An inverse methodology for the characterisation of anisotropic open-cell foams
—
Application to a melamine foam

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Stockholm, 16-18 December 2014

Introduction: anisotropy in porous materials

Anisotropy:

- Isotropic models are OK in most airborne applications
- Isotropic models are NOT OK when the material is excited by vibrations

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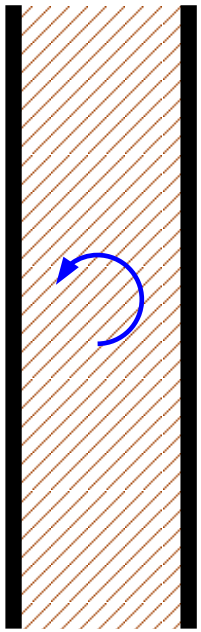

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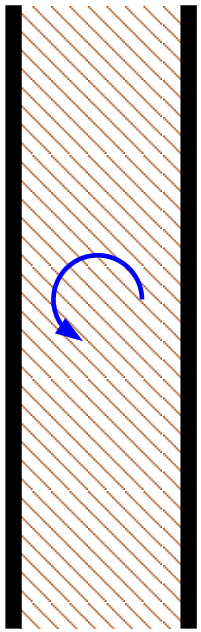

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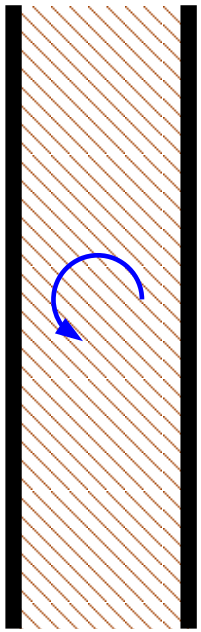

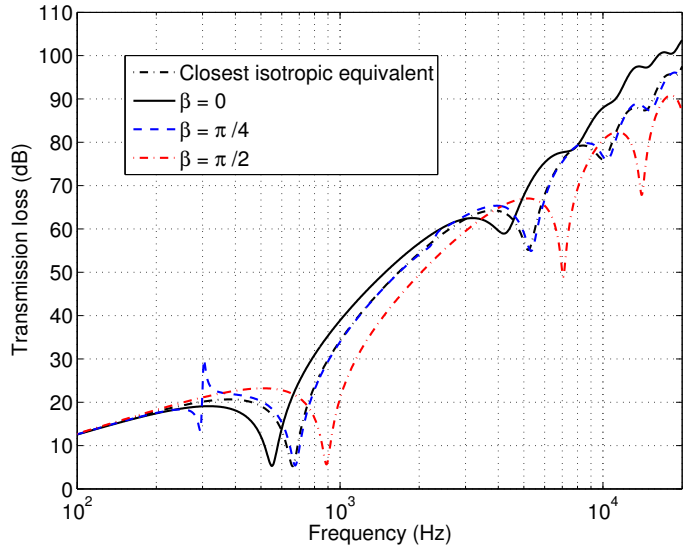

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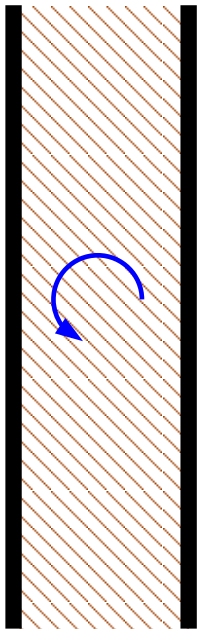

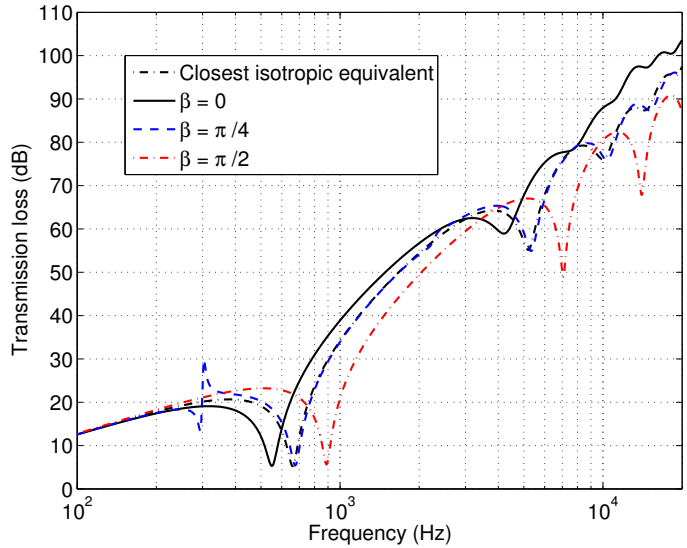

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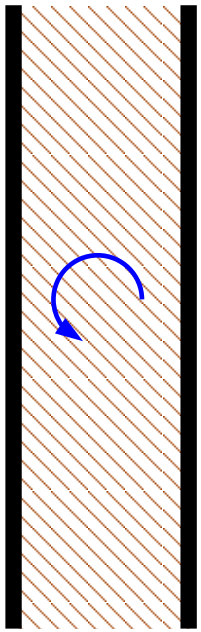

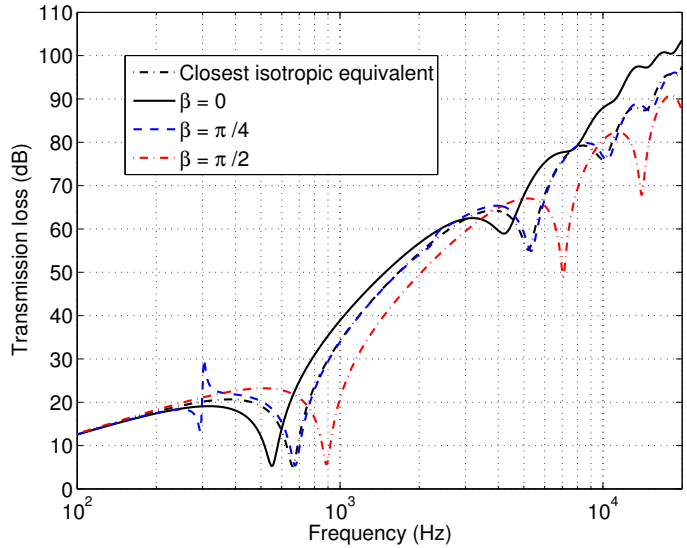

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⇒ Need for characterisation techniques (topic of this talk)

Future = place the materials in their optimal orientation using minimum weight

1. Model of the frame

2. Characterisation of the frame

Stress-strain relations in Biot's theory for the solid frame (s) and the fluid (f):

$$\begin{cases} \sigma_{ij}^{(s)} = H_{ijkl}\varepsilon_{kl}^{(s)} + Q_{ij}u_{k,k}^{(f)} \\ \sigma_{ij}^{(f)} = \left(Q_{kl}\varepsilon_{kl}^{(s)} + Ru_{k,k}^{(f)}\right)\delta_{ij} \end{cases}$$

H_{ijkl}	stiffness matrix of the frame
Q_{ij}	solid-fluid dilatation coupling
R	fluid compressibility modulus

Two possibilities for measuring H_{ijkl}:

- Dynamic tests in vacuo:

$$\boxed{\sigma = \mathbf{H}\,\varepsilon}$$

- Static tests:

$$\boxed{\sigma = \mathbf{C}\,\varepsilon}$$

H: Full frequency-dependent stiffness matrix (elastic and anelastic effects)

C: Fully-relaxed elastic part of the stiffness matrix

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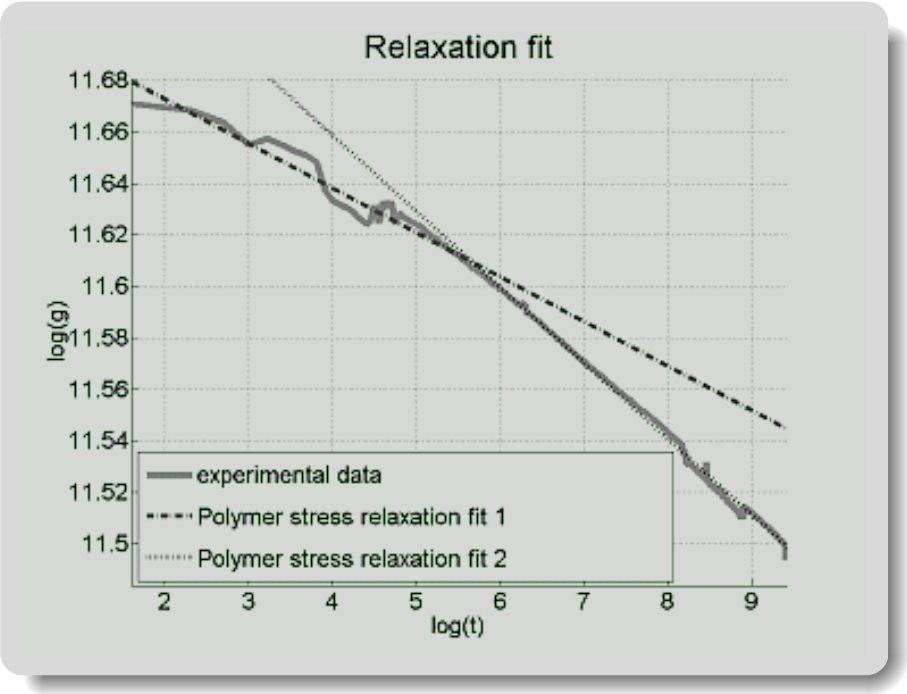

Relaxation test for a melamine foam (time domain)

[Guastavino & Göransson 2005]

Modelling possibilities:

- Add damping as a complex stiffness, e.g. $E = E_0 \cdot (1 - i\eta)$ (not causal)
- Hypothesis of causality $\Rightarrow$ there must be a **constitutive model**

Causal parameterisations (equivalent to each other):

- Discrete superposition of relaxation processes
- Continuous superposition of relaxation processes
- . . . (go to next page)

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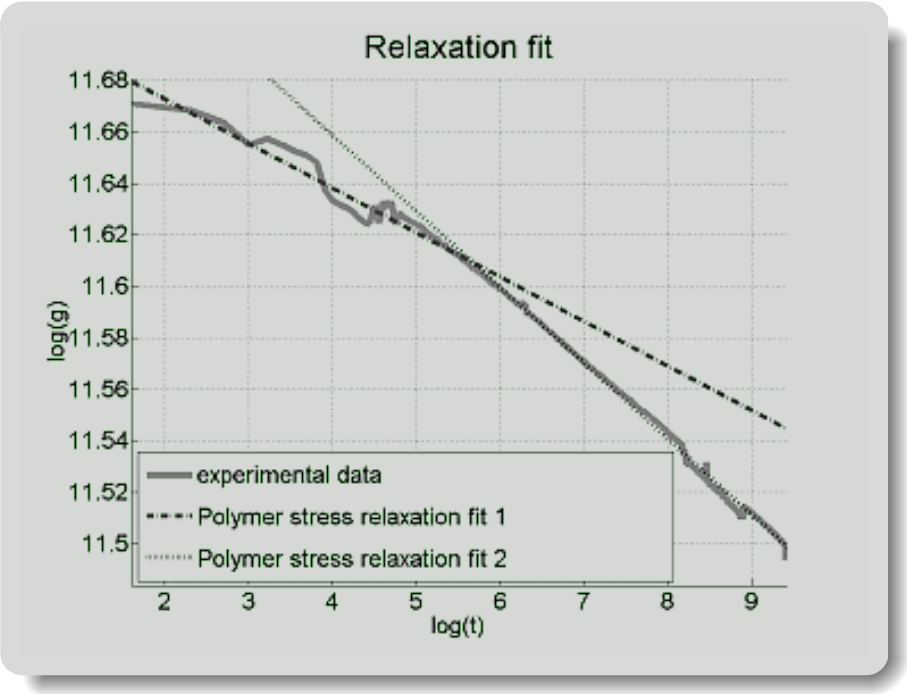

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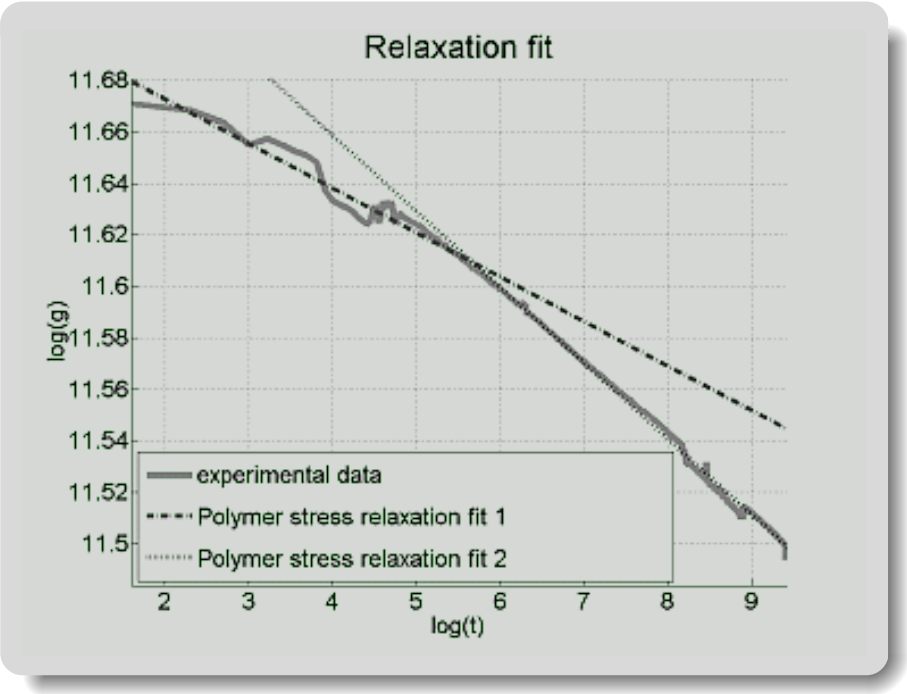

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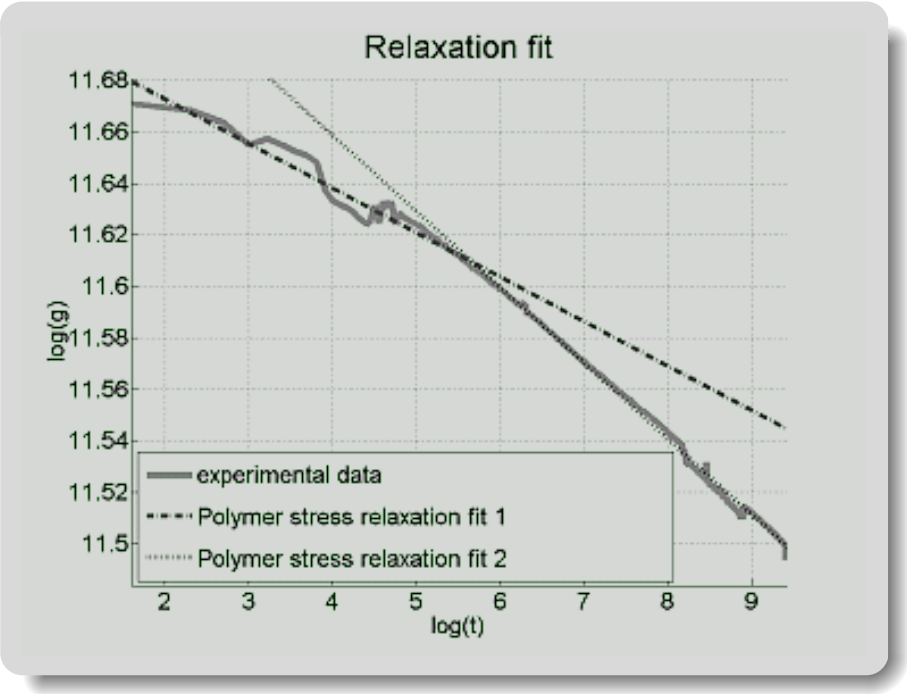

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Fractional differential constitutive equation

$$\left(1 + \frac{\partial^\alpha}{\partial(t/\tau)^\alpha}\right)\boldsymbol{\sigma}(t) = \left(\mathbf{C} + \mathbf{M}\frac{\partial^\alpha}{\partial(t/\tau)^\alpha}\right)\boldsymbol{\varepsilon}(t)$$

$\mathbf{C}$	fully-relaxed stiffness matrix
$\mathbf{M}$	deformation memory
$\alpha \in]0, 1]$	derivative order
τ	relaxation time
β	relaxation frequency

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$$\mathbf{H}(\omega) = \mathbf{C} + \frac{\mathbf{B}\cdot(i\omega/\beta)^\alpha}{1 + (i\omega/\beta)^\alpha}$$

Laplace transform

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$$\mathbf{H}(\omega) = \mathbf{C}\left(1 + \frac{b\,(i\omega/\beta)^\alpha}{1 + (i\omega/\beta)^\alpha}\right)$$

Hypothesis: collinearity of elastic and anelastic contributions (optional)

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Stiffness matrix:

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Compliance matrix (orthotropic symmetry):

$$\left(\mathbf{C}^{0}\right)^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{32}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Orientation of the natural coordinates:

$$\mathbf{C} = \mathbf{A}\mathbf{C}^{0}\mathbf{A}^{\mathsf{T}} \qquad \mathbf{A} = \mathbf{A}(\phi_1, \phi_2, \phi_3)$$

Physical constraints (positive-definiteness of $\mathbf{C}^{0}$):

$$E_i > 0 \qquad G_{ij} > 0 \qquad \nu_{ij}^{2} < E_i/E_j \qquad 2\prod_{ij}\nu_{ij} < 1 - \sum_{ij}\nu_{ij}^{2}E_j/E_i$$

Parameters to estimate

Set of properties to be estimated:

$$\mathbf{X} = \begin{bmatrix} E_1 & E_2 & E_3 & G_{23} & G_{31} & G_{12} & \nu_{21} & \nu_{13} & \nu_{32} & \alpha & \beta & b & \phi_1 & \phi_2 & \phi_3 \end{bmatrix}^\mathsf{T}$$

Non-dimensional form:

$$\mathbf{x} = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 & \xi_7 & \xi_8 & \xi_9 & \xi_{10} & \xi_{11} & \xi_{12} & \xi_{13} & \xi_{14} & \xi_{15} \end{bmatrix}^\mathsf{T}$$

. . . with respect to an arbitrary isotropic material of reference:

$$\mathbf{X}_0 = \begin{bmatrix} E_0 & \nu & \alpha_0 & \beta_0 & b_0 \end{bmatrix}^\mathsf{T}$$

The first 9 non-dimensional parameters are the degrees of anisotropy of the material:

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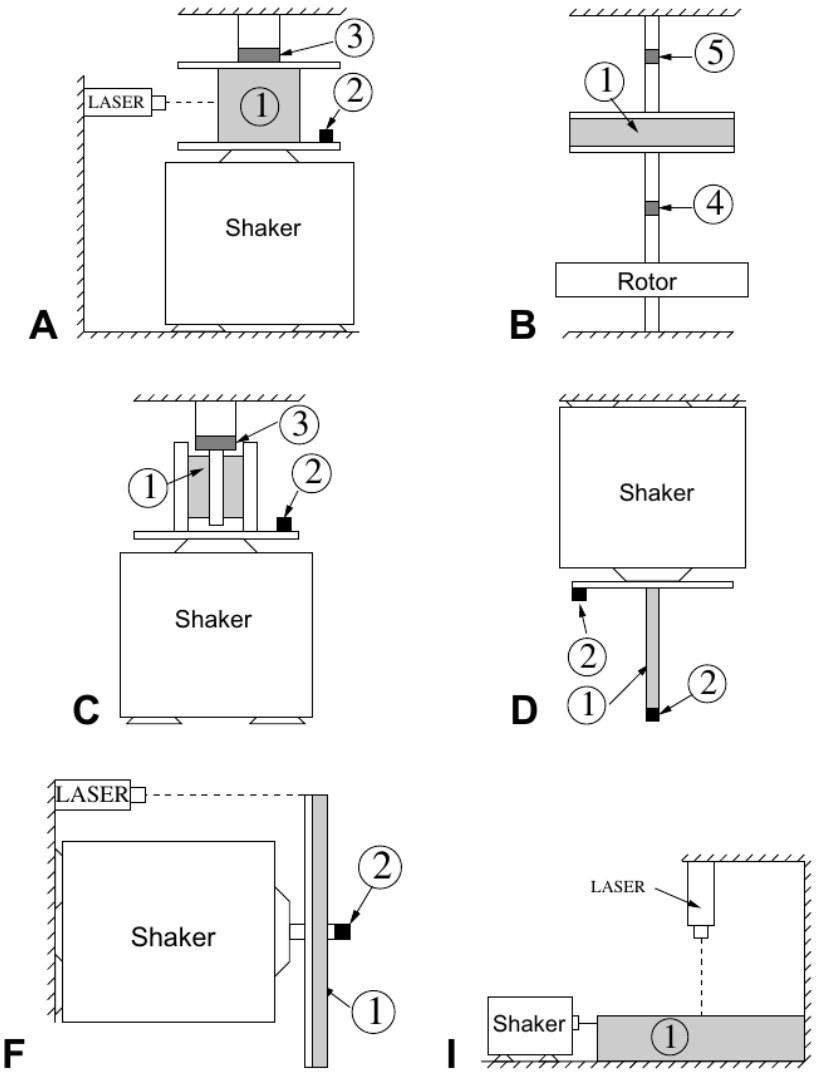

Review by Jaouen et al. 2008

General methodology

$\mathbf{x}_0 =$ unknown

Experimental data

$d(\mathbf{x})$

$\mathbf{x} =$ guess

Minimisation of distance $d(\mathbf{x})$

Fitted model $\mathbf{x} \simeq \mathbf{x}_0$

The parameter search is formulated as an optimisation problem:

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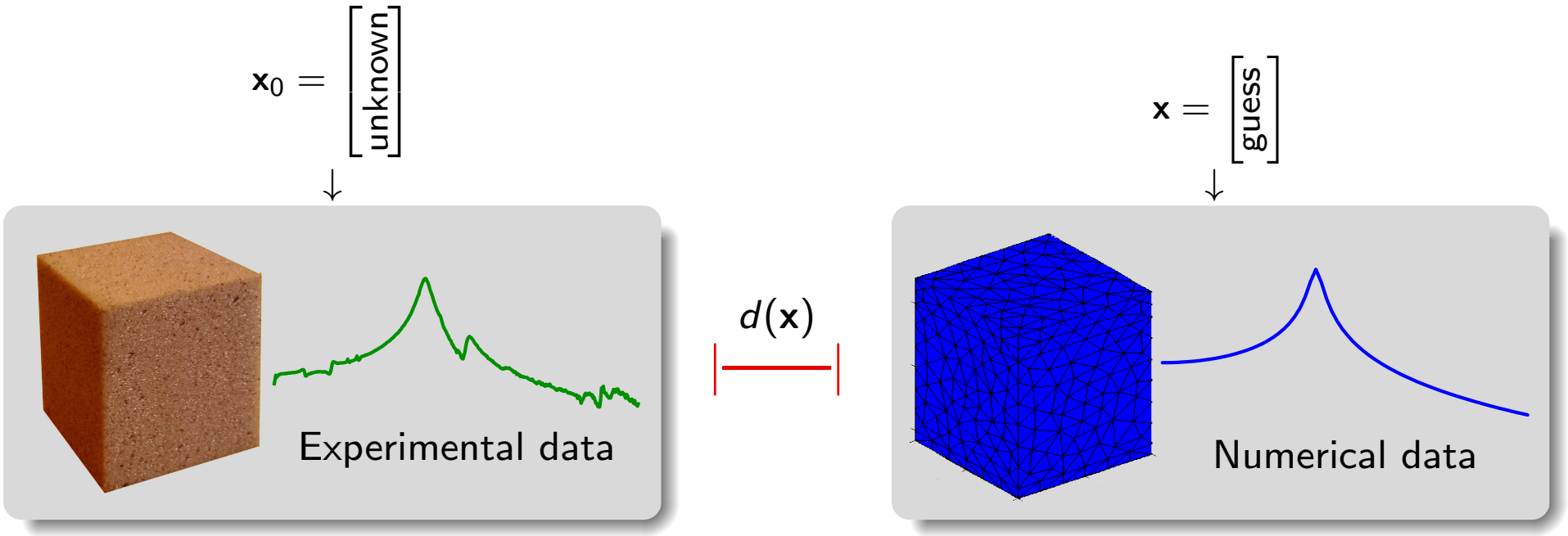

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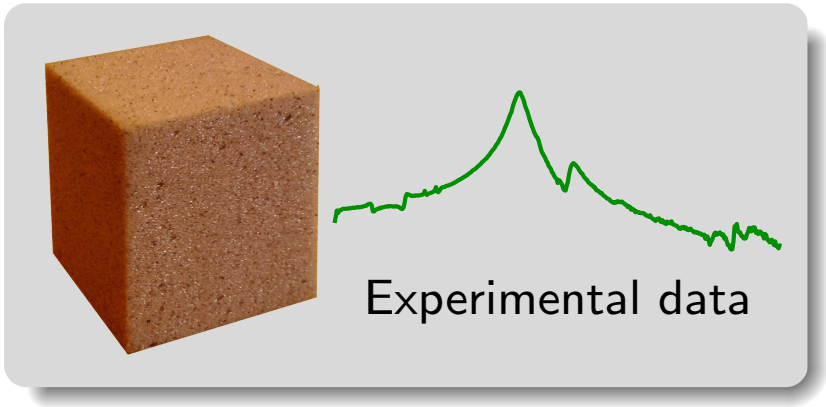

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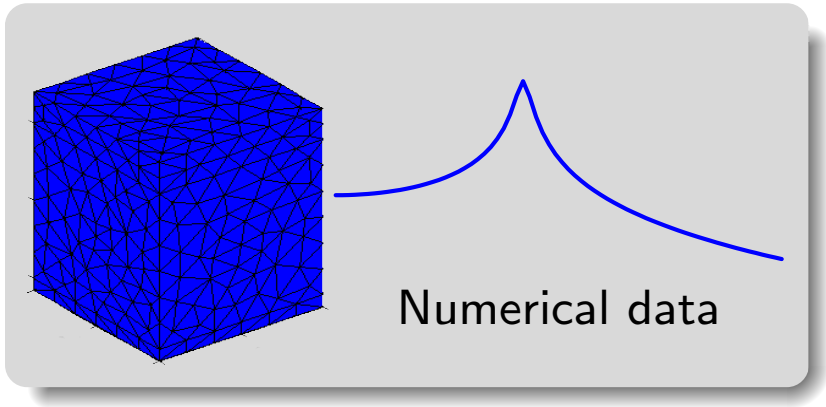

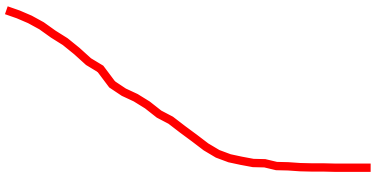

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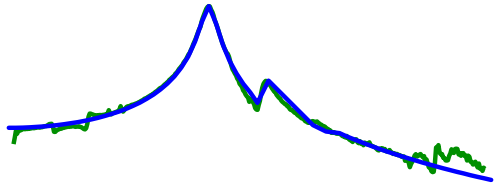

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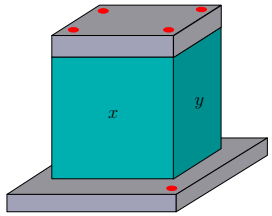
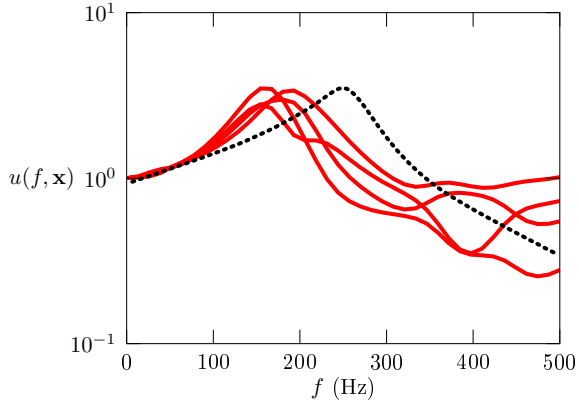
Dynamic measurements $\Rightarrow$ anelastic properties

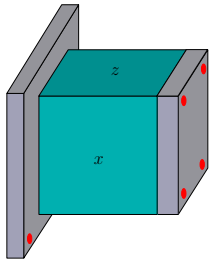
target; ----- model

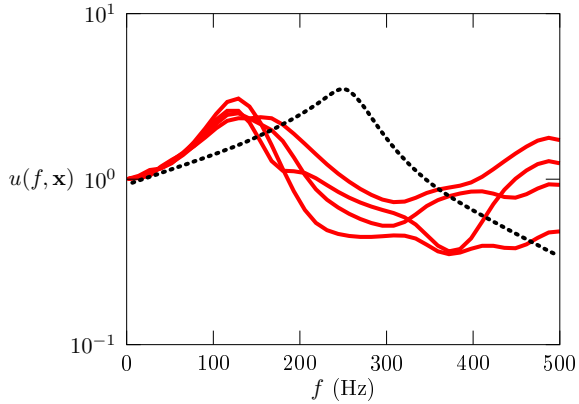
1. Arbitrary isotropic reference
2. Minimisation of

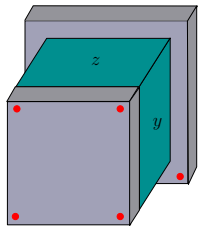
$$d(\mathbf{x}) = 1 + \sum_{m=1}^{M} \sum_{n=1}^{N} \left| \frac{u_m(\omega_n, \mathbf{x}) - u_m^{(\text{exp})}(\omega_n)}{u_m^{(\text{exp})}(\omega_n)} \right|^2$$

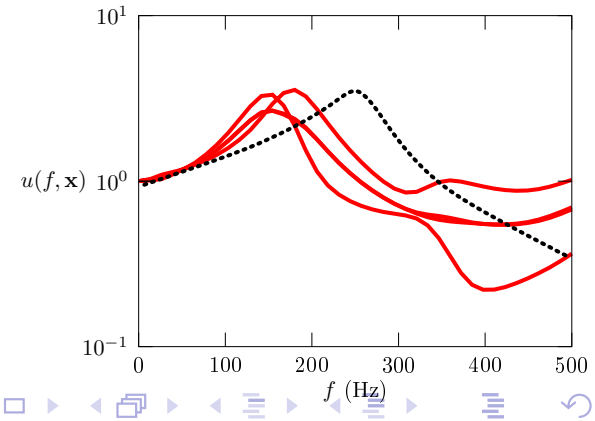










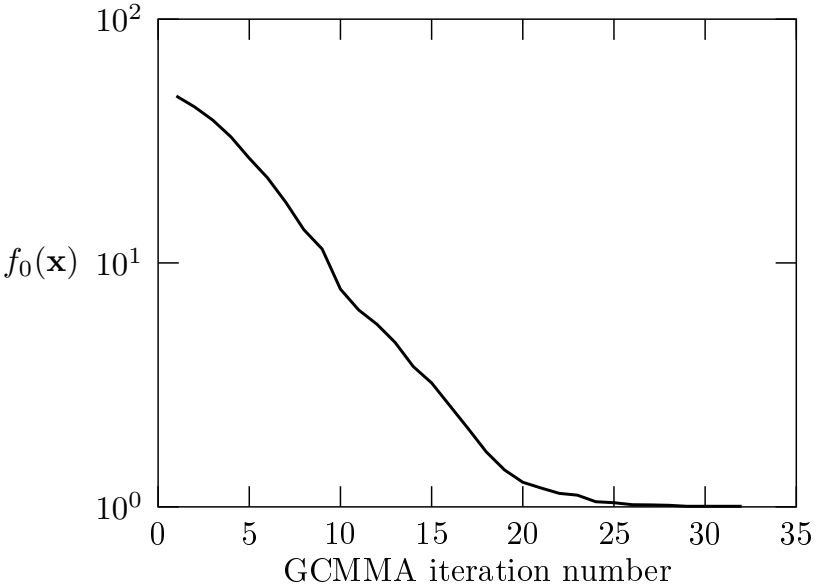


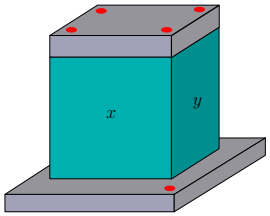
Dynamic measurements $\Rightarrow$ anelastic properties

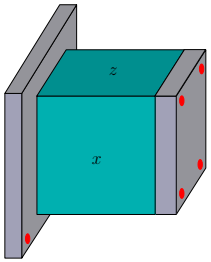
—— target; - - - - - model

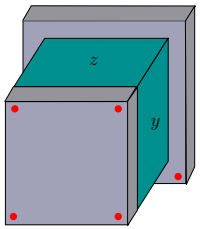
1. Arbitrary isotropic reference
2. Minimisation of

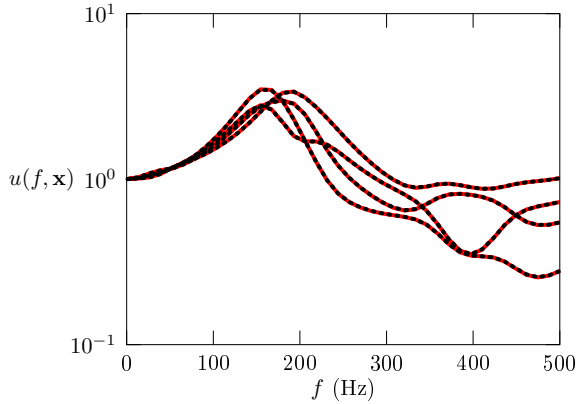
$$d(\mathbf{x}) = 1 + \sum_{m=1}^{M} \sum_{n=1}^{N} \left| \frac{u_m(\omega_n, \mathbf{x}) - u_m^{(\text{exp})}(\omega_n)}{u_m^{(\text{exp})}(\omega_n)} \right|^2$$

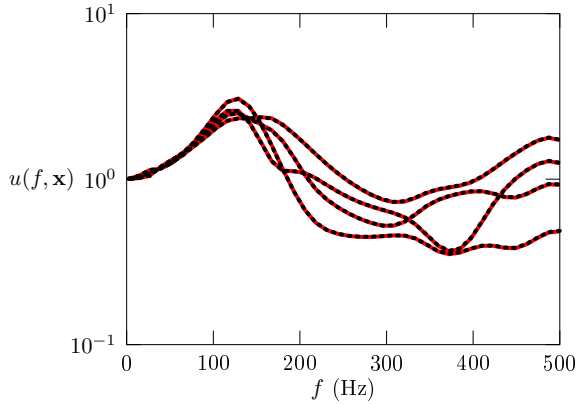


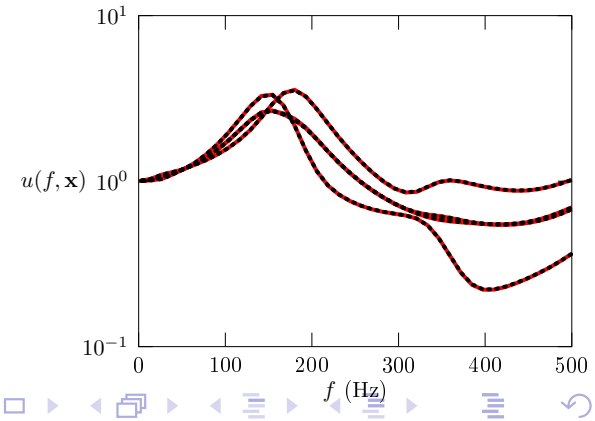




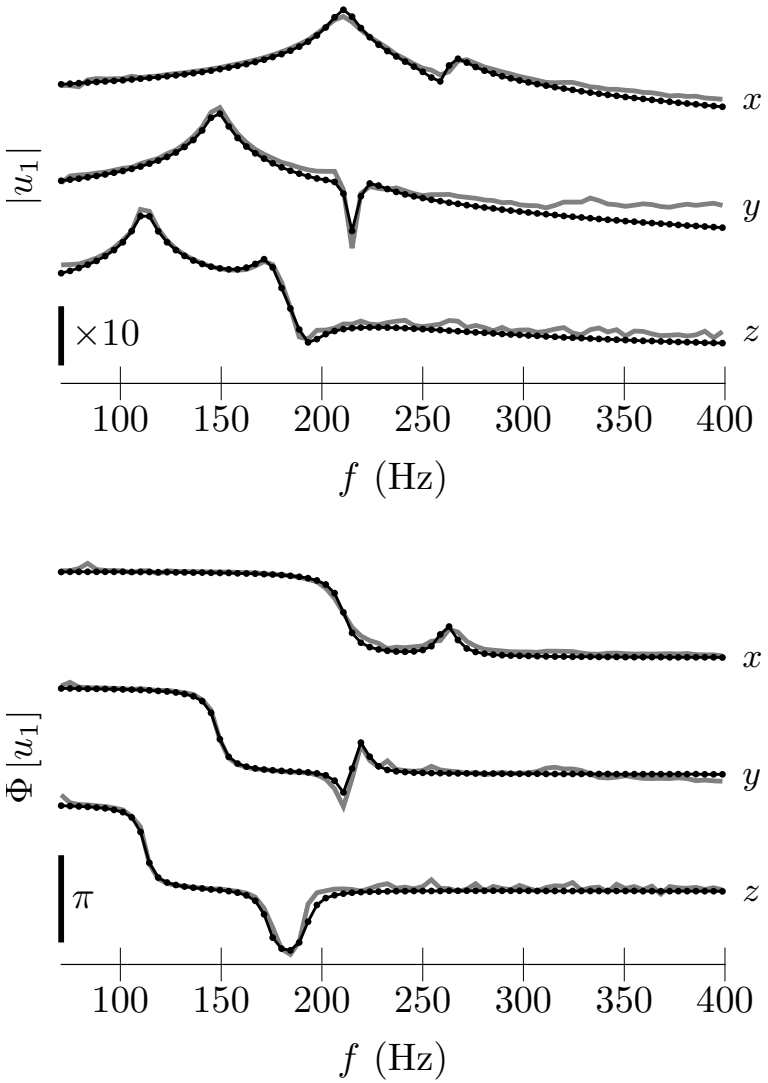








Fitted model



Measured properties

Property	value	unit
E_1	448	kPa
E_2	211	kPa
E_3	170	kPa
G_{23}	104	kPa
G_{31}	124	kPa
G_{12}	101	kPa
ν_{21}	0.445	-
ν_{13}	-0.514	-
ν_{32}	0.433	-
β	813	krad/s
α	0.333	-
b	0.296	-
ϕ_1	-0.118	rad
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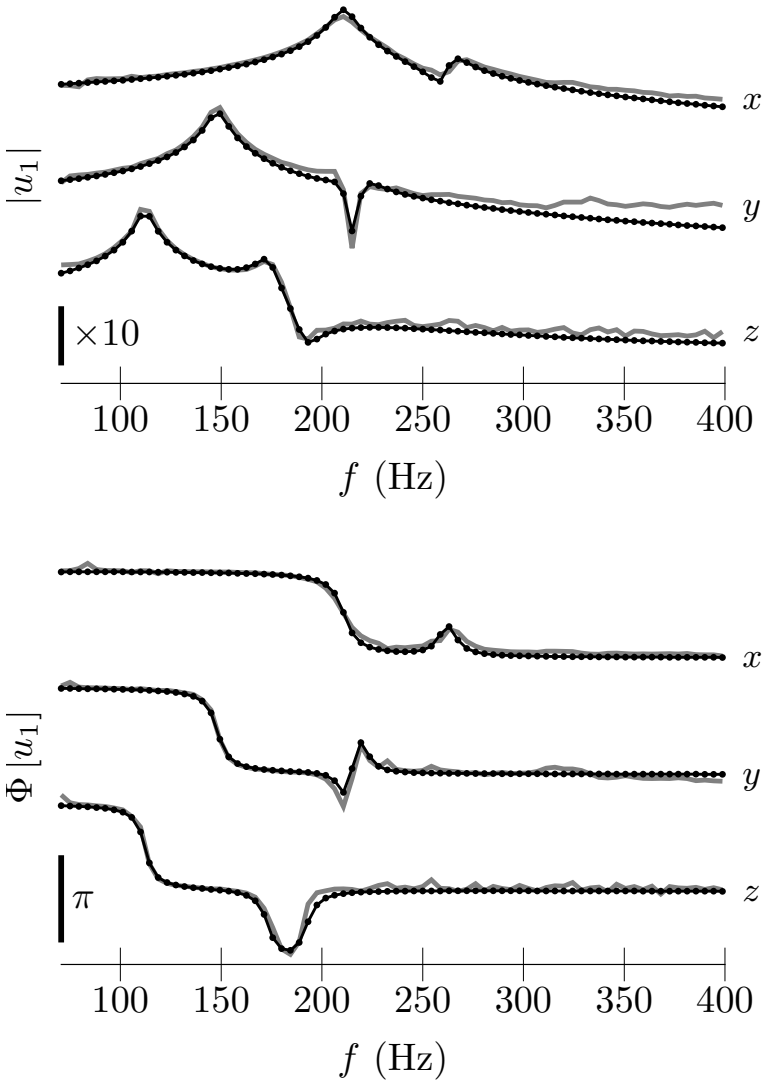
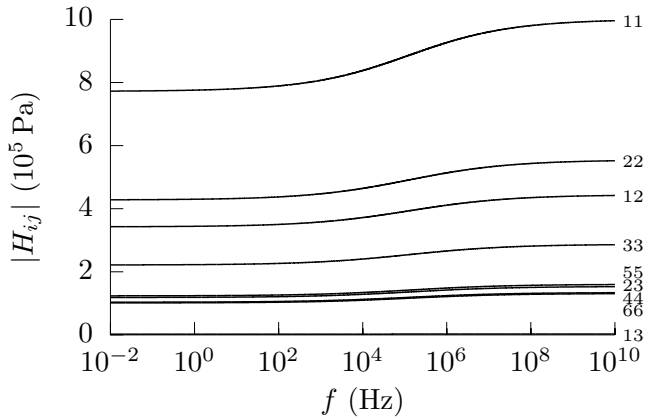
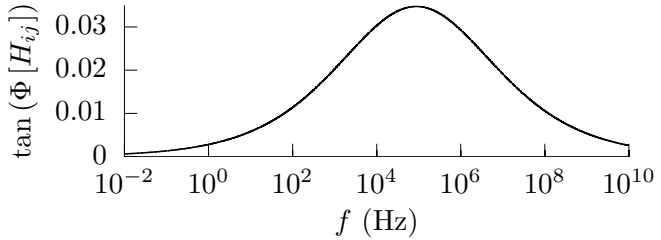
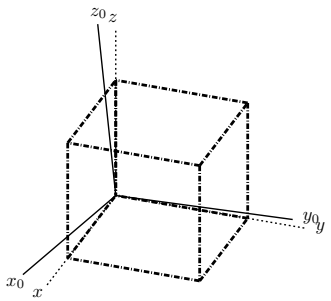
Fitted model



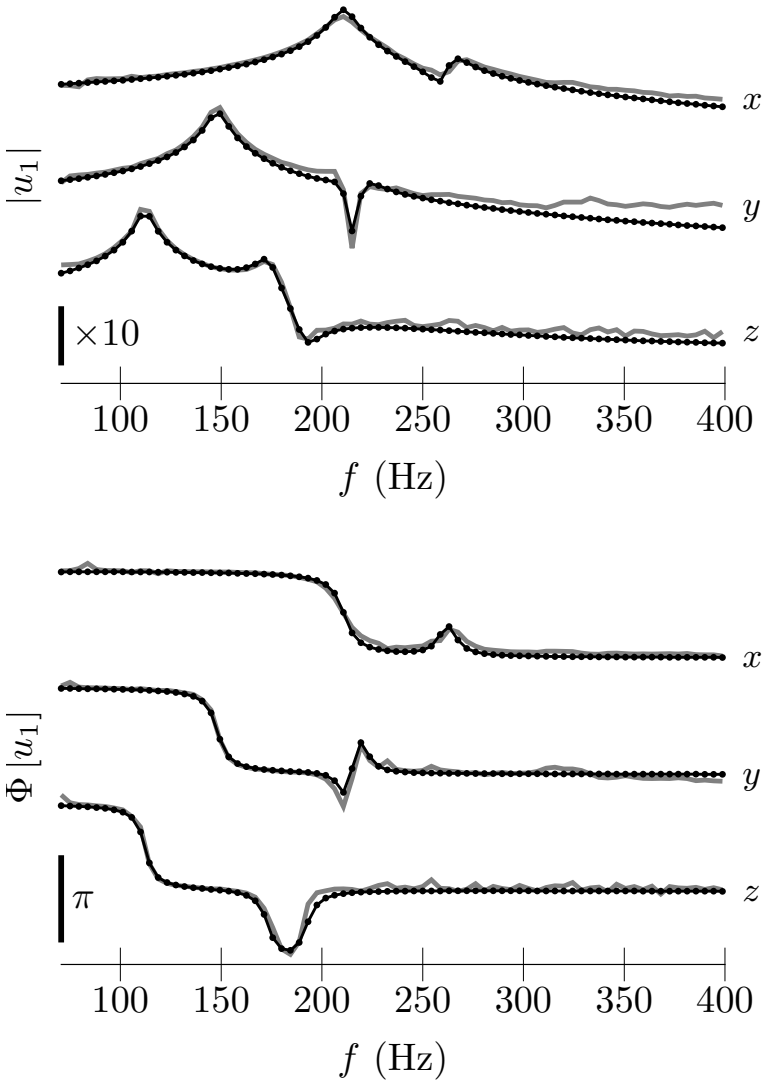
Measured properties



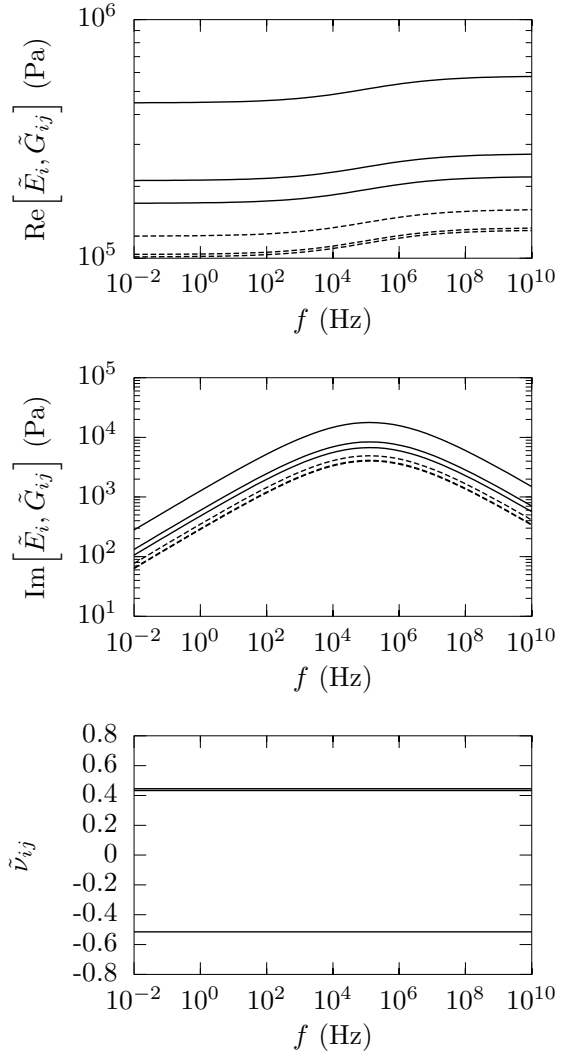




Fitted model



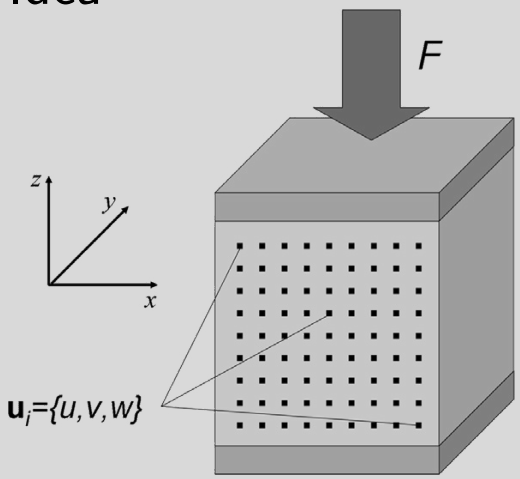
Measured properties



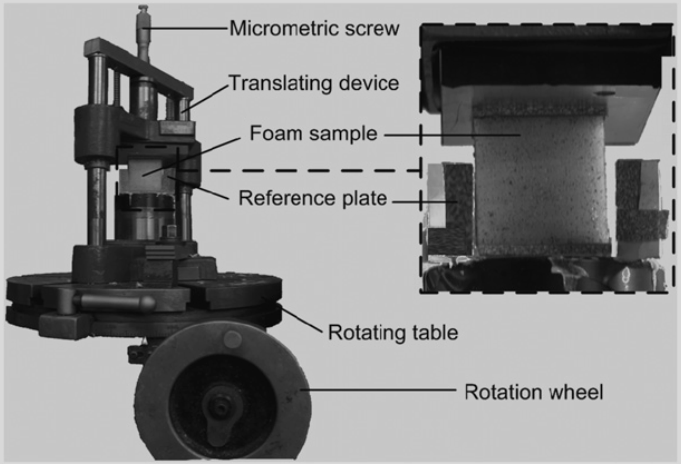
Elastic properties (static)

$$\mathbf{H}(\omega) = \mathbf{C} + \frac{\mathbf{B} \cdot (i\omega/\beta)^{\alpha}}{1 + (i\omega/\beta)^{\alpha}} \quad \text{or} \quad \mathbf{H}(\omega) = \mathbf{C}\left(1 + \frac{b\,(i\omega/\beta)^{\alpha}}{1 + (i\omega/\beta)^{\alpha}}\right)$$

Idea

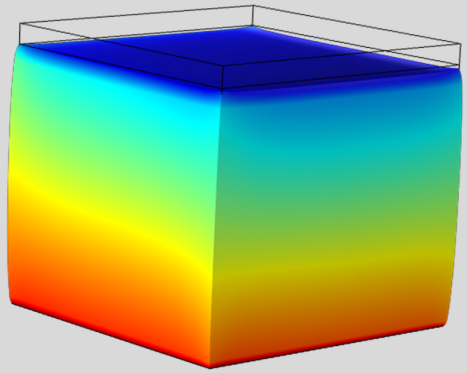


Experiment



Guastavino and Göransson 2007

Model



Data = 3D displacements

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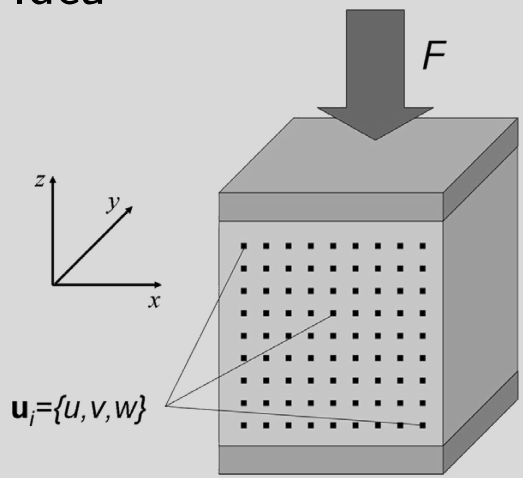
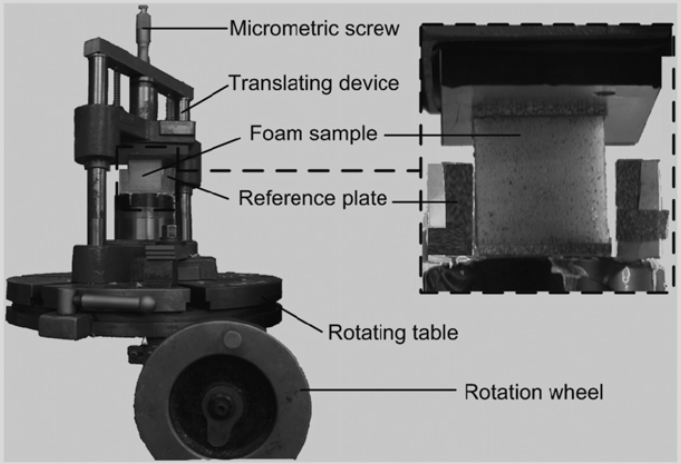


Experiment



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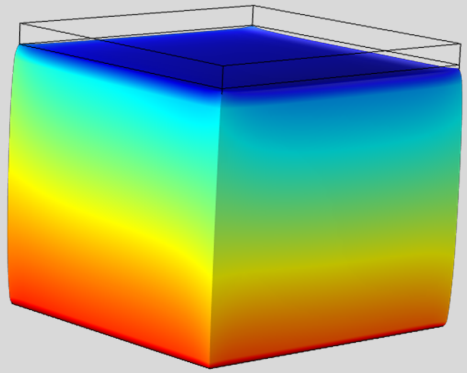

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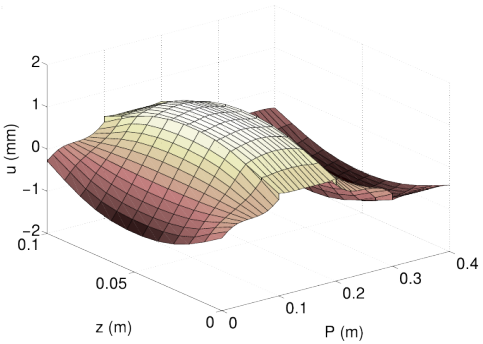

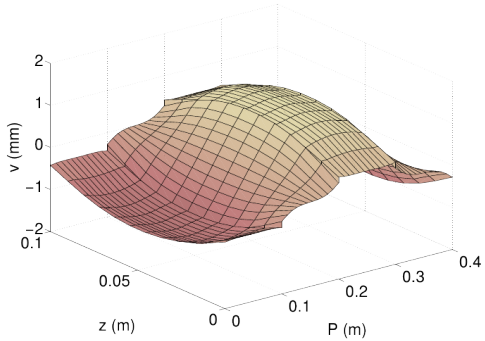

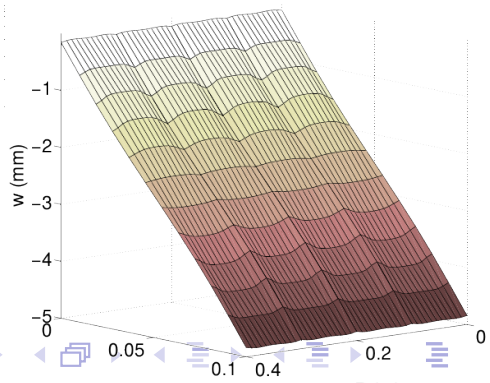

Estimated properties for melamine

Dynamic measurement

Property	value	unit
E_1	448	kPa
E_2	211	kPa
E_3	170	kPa
G_{23}	104	kPa
G_{31}	124	kPa
G_{12}	101	kPa
ν_{21}	0.445	-
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α	0.333	-
b	0.296	-
ϕ_1	-0.118	rad
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ϕ_3	0.131	rad

Static measurement

Property	value	unit
E_1	124.2	kPa
E_2	161.8	kPa
E_3	219.2	kPa
G_{23}	3234.8	kPa
G_{31}	4203.0	kPa
G_{12}	28.3	kPa
ν_{21}	-0.35	-
ν_{13}	0.35	-
ν_{32}	0.63	-
ϕ_1	0.038	rad
ϕ_2	-0.01	rad
ϕ_3	-0.76	rad

Conclusion

- Static and dynamic measurements give different estimations of the elastic (fully-relaxed) moduli
  $\Rightarrow \mathbf{H}(\omega) = \mathbf{C} + \dfrac{\mathbf{B} \cdot (i\omega/\beta)^{\alpha}}{1 + (i\omega/\beta)^{\alpha}}$
- **H** can be used in numerical tools for anisotropic poroelastic materials
  - Hörlin and Göransson 2010 (FEM)
  - Parra et al. 2015 (TMM)
- Fractional-derivative model $\Rightarrow$ few parameters to estimate
- We also have the anisotropic flow resistivity tensor:

$$\min \left| \qquad \right| \Rightarrow \mathbf{R} = \begin{bmatrix} R_{xx} & R_{xy} & R_{xz} \\ & R_{yy} & R_{yz} \\ (\text{sym}) & & R_{zz} \end{bmatrix}$$

[Van der Kelen and Göransson 2013]

- $\Rightarrow$ Towards a comprehensive characterisation method

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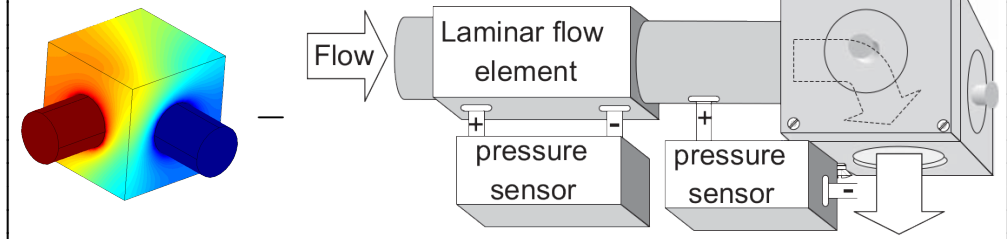


[Van der Kelen and Göransson 2013]

- $\Rightarrow$ Towards a comprehensive characterisation method

Acoustic Response modelling of Multilayered Structures with Anisotropic Porous materials

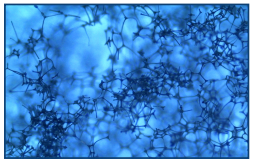
J.P. Parra Martinez[1,2,], P. Göransson[1], O. Dazel[2], J. Cuenca[1]*

[1]KTH Royal Institute of Technology, Teknikringen 8, SE10044 Stockholm, Sweden
[2]LAUM Université du Maine, UMR CNRS 6613, Avenue Olivier Messiaen, F-72085 Le Mans Cedex France
[*]jppm@kth.se

Introduction

Motivation

- Open-cell porous materials are inherently anisotropic due to manufacturing processes, comprising a highly irregular microscopic structure.
- Need of a method for modelling the influence of the inherent mechanical and acoustic anisotropy of poroelastic materials on the acoustic transmission through multilayered structures.



Microscopic picture of a melamine foam (x20)

Objectives

- Create an accurate modelling tool for the broadband acoustic assessment of anisotropic media.
- Investigate the influence of the relative alignment of anisotropic porous materials with respect to the general coordinate system on the acoustic response of multilayered structures.

Approach

Plane wave approximation

- Every physical field can be expressed as

$$\phi(\boldsymbol{r},t) = \hat{\phi} e^{\imath(\omega t - \boldsymbol{kr})}$$

- The variation of the physical fields through a layer, described via a state vector $\boldsymbol{S}$ composed of state variables, can be approximated as a linear combination of the same state variables,

$$\frac{\partial \boldsymbol{S}(z)}{\partial z} = -[\boldsymbol{\alpha}]\boldsymbol{S}(z)$$

- The state vector $\boldsymbol{S}$ depends on the material model

Fluid layer	Solid layer	Porous layer
$\begin{Bmatrix} u_z^f(z) \\ p(z) \end{Bmatrix}$	$\begin{Bmatrix} u_x(z) \\ u_y(z) \\ u_z(z) \\ \sigma_{xz}(z) \\ \sigma_{yz}(z) \\ \sigma_{zz}(z) \end{Bmatrix}$	$\begin{Bmatrix} u_x^s(z) \\ u_y^s(z) \\ u_z^s(z) \\ u_z^t(z) \\ \hat{\sigma}_{xz}(z) \\ \hat{\sigma}_{yz}(z) \\ \hat{\sigma}_{zz}(z) \\ p(z) \end{Bmatrix}$

Material models

- Fluid layer

$$p = -K.\nabla u_z^f, \qquad \frac{\partial p}{\partial z} = \omega^2 \rho u_z^f.$$

- Anisotropic solid Layer

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\epsilon}, \qquad \nabla \cdot \boldsymbol{\sigma} = -\omega^2 \rho \boldsymbol{u}.$$

- Anisotropic porous layer

$$\nabla \cdot \hat{\boldsymbol{\sigma}} = -\omega^2 \tilde{\boldsymbol{\rho}}_s \boldsymbol{u}^s - \omega^2 \tilde{\boldsymbol{\rho}}_{eq} \tilde{\gamma} \boldsymbol{u}^t,$$

$$-\nabla p = -\omega^2 \tilde{\boldsymbol{\rho}}_{eq} \tilde{\gamma} \boldsymbol{u}^s - \omega^2 \tilde{\boldsymbol{\rho}}_{eq} \boldsymbol{u}^t,$$

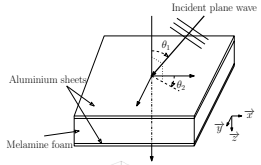
$$\hat{\boldsymbol{\sigma}} = \hat{\boldsymbol{H}}(\omega).\boldsymbol{\epsilon},$$

$$p = -\tilde{K}_{eq} \nabla . \boldsymbol{u}^t.$$

Results

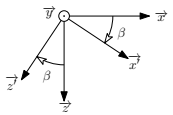
Setup

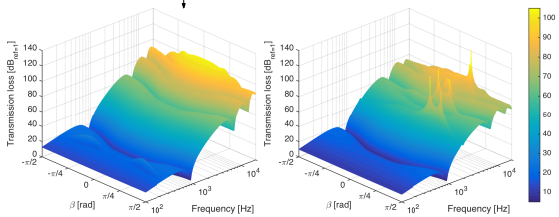
- Isotropic Aluminium facesheets
- Anisotropic melamine foam



Relative alignment

- The porous layer is rotated with respect to the general coordinate system





Normal Incidence

- Symmetry with respect to β=0 rad.
- Rotation of the poroelastic material shifts resonances and anti-resonances in frequency
- Increase of transmission loss of the overall structure for rotations around β=±π/4 rad.

Oblique Incidence

- No symmetry plane
- Rotation of the poroelastic material shifts resonances and anti-resonances in frequency
- Dramatic increase of transmission loss in mid and high frequencies for particular rotations

Conclusion / Quo vadis?

- A broadband plane wave method for the acoustic characterisation of multilayered structures composed of anisotropic poroelastic materials has been successfully implemented.
- The relative alignment of the porous layer has important influence on the acoustic transmission through the system.
- The methodology holds a potential to identify the physical phenomena behind the resonant behaviour.