SYMPOSIUM ON THE ACOUSTICS OF PORO-ELASTIC MATERIALS
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Numerical Investigation of the Different Dissipation Mechanisms within a Typical Multilayered Trim Panel

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Context & objectives

- Why investigate damping mechanisms in Porous materials?
  - Porous Materials have multiple energy dissipation mechanisms, some specific to their biphasic nature
  - These dissipation mechanisms allows to add a lot of damping for little added mass, making porous trim materials an excellent choice for limiting sound and vibration
  - Asserting the relative importance of the different damping mechanisms of a trim is a powerful tool to improve Trim design and help choosing better porous materials for specific application
- Biot model is extensively used for modelling vibro acoustic behavior of isotropic porous materials, and a numerically attractive u-p formulations is available in multiple FE codes
  - Reduced CPU cost compared to u-w formulation
  - Coupling conditions with solids, acoustic fluids and other porous u-p easily implemented for FE
  - Anisotropic formulations available
- Is it possible to easily evaluate energy dissipation(s) with this model?

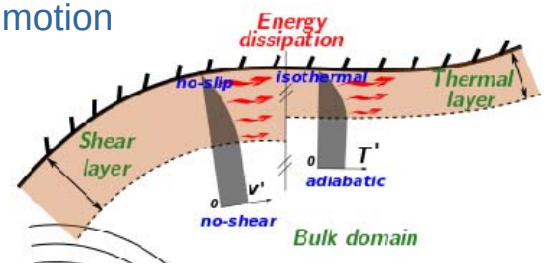
Reference: « Theory of propagation of elastic waves in a fluid-saturated porous solid », M.A. Biot, JASA, 28(2), 1956.

Summary

- Context & objectives
- Biphasic Porous materials: 3 energy dissipation mechanisms
- (Anistropic) Biot theory and FE formulation
- Energy dissipation evaluation from the Weak form
- Simple Numerical application
- Conclusion & perspectives

Basic assumptions and energy dissipation mechanisms

- Basic Biot assumptions:
  - Heterogeneous porous medium is treated as an equivalent homogeneous medium
    - Solid phase volume fraction : $1-\Omega$ Fluid phase volume fraction: Ω
    - Elastic stresses and pressure are defined at every points
    - Wavelengths λ are much larger than any microstructure of the porous medium
  - Closed pores are considered part of the skeleton (fluid motion effects within closed pores are neglected)
  - Displacement and pressure fluctuation are small so a linear theory apply
- Three mechanisms for energy dissipation
  - « Solid »-like:
    - Friction between fibers and internal losses (elastic hysteresis)
    - Would also happen in vacuum
  - Visco-Thermal dissipation in fluid:
    - Thermal exchange between fluid and skeleton
      - Modeled as fluid pressure-density hysteresis
      - « Fluid »-like when the skeleton can be assumed isothermal or non-conducting
    - Viscous forces damping fluid-skeleton RELATIVE motion
      - No damping when fluid and skeleton move identically



Derivation of [aniso] biot model (1)

- Momentum equation for the fluid and skeleton

$$\partial_j \sigma_{ij}^S = (1-\Omega)\rho_S \ddot{u}_i - f_i^{FS} \quad (1)$$

$$\partial_j \sigma_{ij}^F = -\Omega \partial_i p = \Omega \rho_F \ddot{U}_i - f_i^{SF} \quad (2)$$

- Interaction forces between fluid and skeleton come from inertia coupling, viscous coupling, both possibly anisotropic

$$f_i^{FS} = -f_i^{SF} = \rho_{ij}^{12}\left(\ddot{u}_j - \ddot{U}_j\right) - \Omega R_{ij}\left(\dot{u}_j - \dot{U}_j\right) \quad (3)$$

$$f_i^{FS} = \left[\omega^2 \rho_{ij}^{12} + i\omega\Omega R_{ij}\right]\left(U_j - u_j\right) = \omega^2 \tilde{\rho}_{ij}^{12}\left(U_j - u_j\right) \quad (3\text{ h})$$

- Introduce harmonic interaction forces (3h) into harmonic momentum equations

$$\partial_j \sigma_{ij}^S + \omega^2\left(\tilde{\rho}_{ij}^{11} u_j + \tilde{\rho}_{ij}^{12} U_j\right) = 0 \quad (4)$$

$$-\Omega \partial_i p + \omega^2\left(\tilde{\rho}_{ij}^{12} u_j + \tilde{\rho}_{ij}^{22} U_j\right) = 0 \quad (5)$$

- Express U in term of u and p using (5)

Isotropic Biot

$$U_i = u_i + \frac{\Omega}{\tilde{\rho}_{22}}\left(\frac{\partial_i p}{\omega^2} - \rho_F u_i\right)$$

$$U_i = \left(\tilde{\rho}_{ij}^{22}\right)^{-1}\left(\frac{\Omega \partial_j p}{\omega^2} - \tilde{\rho}_{jl}^{12} u_l\right) = u_i + \Omega\left(\tilde{\rho}_{ij}^{22}\right)^{-1}\left(\frac{\partial_j p}{\omega^2} - \rho_F u_j\right) \quad (6)$$

Derivation of [aniso] biot model (2)

- Elimination of U from skeleton momentum equation (4) using (6)

$$\partial_j \sigma_{ij}^S + \omega^2 \left(\tilde{\rho}_{ij}\right) u_j + \Omega \left(\tilde{\gamma}_{ij}\right) \partial_j p = 0$$

Anisotropic density and pressure coupling term

$$\tilde{\gamma}_{ij} = \tilde{\rho}_{ik}^{12} \left(\tilde{\rho}_{kj}^{22}\right)^{-1}$$

$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}^{11} - \tilde{\rho}_{ik}^{12} \left(\tilde{\rho}_{kl}^{22}\right)^{-1} \tilde{\rho}_{lj}^{12}$$

$$= \tilde{\rho}_{ij}^{11} - \tilde{\gamma}_{il} \tilde{\rho}_{lj}^{12}$$

- Material constitutive relations

$$-\Omega p = \tilde{R} \partial_i U_i + \tilde{Q} \partial_i u_i \qquad \text{(fluid)}$$

$$\sigma_{ij}^S = \tilde{Q} \delta_{ij} \partial_k U_k + \tilde{C}_{ijkl} \varepsilon_{kl} \qquad \text{(skeleton)}$$

- Inserting (fluid) into (skeleton) eliminate div(U) from skeleton constitutive equation

$$\partial_k U_k = -\frac{\Omega}{\tilde{R}} p - \frac{\tilde{Q}}{\tilde{R}} \partial_k u_k \qquad \beta = \frac{\tilde{Q}}{\tilde{R}}$$

$$\sigma_{ij}^S = \underbrace{\left(\tilde{C}_{ijkl} - \tilde{Q} \beta \delta_{ij} \delta_{kl}\right)}_{C_{ijkl}} \varepsilon_{kl} - \Omega \tilde{\beta} p \delta_{ij} = \underbrace{C_{ijkl} \varepsilon_{kl}}_{\hat{\sigma}_{ij}^S} - \Omega \beta p \delta_{ij}$$

- Skeleton stress definition

$$C_{ijkl} \varepsilon_{kl} = \hat{\sigma}_{ij}^S = \sigma_{ij}^S + \Omega \beta p \delta_{ij} \qquad (7)$$

Derivation of [aniso] biot model (3)

- Inserting total stress definition into momentum of skeleton

$$\partial_j\left(C_{ijkl}\varepsilon_{kl}\right)+\omega^2\left(\tilde{\rho}_{ij}\right)u_j+\Omega\left(-\beta\delta_{ij}+\tilde{\gamma}_{ij}\right)\partial_j p=0$$

(skeleton biot)

- Multiplying the fluid momentum equation by $\Omega\left(\tilde{\rho}_{ij}^{22}\right)^{-1}$ and taking the divergence

$$-\Omega^2\partial_i\left(\left(\tilde{\rho}_{ij}^{22}\right)^{-1}\partial_j p\right)+\omega^2\Omega\left(\partial_i\left(\tilde{\gamma}_{ij}u_j\right)+\partial_i U_j\right)=0$$

- …and replacing $\partial_i U_i$ by u, p using fluid constitutive relation…

$$-\Omega p=\tilde{R}\partial_i U_i+\tilde{Q}\partial_i u_i$$

$$\Rightarrow\quad \partial_i U_i=-\frac{\Omega}{\tilde{R}}p-\beta\partial_i u_i$$

- Gives

$$\Omega^2\partial_i\left(\left(\tilde{\rho}_{ij}^{22}\right)^{-1}\partial_j p\right)+\omega^2\frac{\Omega^2}{\tilde{R}}p+\Omega\omega^2\partial_i\left(\left(\tilde{\gamma}_{ij}-\beta\delta_{ij}\right)u_j\right)=0$$

(fluid biot)

- Symmetric weak form using green theorem and $p=i\omega\psi$

$$\int\delta\varepsilon_{ij}C_{ijkl}\varepsilon_{kl}-\omega^2\delta u_i\left(\tilde{\rho}_{ij}\right)u_j+i\omega\delta u_i\left(\tilde{c}_{ij}\right)\partial_j\psi\quad dV=0$$

$$\int\partial_i\delta\psi\left(\Omega^2\left(\tilde{\rho}_{ij}^{22}\right)^{-1}\right)\partial_j\psi-\omega^2\delta\psi\left(\frac{\Omega^2}{\tilde{R}}\right)\psi+i\omega\partial_i\delta\psi\left(\tilde{c}_{ij}\right)u_j\ dV=0$$

[Aniso] biot model – Galerkin formulation

- Using a galerkin formulation, Finite element impedance matrices reads*:

$$\sum_{elem} K_{ij}^{e,S} u_j^e - \omega^2 M_{ij}^{e,S} u_j^e + i\omega C_{ij}^e \psi_j^e = 0$$

$$\sum_{elem} K_{ij}^{e,F} \psi_j^e - \omega^2 M_{ij}^{e,F} \psi_j^e + i\omega C_{ji}^e u_j^e = 0$$

$$K_{ij}^{e,S} = \int_{V_e} B_{i,kl}^{e,S}(x) C_{klmn} B_{j,mn}^{e,S}(x)\, dV_e$$

$$M_{ij}^{e,S} = \int_{V_e} N_i^{e,S}(x) \tilde{\rho}_{(i\%3)(j\%3)} N_j^{e,S}(x)\, dV_e$$

$$C_{ij}^e = \int_{V_e} N_i^{e,S}(x) \tilde{c}_{(i\%3)k} \frac{\partial N_j^{e,F}}{\partial x_k}(x)\, dV_e$$

$$K_{ij}^{e,F} = \int_{V_e} \frac{\partial N_i^{e,F}}{\partial x_k}(x) \left(\Omega^2 \left(\tilde{\rho}_{kl}^{22} \right)^{-1} \right) \frac{\partial N_j^{e,F}}{\partial x_l}(x)\, dV_e$$

$$M_{ij}^{e,F} = \int_{V_e} N_i^{e,F}(x) \left(\frac{\Omega^2}{\tilde{R}} \right) N_i^{e,F}(x)\, dV$$

*Reference: « Horlin, Goransson: Weak, anisotropic symmetric formulations of biot's equations for vibro-acoustic modelling of porous elastic materials. », Int. J. Num. Meth. Eng., 2010.

Dissipated powers: Extracted from Galerkin formulation

- Example: the viscous dissipation*:

$$W_{viscous} = Average\left(f_i^{FS}\left(\dot{u}_i - \dot{U}_i\right)\right) = \omega^3 \operatorname{Im}\left(\left(U_i - u_i\right)^* \tilde{\rho}_{ij}^{12}\left(U_j - u_j\right)\right)$$

  - Could be evaluated directly from solid+Fluid displacements using

$$U_i - u_i = \Omega\left(\tilde{\rho}_{ij}^{22}\right)^{-1}\left(\frac{\partial_j p}{\omega^2} - \rho_F u_j\right)$$

- Can be expressed from the weak form, or directly from finite element discretization
  - This can be done for other FE materials (fluids, solids, NRBC, BC,…)
  - Leads to exact power balance for the discrete models
    - ➔ Power balance is closed even for coarse meshes

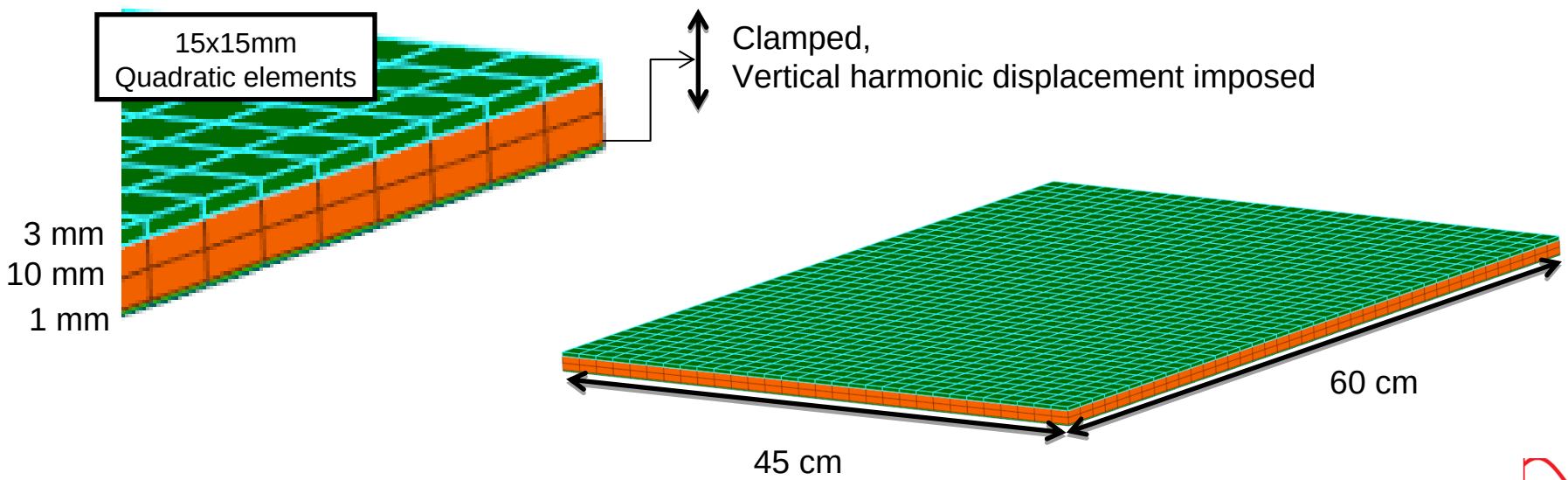
$$2W_{viscous} = \omega \operatorname{Im}\underbrace{\left(\Omega^2 \partial_i \psi^* \left(\tilde{\rho}_{ij}^{22}\right)^{-1} \partial_j \psi\right)}_{\text{Fluid mass}} - \omega^3 \operatorname{Im}\underbrace{\left(u_i^* \left(\tilde{\rho}_{ij}\right) u_j\right)}_{\text{solid mass}} + \omega^2 \operatorname{Re}\underbrace{\left(u_i^* \left(\tilde{c}_{ij}\right) \partial_j \psi + \partial_i \psi^* \left(\tilde{c}_{ij}\right) u_j\right)}_{\text{coupling}}$$

$$2W_{viscous}^{element} = \omega \operatorname{Im}\underbrace{\left(\underline{\psi}^* \underline{\underline{K}}^F \underline{\psi}\right)}_{\text{Fluid mass}} - \omega^3 \operatorname{Im}\underbrace{\left(\underline{u}^* \underline{\underline{M}}^S \underline{u}\right)}_{\text{solid mass}} + \omega^2 \operatorname{Re}\underbrace{\left(\underline{u}^* \underline{\underline{C}} \underline{\psi} + \underline{\psi}^* \underline{\underline{C}} \underline{u}\right)}_{\text{coupling}}$$

*Reference: « Dazel, Sgard, Becot, Atalla, Expressions of dissipated powers and stored energies in poroelastic media modeled by u-U and u-p formulations », JASA, 123(4), 2008.

Numerical illustration

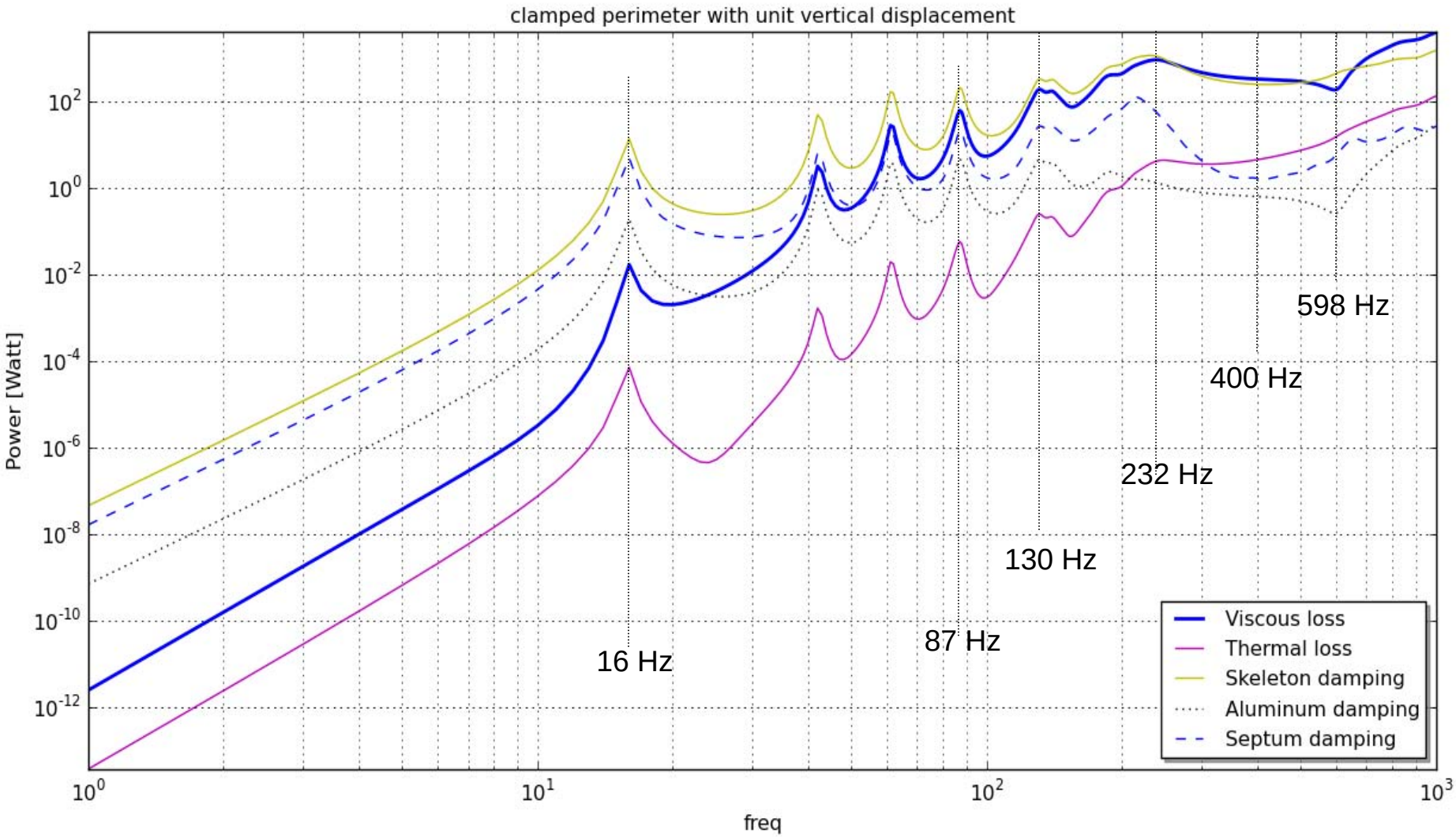
- RTC III test (structure only)
  - 60x45cm 1mm thick aluminum plate is clamped, and an harmonic vertical displacement is imposed at the clamping
    - A 1cm Porous + 3mm heavy layer trim is glued on top surface of the plate
    - Sides and top of the trim are free
  - Dissipated powers are investigated:
    - Check frequency dependence of the different dissipated powers
    - Check spatial dependence of viscous loss for specific frequencies



Material Properties….

- 1mm aluminum plate
  - Young modulus:7^{e}10+7^{e}7j
  - Poisson ratio: 0.3
  - Density: 2700
- 3mm septum (charged polymer)
  - Young modulus:7.73^{e}7+ 7.73^{e}6j
  - Poisson ratio: 0.35
  - Density: 3800
- 10mm isotropic porous (polyurethane foam)
  - Flow resistivity: 22000
  - Porosity: 0.97
  - Tortuosity: 1.38
  - Biot factor: 1.0
  - Champoux-Allard micromodel
    - Viscous length:1.7^{e}-5
    - Thermal length:4^{e}-5
  - Skeleton young modulus: 192^{e}3+24.96^{e}3j
  - Skeleton poisson ratio: 0.23
  - Skeleton bulk density: 827
  - Air:
    - Viscosity 1.82e-05
    - Cp: 1004.0, cv: 716.0
    - Thermal conductivity: 0.0256
    - P0: 101300

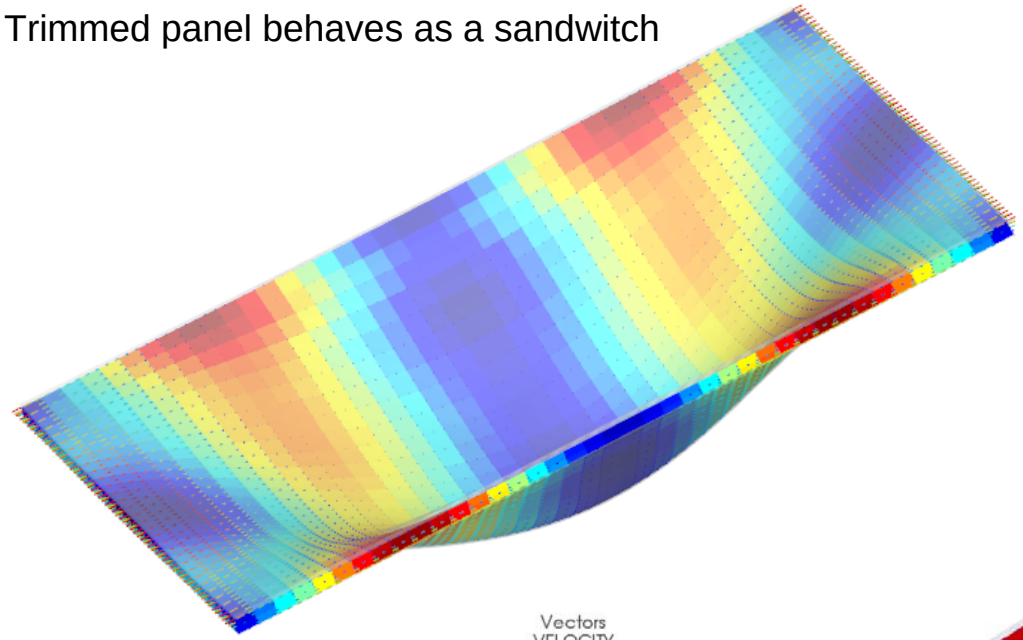
Dissipated powers



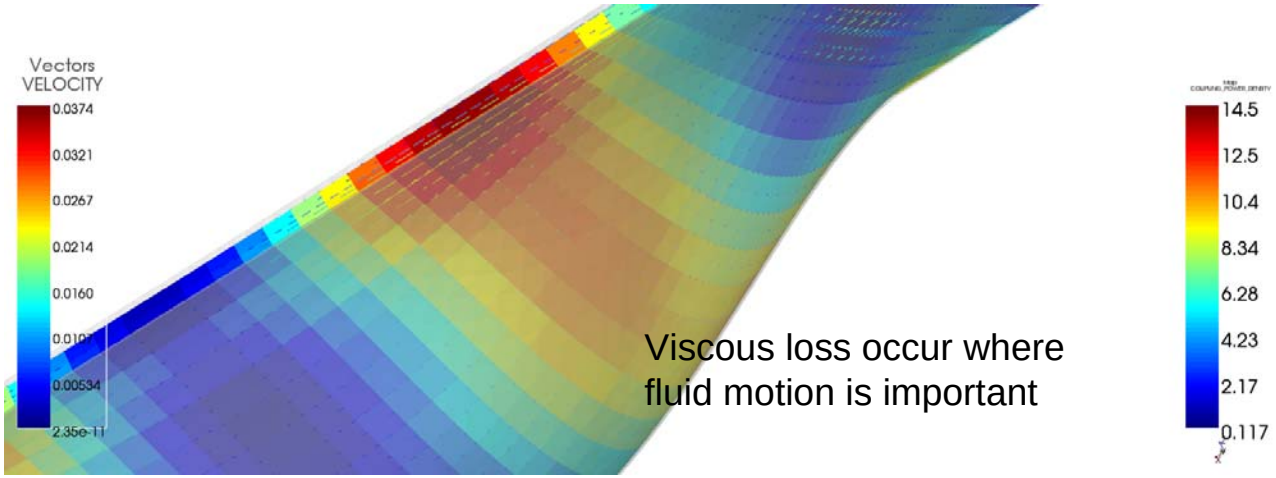
➔ Check Global deformation, fluid motion and viscous dissipated power at those freqs….

16 Hz, first sandwitch mode

Trimmed panel behaves as a sandwitch



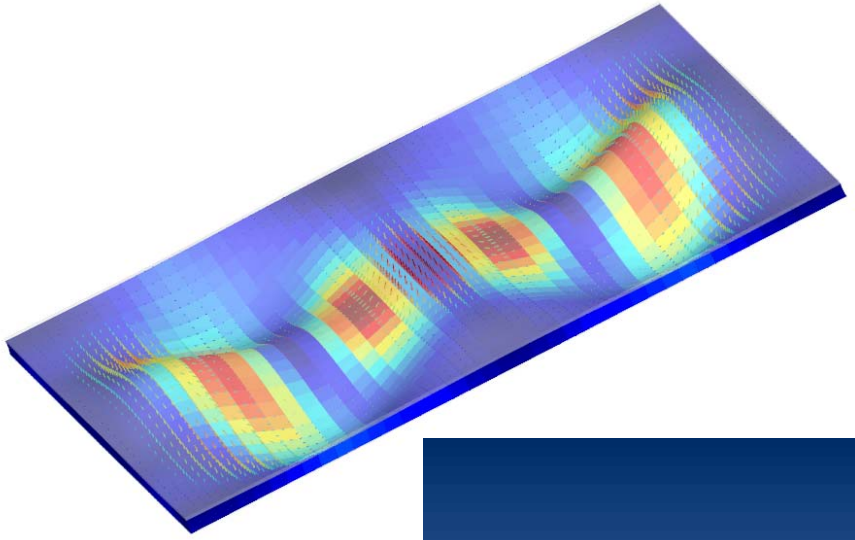
- Trim porous nature not so important
  - Structural damping key factor
  - Top layer and skeleton core



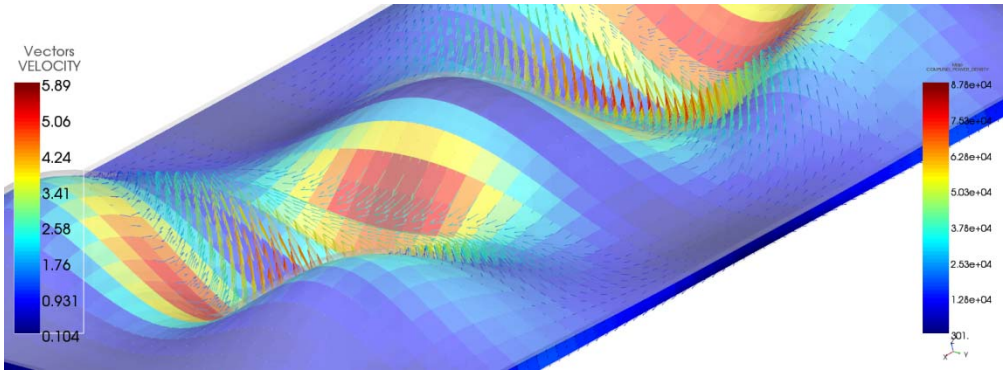
Viscous loss occur where fluid motion is important

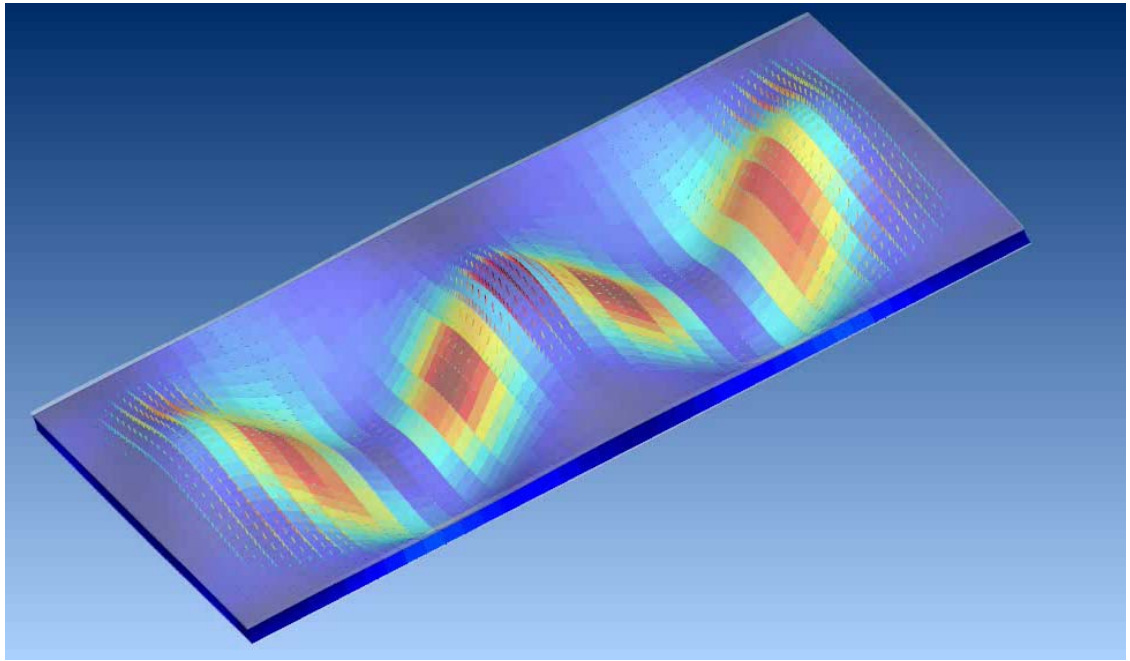
87 Hz, longitudinal sandwitch mode

Panel still behaves as a sandwitch



- Structural damping still dominates, but viscous effects increase

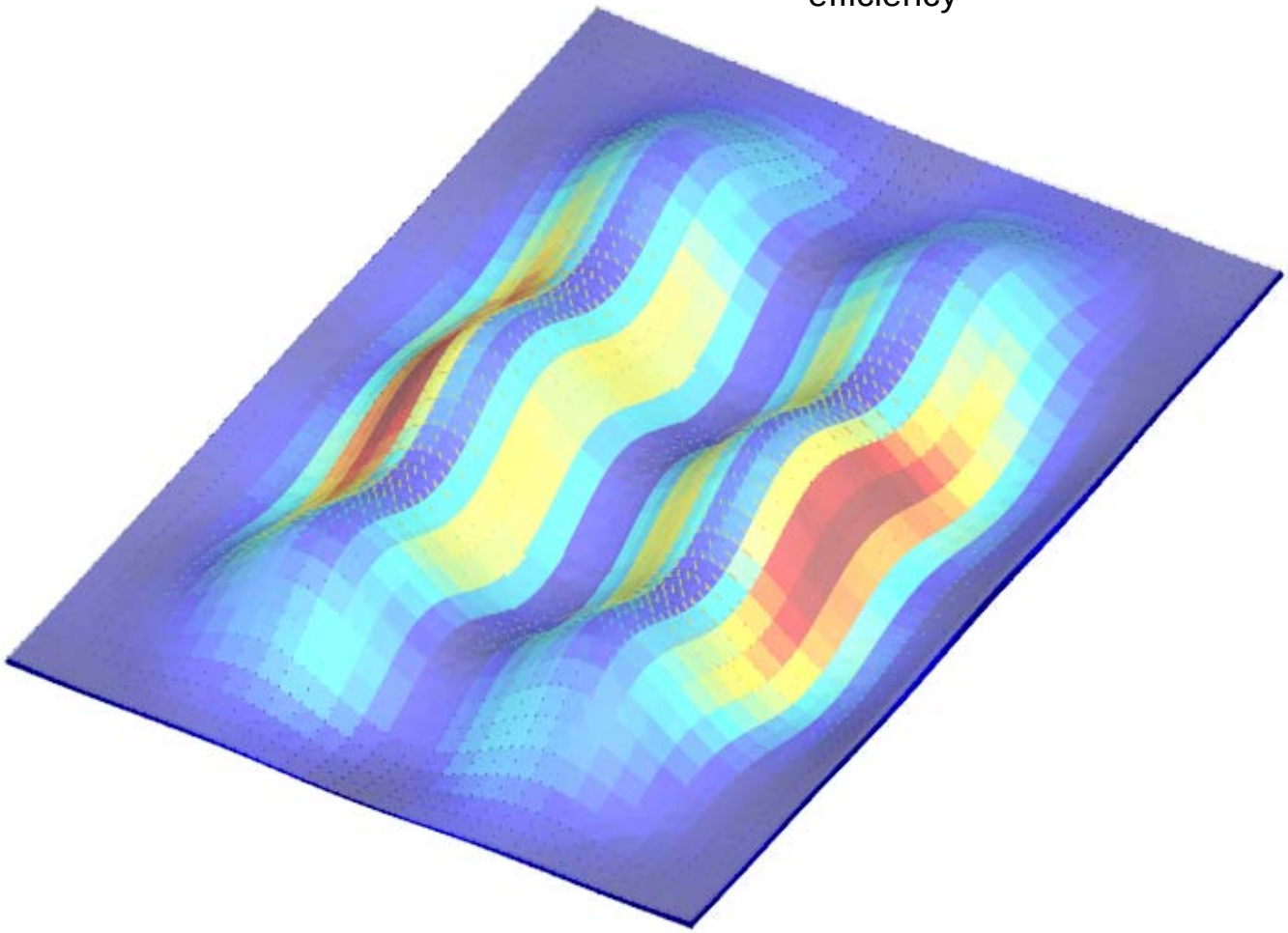




130 Hz, Transversal sandwitch mode

Damping increases, modes becomes less distinct

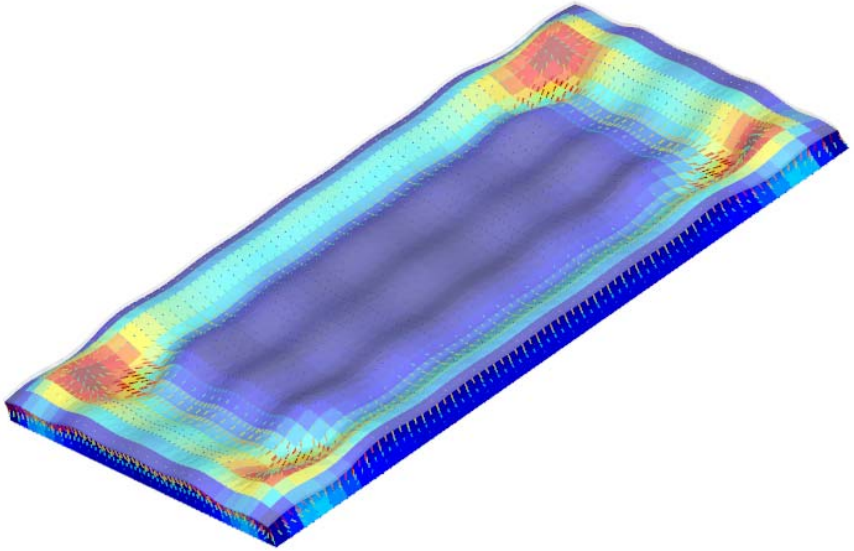
- Structural damping slightlydominates, viscous effects a close second in damping efficiency

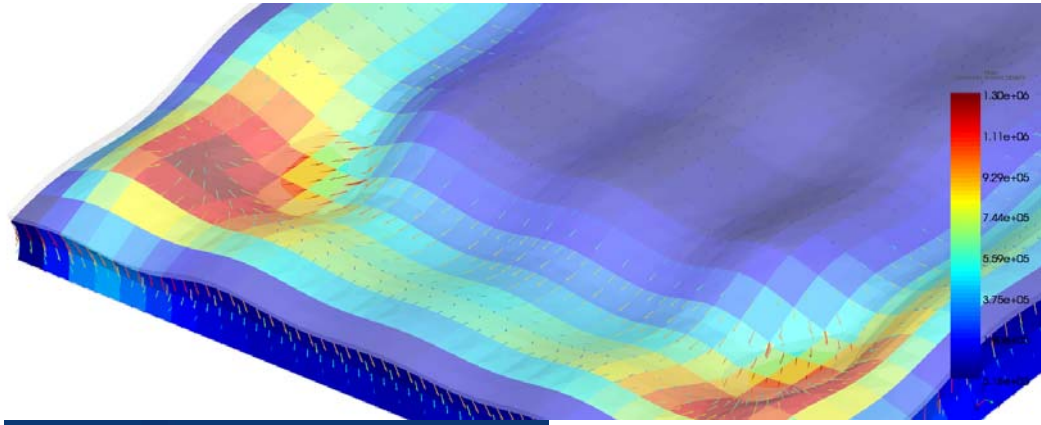


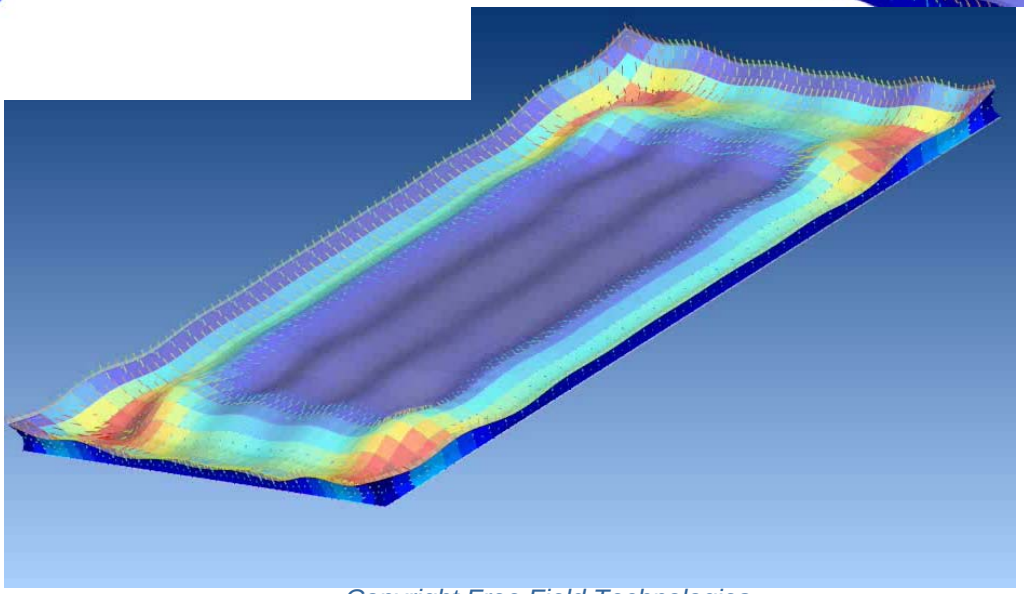
232 Hz, Top and bottom start to decouple

Heavy layer decouple from aluminum plate near the edges

- Viscous effects dominates on the edges
- Top and bottom plate still (sligthly) modal



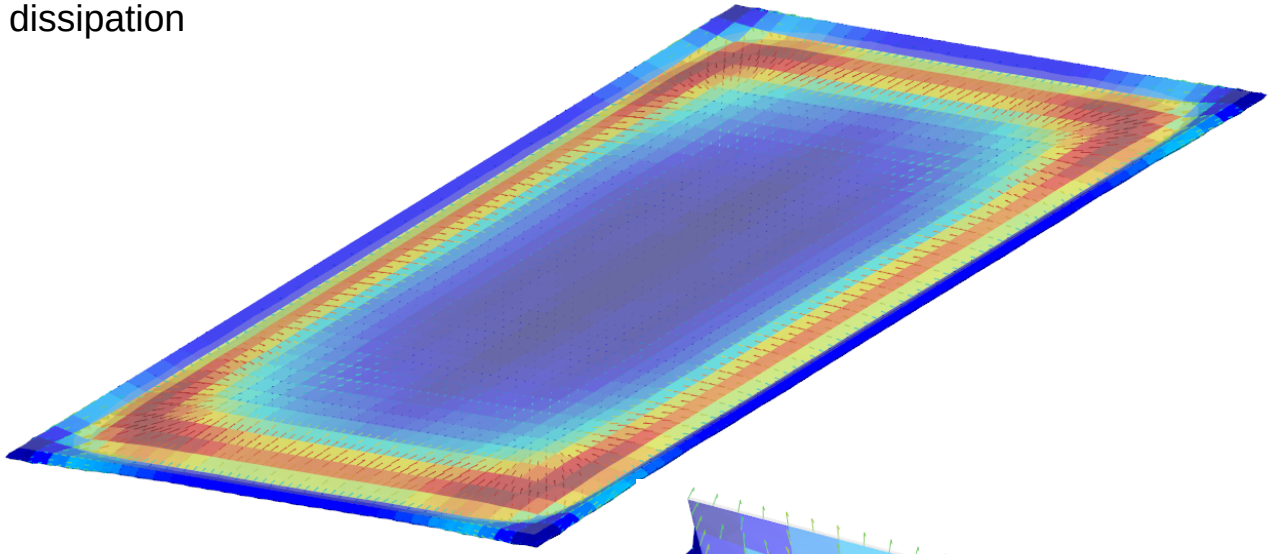


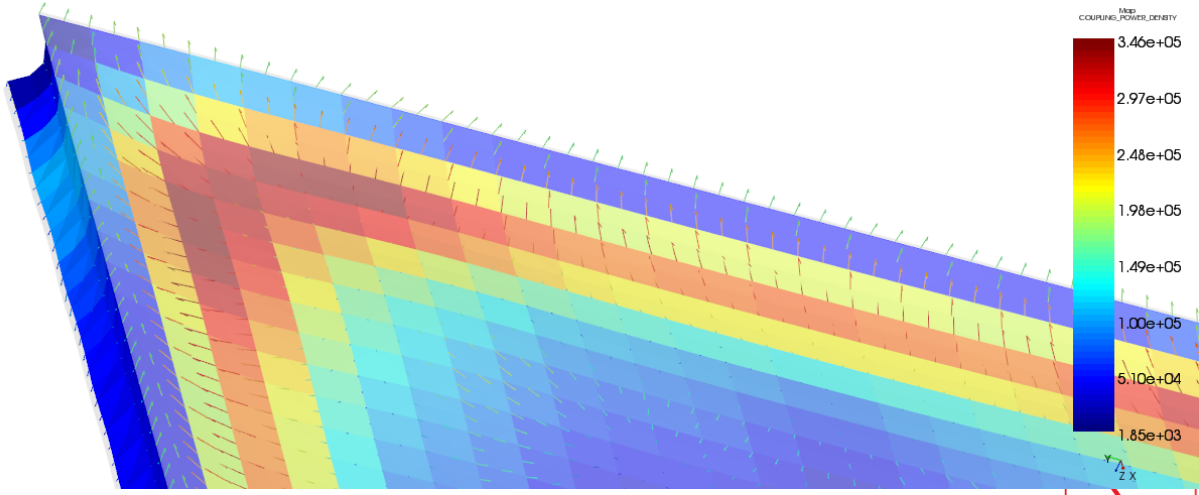


400 Hz Heavy layer decoupled from plate

- Heavy layer decouple from aluminum plate
- Free edges pumps and inject fluid-dominated wave towards the center. Characteric « ring » viscous dissipation

- Viscous effects dominates
- Top and bottom plate non modal





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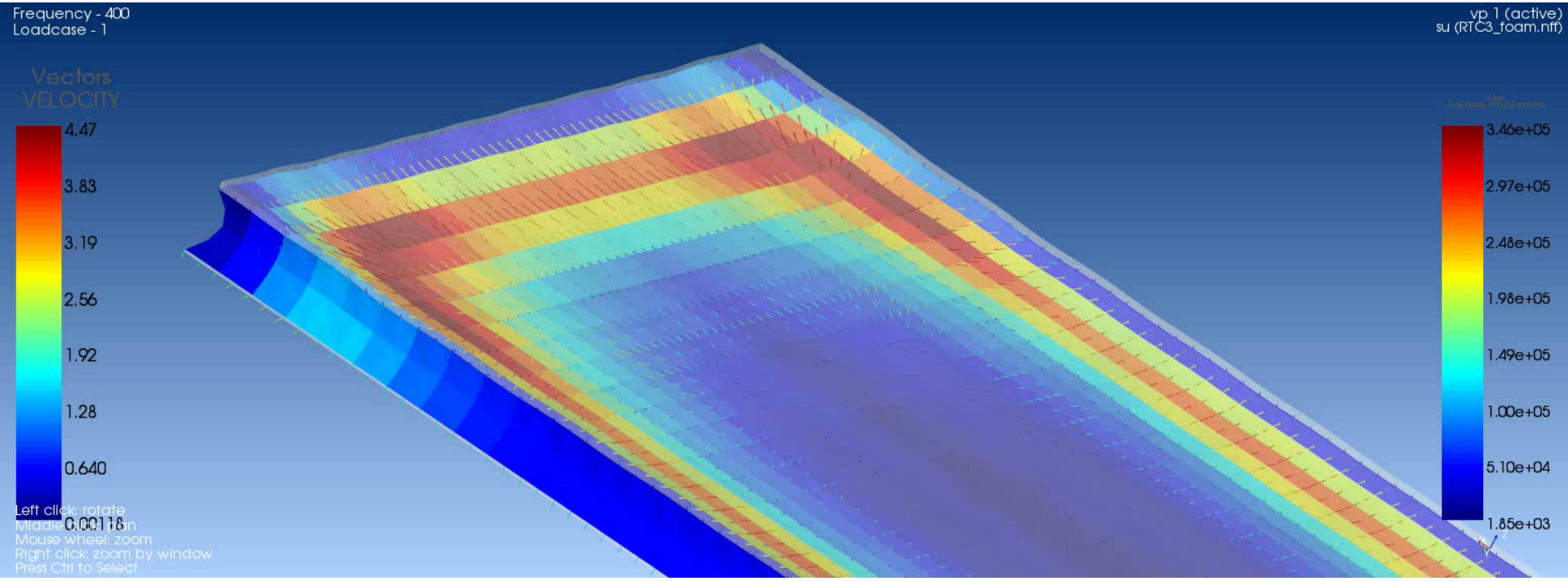
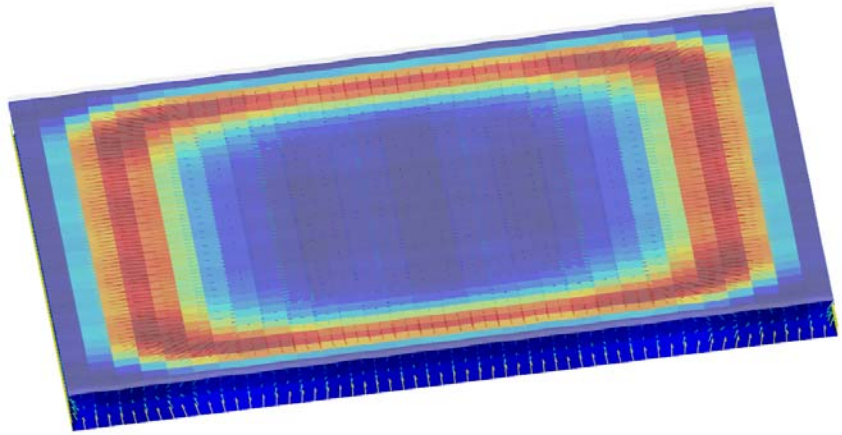
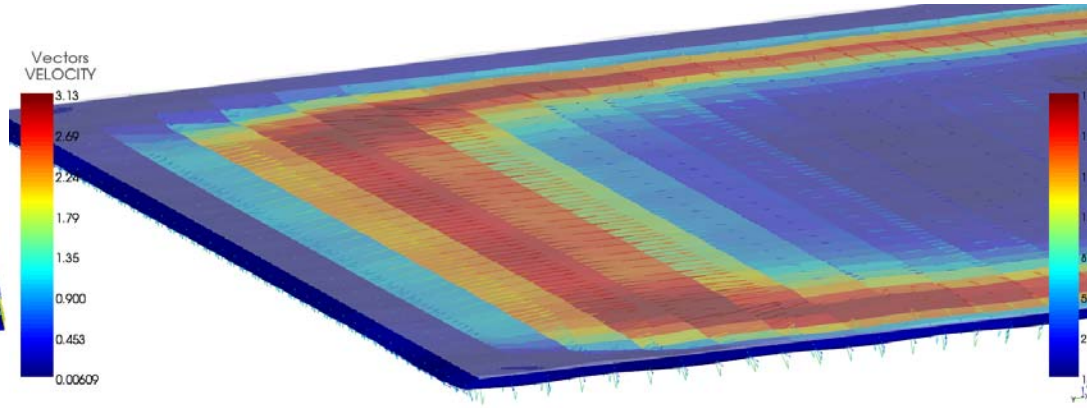


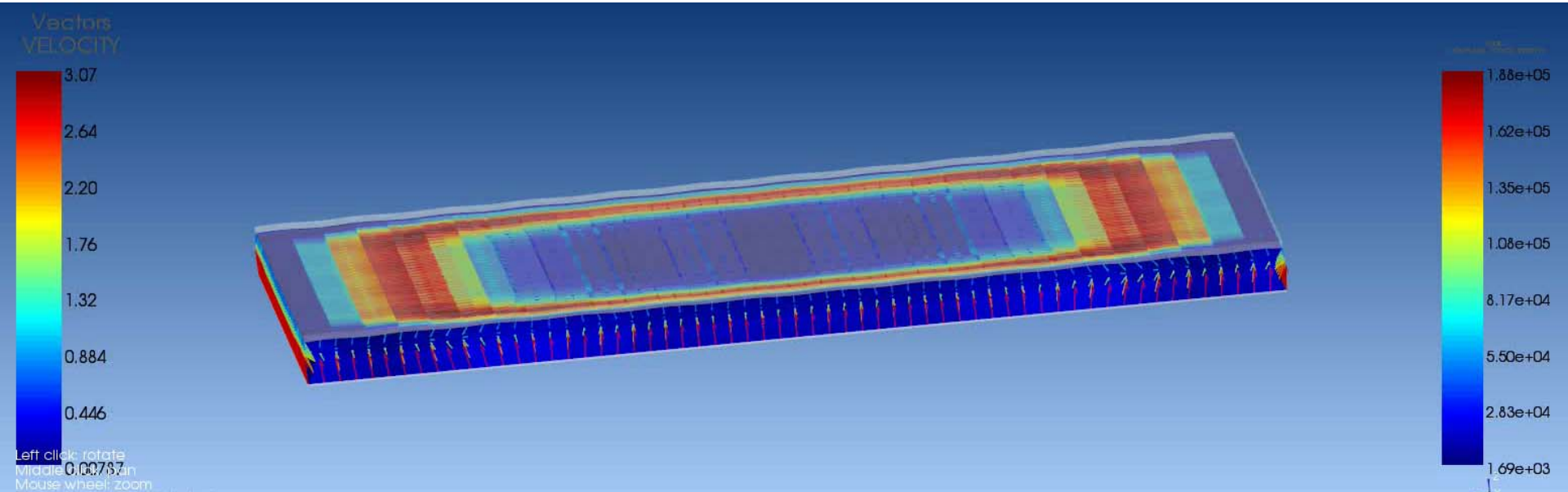
598 Hz 1D pumping

- The first 1D resonnance is reached: heavy layer and plate act as two masses separated by a porous spring
- Edge effect still present

- Viscous effects decrease, as 1D motion do not allow for much transverse fluid motion.
- But still present in the ring region



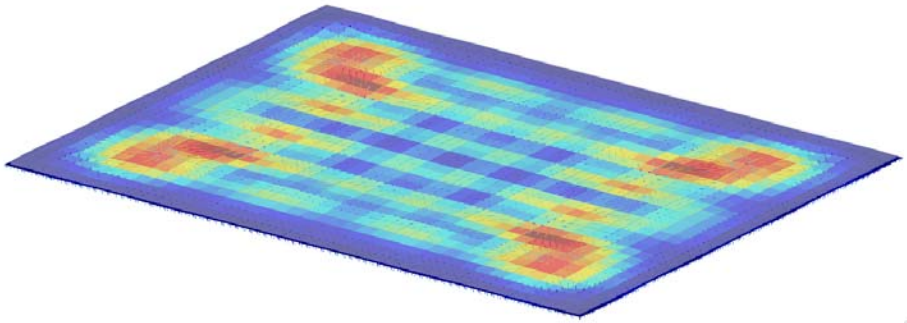


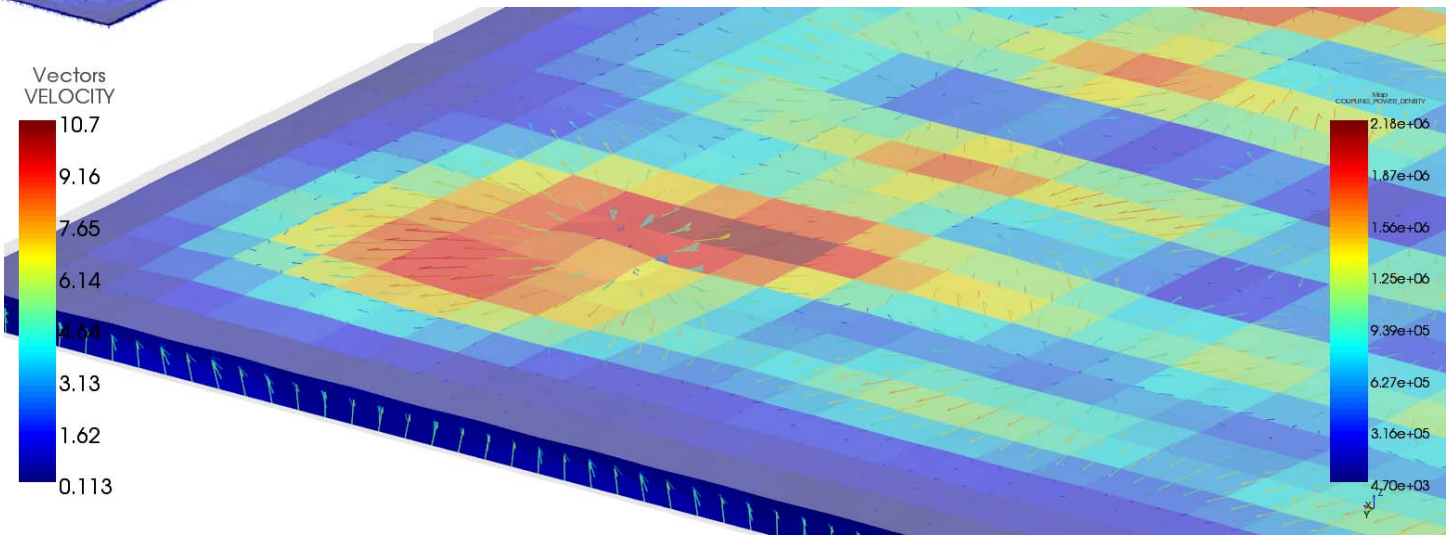


800 Hz fluid modes

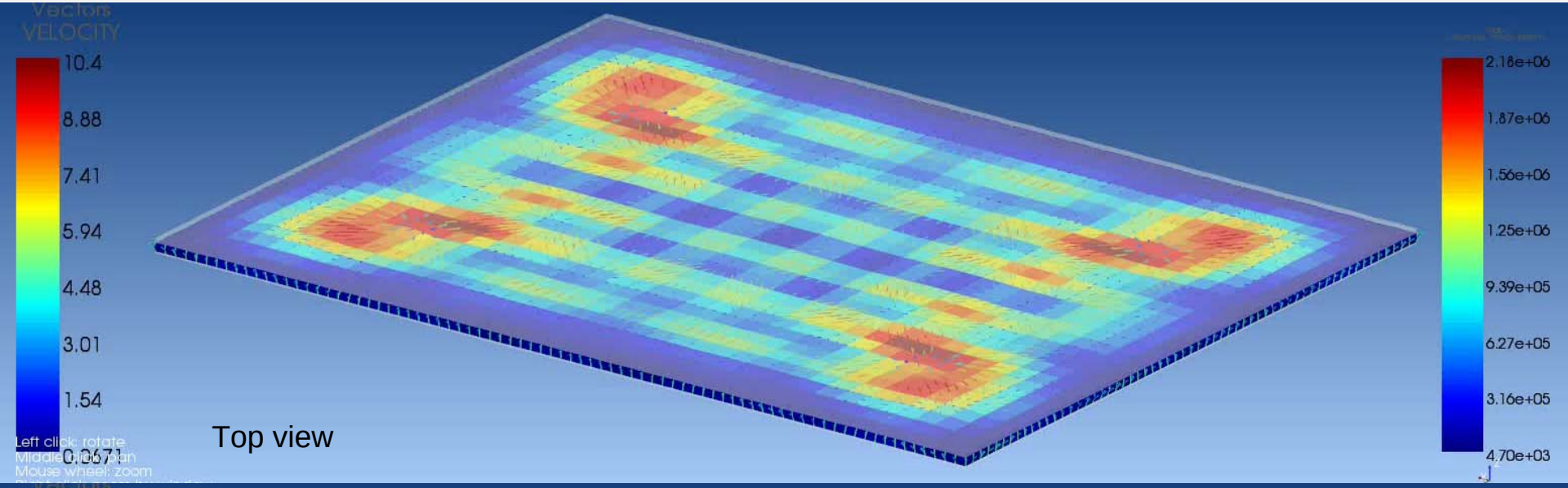
- Resonance of fluid in the porous
- No edge effect, viscous damping show a complex pattern relecting internal « acoustic » modes

- Viscous effects again dominates.
- Heavy layer well isolated, trim very efficient for reducing acoustic transmission

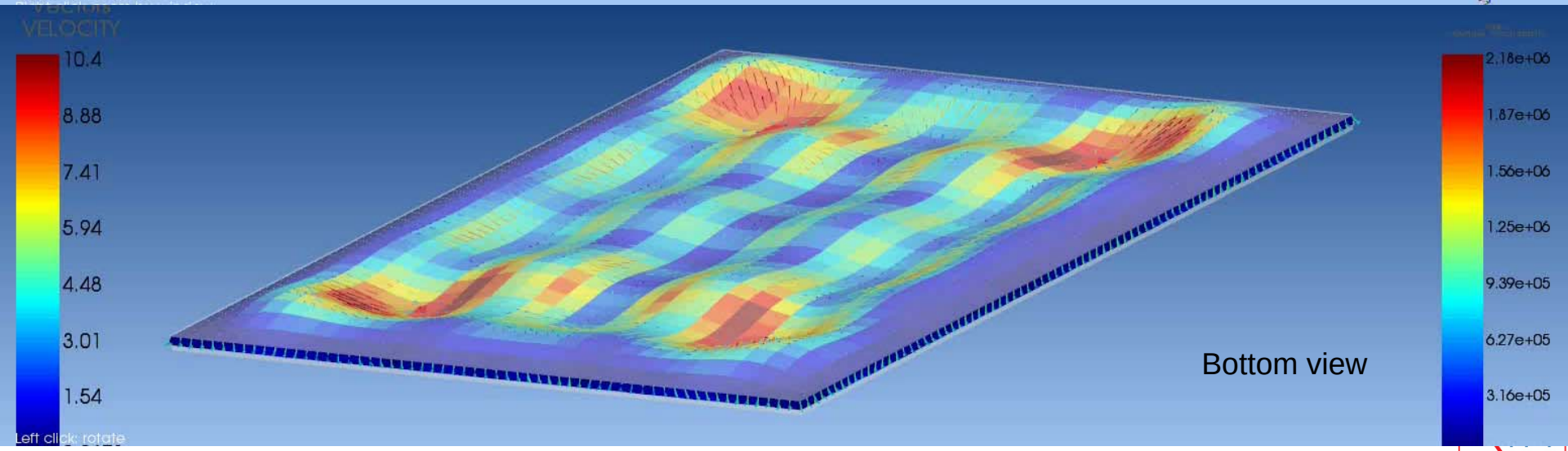




800 Hz fluid modes



Top view



Bottom view

Conclusions & perspectives

- U-P formulation for [anisotropic] Biot
  - Efficient formulation allowing easy implementation of boundary condition and fluid/solid coupling
  - Allows for evaluation of stored energies and dissipative powers, including discrimination between the 3 dissipation mechanisms typical of porous materials
  - Fully integrated in the Actran commercial vibro-acoustic simulation software

- Investigation of the behavior of the porous material in term of dissipated power helps understanding complex behavior a lot!
  - In frequency, to understand main dissipative mechanisms at play and adjust material parameters
  - In space, to tune porous trim per patch

- Perspective
  - Production of meaningfull graphs demands some expertise
  - Scripts should produce directly relevant quantities and automatically produce maps and power balances