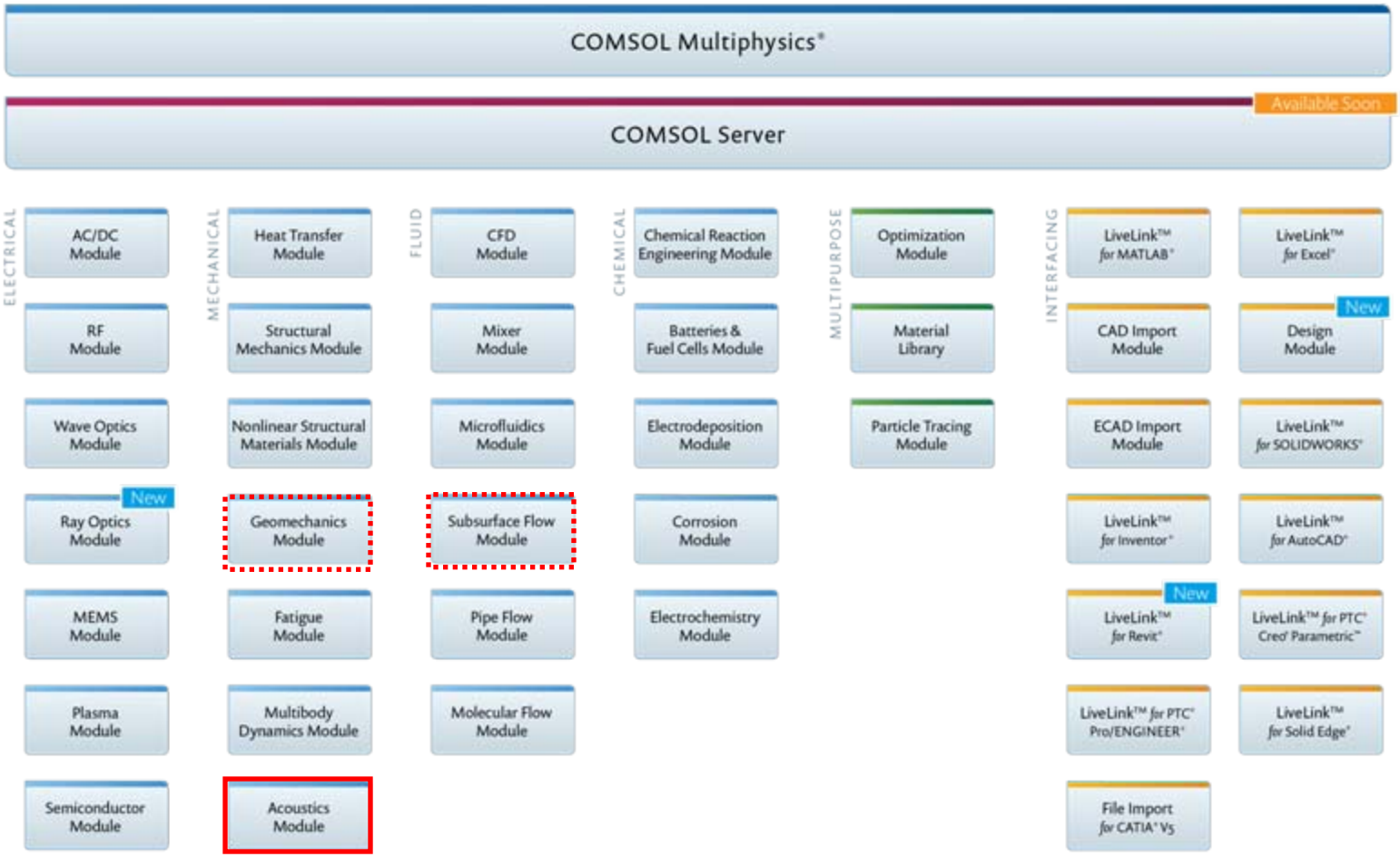
Product Suite – COMSOL 5.0



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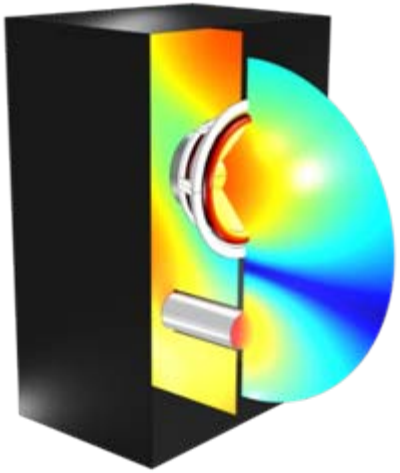
More Acoustics in COMSOL

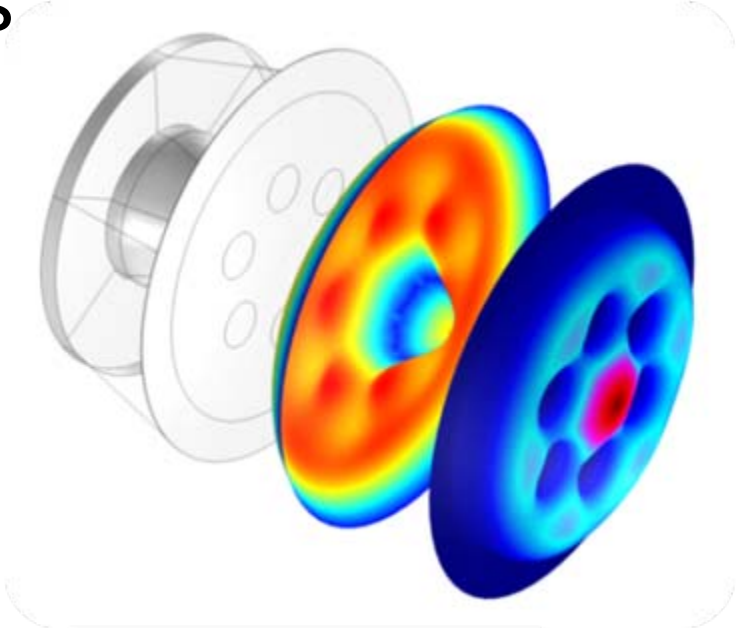
- Electroacoustics
  - Speakers



SPL in and outside a vented loudspeaker cabinet.

More Acoustics in COMSOL

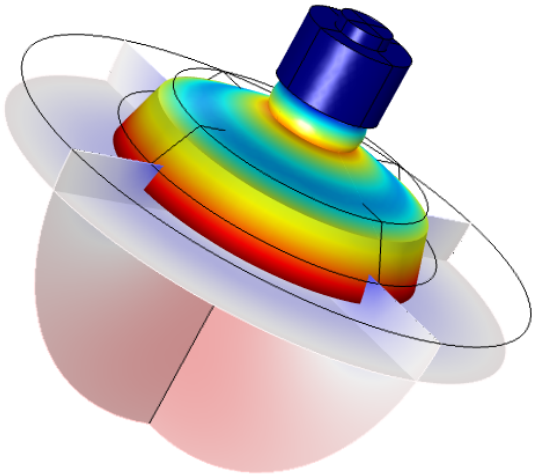
- Electroacoustics
  - Speakers
  - Microphones



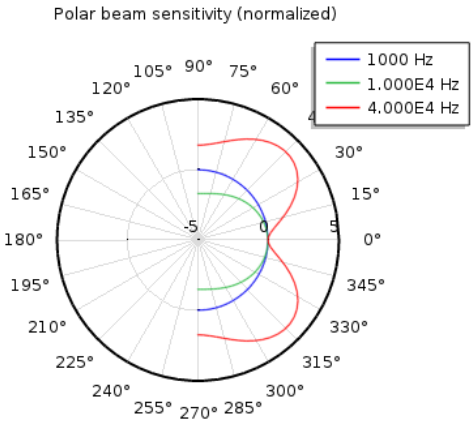
Brüel and Kjær type 4134 condenser microphone. Model courtesy: Brüel and Kjær.

More Acoustics in COMSOL

- Electroacoustics
  - Speakers
  - Microphones
  - Transducers



Tonpilz Piezoelectric transducer



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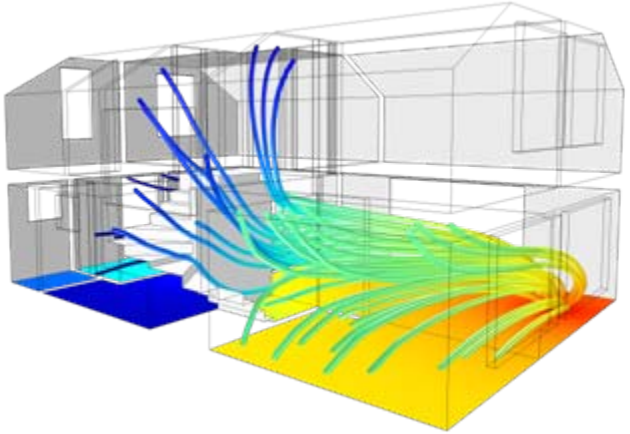
More Acoustics in COMSOL

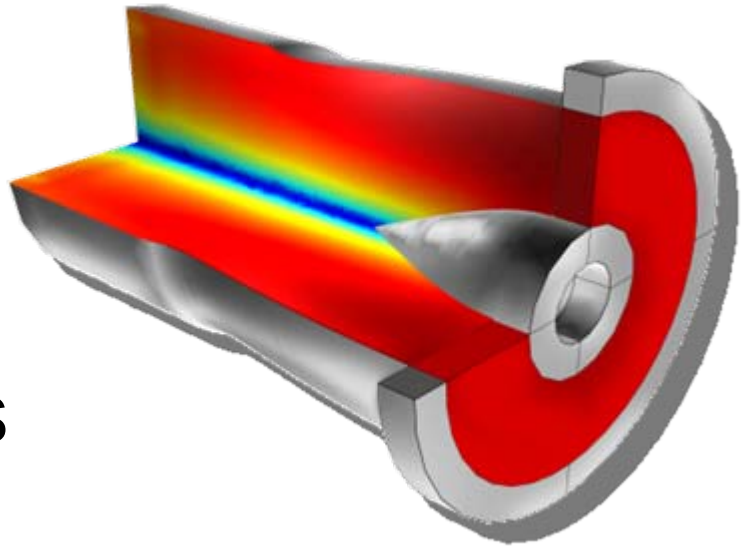
- Electroacoustics
  - Speakers
  - Microphones
  - Transducers
- Geometrical acoustics



Acoustic diffusion inside a two story single-family home

More Acoustics in COMSOL

- Electroacoustics
  - Speakers
  - Microphones
  - Transducers
- Geometrical acoustics
- Aeroacoustics



Noise emitted from a jet engine

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More Acoustics in COMSOL

- Electroacoustics
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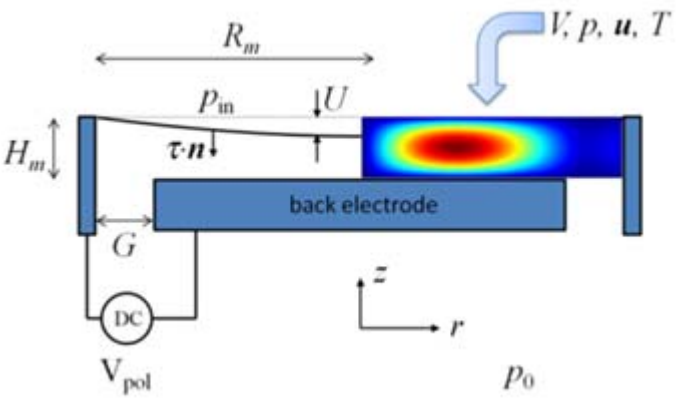
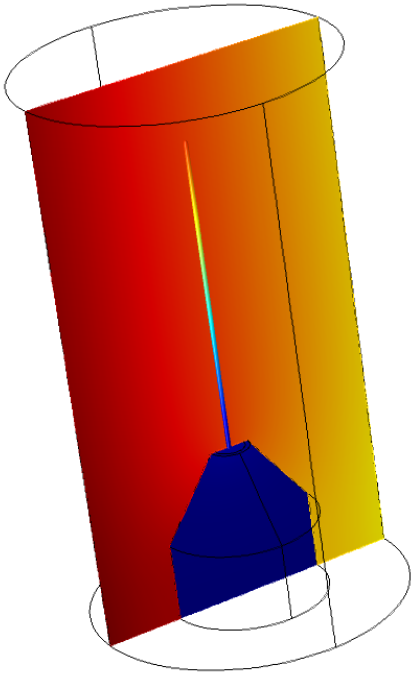
  - Microphones
  - Transducers
- Geometrical acoustics
- Aeroacoustics
- Thermoacoustics



Condenser microphone model

More Acoustics in COMSOL

- Electroacoustics
  - Speakers
  - Microphones
  - Transducers
- Geometrical acoustics
- Aeroacoustics
- Thermoacoustics
- Pipe Acoustics



Probe Tube Microphone, modeled part in 1D pipe, part in 3D volume

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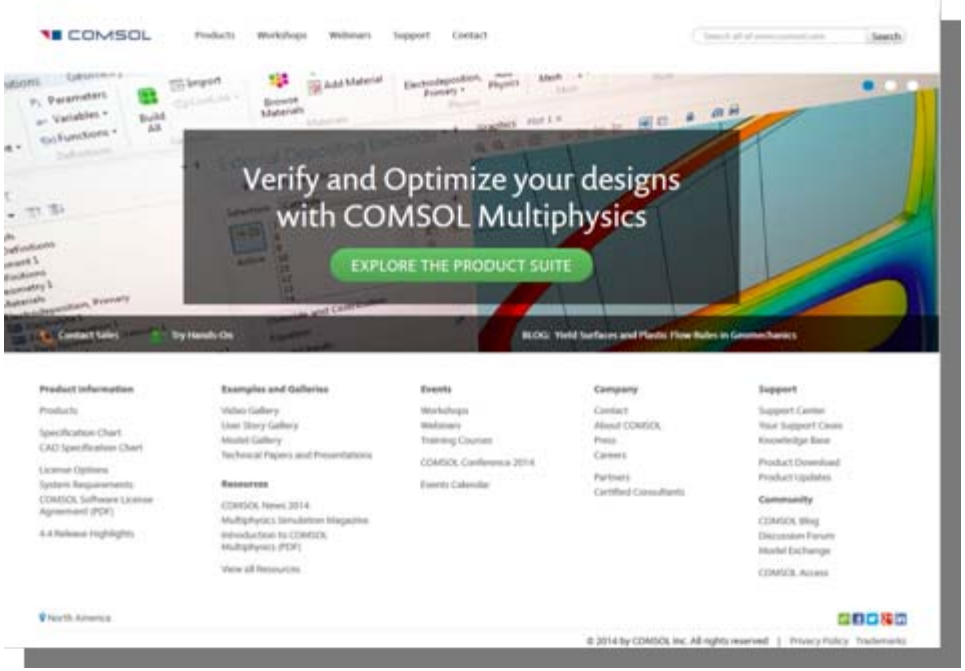
  - User Community Forum
  - Models



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