An ultrasonic propagation model suited to inversion methods (with some application to porous materials)

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Le Mans
Acoustique

Inversion methods for non-destructive testing and evaluation

- NDE : evaluation of physical parameters characterizing a given material
    celerity, attenuation, density, size, . . .

- NDT : detection of flaws or of abnormal behaviours

- **Inversion methods :**

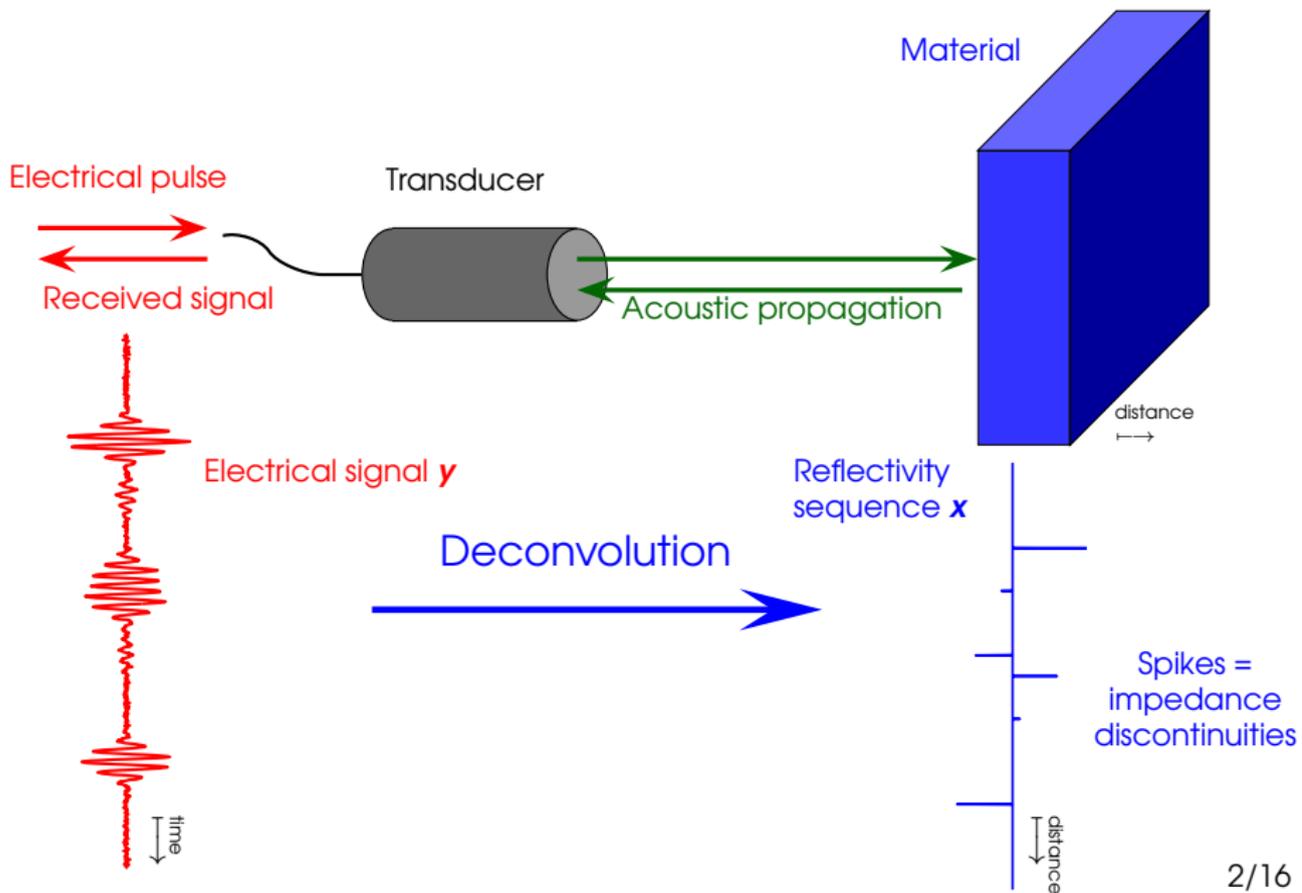 1. Modelling of the acquisition process (sensors, physics) :

     $$\boldsymbol{y} = \Phi(\boldsymbol{x}, \text{noise})$$

  2. Prior information about the searched object

  3. Define an estimation procedure :

     $$\widehat{\boldsymbol{x}} = {}^{"}\Phi^{-1"}(\boldsymbol{y})$$

     $\rightarrow$ Image reconstruction (Fatemi and Kak, 1980)

     $\rightarrow$ Time-of-flight tomography (Glover and Sharp, 1977)

     $\rightarrow$ Deconvolution (O'Brien et al., 1994)

Deconvolution for NDT&E

Introduction
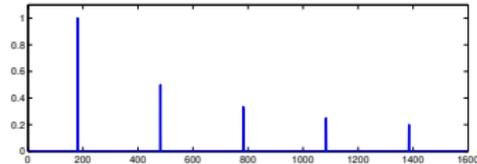○○●○

Propagation model
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Validation and deconvolution
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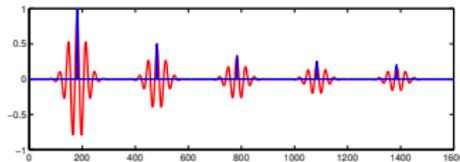Application to porous materials
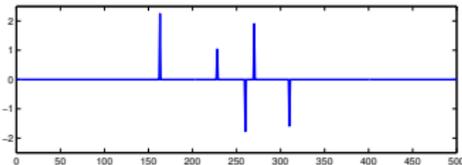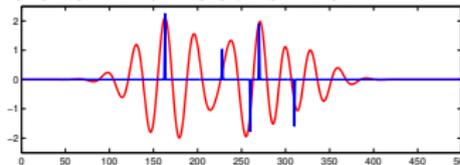○○○○

Deconvolution with a linear model

Objective : build a <u>linear</u> model between ultrasonic data and the reflectivity sequence

$$\boldsymbol{y} = \mathbf{H}(\boldsymbol{\theta})\boldsymbol{x} + \text{ noise}, \quad \boldsymbol{\theta} = \text{parameters of the propagation media}$$

- linearity $\Rightarrow$ robust estimation methods
- $\boldsymbol{x}$ "simple" $\rightsquigarrow$ identification methods to find $\boldsymbol{\theta}$

- $\boldsymbol{\theta}$ known (material properties)
 $\rightsquigarrow$ deconvolution methods to find $\boldsymbol{x}$

- Original work on homogeneous and isotropic media
  (plastics, composite materials, oils, ...)
- **Applicability to porous materials ?**

1. Time-domain propagation model with frequency-dependent attenuation

2. Validation on experimental data and deconvolution examples

3. Application to porous materials

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Frequency model for a single point source

- Two measurement configurations with target located at r_T :
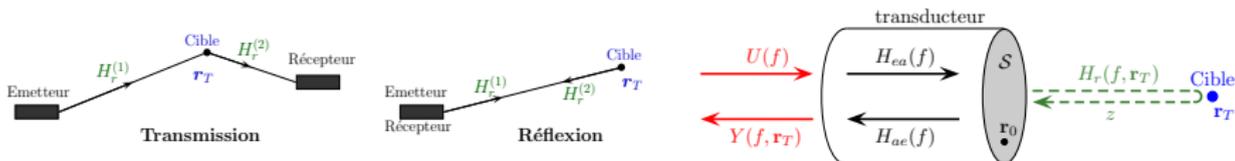
- In both cases, the received signal reads (Fink and Cardoso, 1984) :

$$Y(f, r_T) = U(f)H_{ea}(f)H_r^{(1)}(f, r_T)H_r^{(2)}(f, r_T)H_{ae}(f) \implies Y(f, r_T) = H_i(f)H_r(f, r_T)$$

- $H_i(f)$: instrumental effects (transducer bandwidth)
- $H_r(f, r_T)$: <u>propagation transfer function</u> (Stephanishen, 1971) :

$$\begin{aligned} H_r^{(1)}(f, r_T) &= \int_{r_0 \in S} \frac{e^{-jk(f)|r_T - r_0|}}{2\pi|r_T - r_0|}dS \\ &\simeq b(z)e^{-jk(f)z} \qquad (|r_T - r_0| \simeq z) \end{aligned}$$
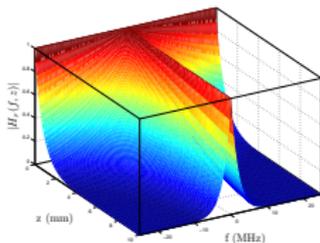
Propagation transfer function

- $k(f) = \beta(f) - j\alpha(f) \implies H_r(f, z) = b(z) \underbrace{e^{-\alpha(f)z}}_{\text{attenuation}} \underbrace{e^{-j\beta(f)z}}_{\text{dispersion}}$

- <u>Attenuation</u> : frequency power law $\alpha(f) = \alpha_0 |f|^\gamma$ (Ophir and Jaeger, 1982)

  cf. Geophysics : $\alpha(f) = \dfrac{1}{Q(f)} \dfrac{\pi f}{c}$

  $\rightsquigarrow |H_r(f, z)| = b(z)e^{-\alpha_0 |f|^\gamma z}$: cumulative low-pass filtering effect = loss of resolution

- <u>Dispersion</u> : $\beta(f) = \dfrac{2\pi f}{c_\infty} + \epsilon(f)$ (Gurumurthy and Arthur, 1982)

  For discrete-time *causal* signals, $\epsilon(f) = \dfrac{1}{f_S} \mathcal{P} \displaystyle\int_{-\frac{f_S}{2}}^{\frac{f_S}{2}} \alpha(f) \cot\left(\dfrac{\pi}{f_S}(g - f)\right) \mathrm{d}g$

  cf. Kramers-Kronig relations (Kuc, 1984, Oppenheim and Schafer, 1989)

Linear time-model with attenuation and dipersion

- $H_r(f, z) \rightsquigarrow h_r(t, z)$ impulse response for a single target at z
- Now, for any target distribution $b(z)$:

$$y(t) = \int_z b(z) h_i(t) \star h_r(t, z) \mathrm{d}z = \int_z b(z) \left(\int_\tau h_i(t - \tau) h_r(\tau, z) \mathrm{d}\tau \right) \mathrm{d}z$$

With sampled data, discretization of space and time :

$$\boxed{\boldsymbol{y} = \mathbf{H}_i \mathbf{H}_r \boldsymbol{x}}$$

where :

- $\mathbf{H}_i$: convolution matrix (transducer impulse response)
- $\mathbf{H}_r$: attenuation matrix (depending on $\alpha(f)$)
- $\boldsymbol{x}$: <u>unknown discretized reflectivity sequence</u>

- Equivalently : $\boldsymbol{y} = \mathbf{H}(\boldsymbol{\theta})\boldsymbol{x}$, with $\boldsymbol{\theta}$ the parameters of the attenuation function

(Carcreff *et al.*, IEEE Trans. Ultrason., Ferroelectr., Freq. Control., 2014)

Linear time-model with attenuation and dispersion (2)
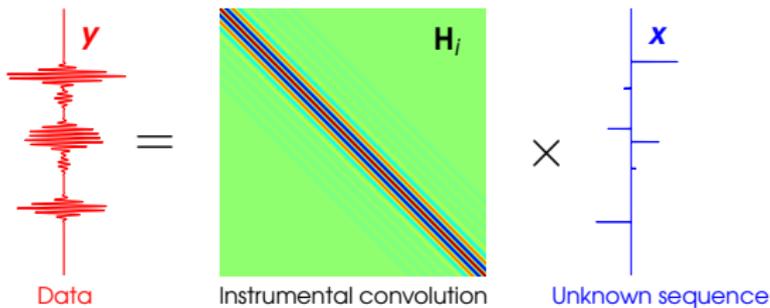
- <u>Classical convolution model</u> : $\boxed{\boldsymbol{y} = \mathbf{H}_i \, \boldsymbol{x}}$

Data   Instrumental convolution   Unknown sequence

Linear time-model with attenuation and dispersion (2)

- <u>Classical convolution model</u> : $\boxed{\boldsymbol{y} = \mathbf{H}_i \, \boldsymbol{x}}$

Data   Instrumental convolution   Unknown sequence
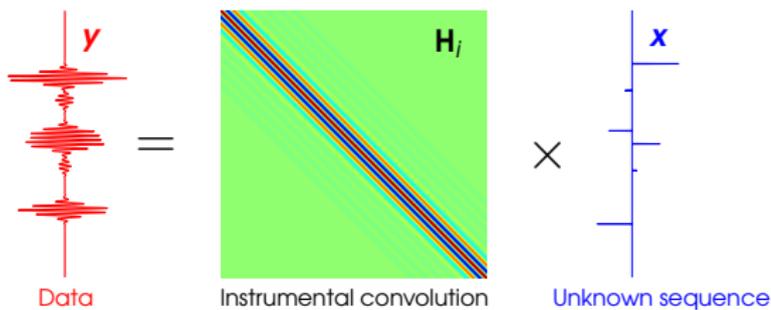
- <u>Model with attenuation and dispersion</u> : $\boxed{\boldsymbol{y} = \mathbf{H}_i \, \mathbf{H}_r \, \boldsymbol{x} = \mathbf{H}(\boldsymbol{\theta}) \, \boldsymbol{x}}$

Data   Instrumental convolution   Propagation "filtering"   **Equivalent model**

8/16
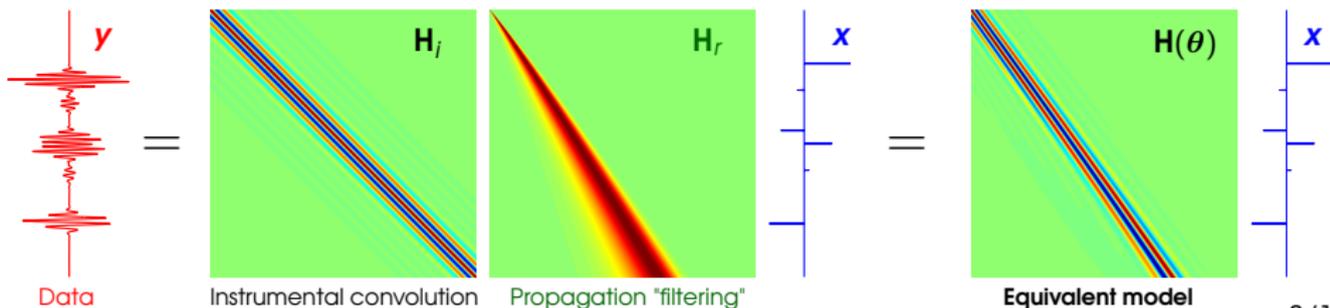
Modeling wave propagation

Model without dispersion

$\rightsquigarrow$ **Classical model**, invariant with the propagation

Model with dispersion (normalized amplitude)

$\rightsquigarrow$ **Realistic model**, attenuation and dispersion with the propagation

Experimental validation : Polycarbonate (linear attenuation)
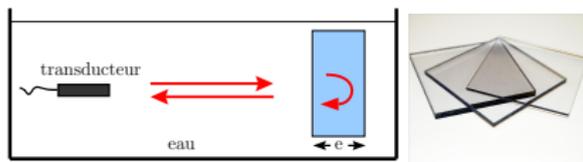
Pulse/echo measurement, 2.25 MHz, plate thickness 10.2 mm

1. Two separated echoes $y_1(t)$, $y_2(t)$

2. $\alpha(f) \simeq -\dfrac{1}{e} \log \left| \dfrac{Y_2(f)}{Y_1(f)} \right| +$ constant

3. Phase computed from $\alpha(f) \rightsquigarrow \mathbf{H}(\boldsymbol{\theta})$

4. Prediction of y_2 from y_1

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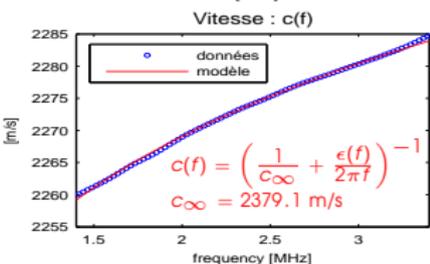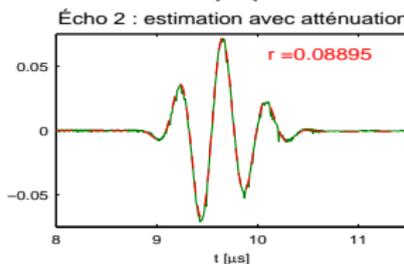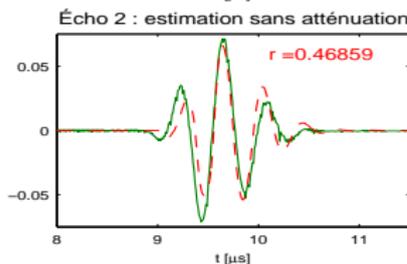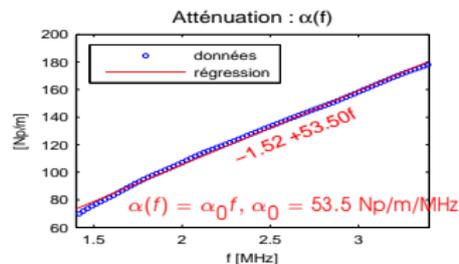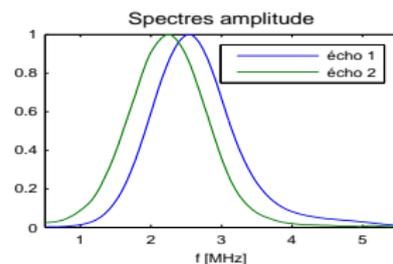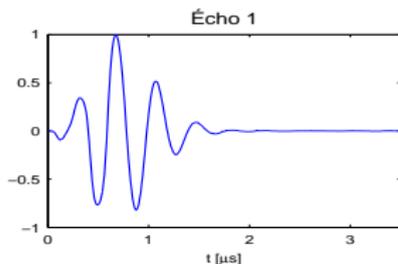
Application to spike deconvolution (1/2)

Estimation of a *sparse* **sequence :**

$$\min_{\boldsymbol{x}} \|\boldsymbol{y} - \mathbf{H}(\boldsymbol{\theta})\boldsymbol{x}\|^2 + \lambda R(\boldsymbol{x}), \ R(\boldsymbol{x}) \text{ enforcing sparsity (lots of zeros in } \boldsymbol{x})$$

- Thickness measurement of a thin plate
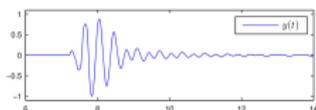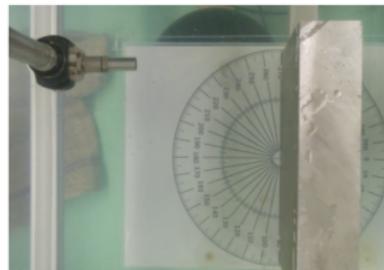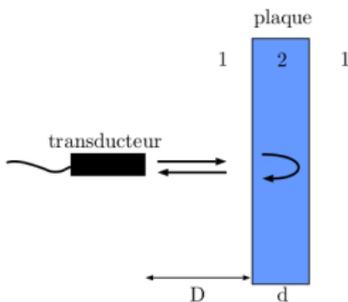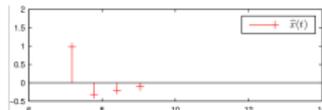
data   estimated sequence   data and model
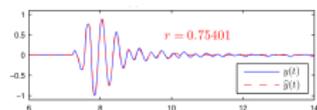
Application to spike deconvolution (2/2)
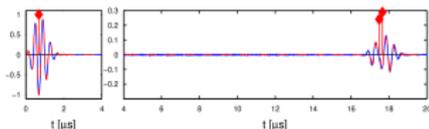
Estimation of a sparse sequence :

$$\min_{x} \|y - H(\theta)x\|^2 + \lambda R(x), \ R(x) \text{ enforcing sparsity (lots of zeros in } x)$$

- Detection of a Flat Bottom Hole
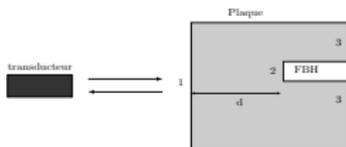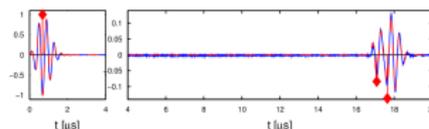

($\varnothing$ 1/2", 2.25 MHz transducer)
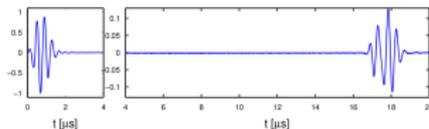
data

estimation with invariant model

estimation with propagative model

Appropriate model + efficient inversion methods → retrieve precise information

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Attenuation model from Equivalent Fluid Model

EFM **at high frequency**, convention $e^{-i\omega t}$ (Johnson-Champoux-Allard)

- Effective density : $\rho(\omega) = \rho_0 \dfrac{\alpha_\infty}{\Phi} \left(1 + \dfrac{2}{\Lambda} \sqrt{\dfrac{i\eta}{\omega \rho_0}} \right)$.

- Effective bulk modulus : $K(\omega) = \dfrac{\gamma P_0}{\Phi} \left(1 + \dfrac{2(1-\gamma)}{\Lambda'} \sqrt{\dfrac{i\eta}{\omega P_r \rho_0}} \right)$.

  ρ_0 : fluid density
  α_∞ : dynamic tortuosity,
  Φ : porosity,
  Λ : characteristic viscous length,
  Λ' : characteristic thermal length,
 η : fluid viscosity,
  P_r : Prandtl number.

- The wavenumber k is defined as : $k(\omega) = \omega \sqrt{\dfrac{\rho(\omega)}{K(\omega)}}$.

Attenuation model from EFM

Writing the wavenumber into real and imaginary parts, $k(\omega) = \beta(\omega) + i\alpha(\omega)$ leads to :

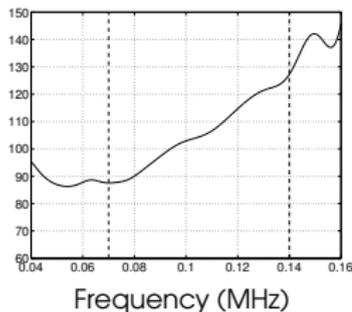$$\alpha(\omega) = \frac{\sqrt{2}}{2} \sqrt{\frac{\omega\eta}{\rho_0 c_0^2}} \left(\frac{\gamma - 1}{\Lambda' \sqrt{P_r}} + \frac{1}{\Lambda} \right) ,$$

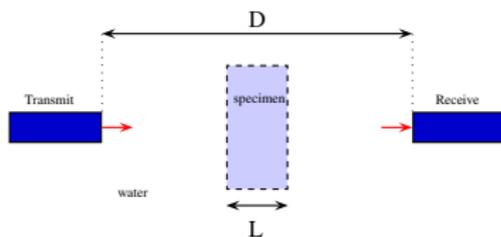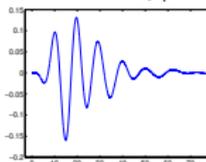which indicates attenuation in $\omega^{1/2}$.
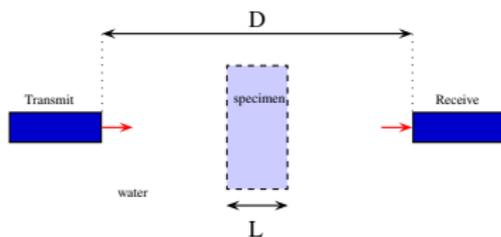
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Results with polyurethan foam, L = 20 mm, $f_c = 100$ kHz

D

Transmit

specimen

Receive

water

L

Transmitted signal
in the air (y_1)

Time (μs)

Transmitted signal
through material (y_2)

Time (μs)

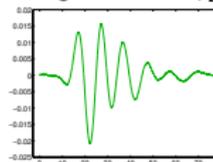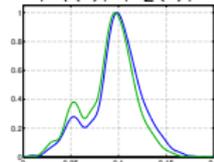Fourier transforms
$|Y_1(f)|, |Y_2(f)|$

Frequency (MHz)

Results with polyurethan foam, L = 20 mm, $f_c = 100$ kHz

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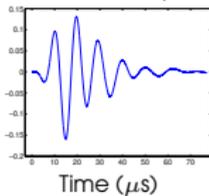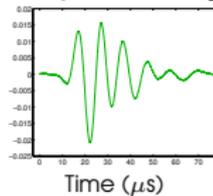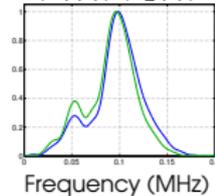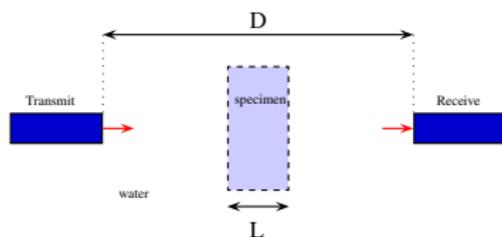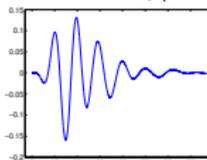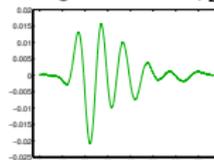
Fourier transforms $|Y_1(f)|, |Y_2(f)|$
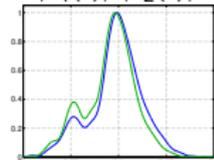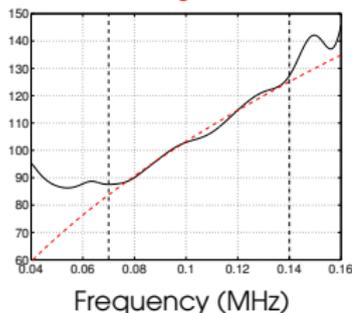
Frequency (MHz)

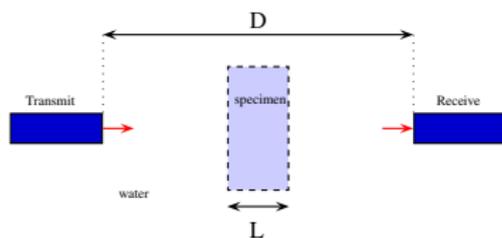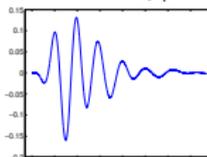$$\alpha(f) \simeq -\frac{1}{L} \log \left| \frac{Y_2(f)}{Y_1(f)} \right| + \text{cst}$$
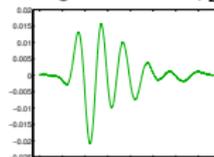

Frequency (MHz)

Results with polyurethan foam, L = 20 mm, $f_c = 100$ kHz

Transmitted signal in the air (y_1)

Transmitted signal through material (y_2)

Fourier transforms $|Y_1(f)|, |Y_2(f)|$

Time (μs)

Time (μs)

Frequency (MHz)

$$\alpha(f) \simeq -\frac{1}{z} \log \left| \frac{Y_2(f)}{Y_1(f)} \right| + \text{cst}$$
$$\simeq \alpha_0 f^{0.5}$$

Frequency (MHz)

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D

Transmit

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through material (y_2)

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Fourier transforms
$|Y_1(f)|, |Y_2(f)|$
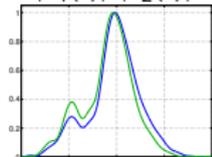
Frequency (MHz)

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$\simeq \alpha_0 f^{0.5}$

Frequency (MHz)

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$\simeq \alpha_0 f$

Frequency (MHz)

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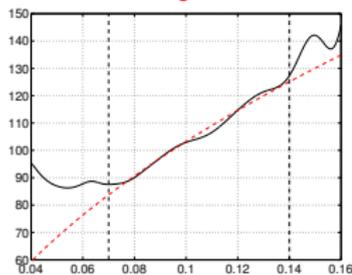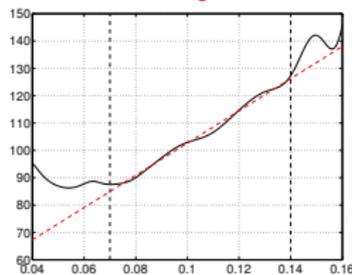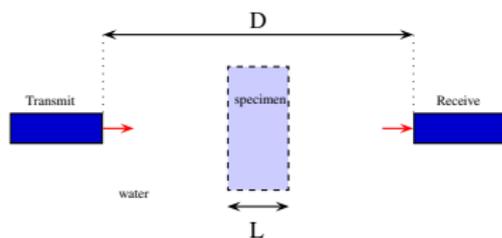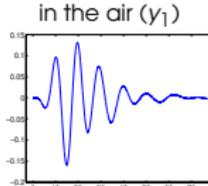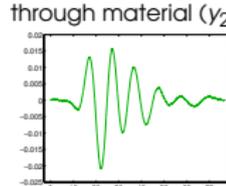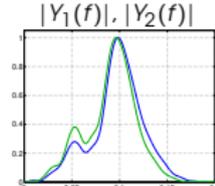$$\simeq \alpha_0 f^{0.5}$$

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$$\simeq \alpha_0 f$$

$$\alpha(f) \simeq -\frac{1}{z} \log \left| \frac{Y_2(f)}{Y_1(f)} \right| + \text{cst}$$
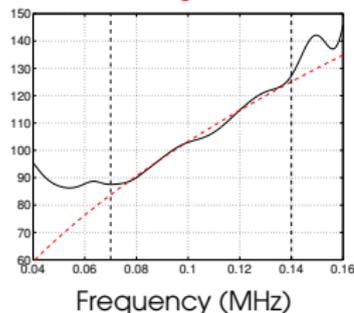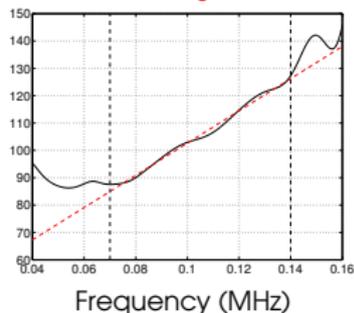$$\simeq \alpha_0 f^{1.3}$$

Results with polyurethan foam, L = 20 mm, $f_c = 100$ kHz

Transmitted signal in the air (y_1)

Transmitted signal through material (y_2)

Fourier transforms $|Y_1(f)|, |Y_2(f)|$

Time (μs)   Time (μs)   Frequency (MHz)

In the time domain...

no attenuation

$\alpha(f) = \alpha_0 f^{1/2}$

$\alpha(f) = \alpha_0 f$

$\alpha(f) = \alpha_0 f^{1.3}$

Time (μs)   Time (μs)   Time (μs)   Time (μs)
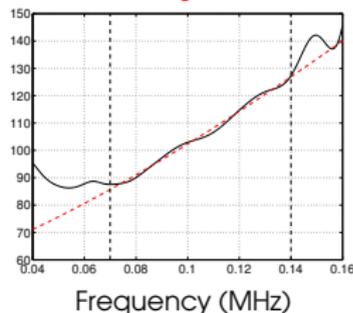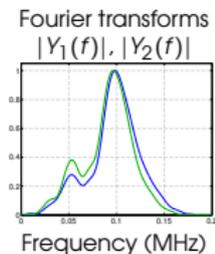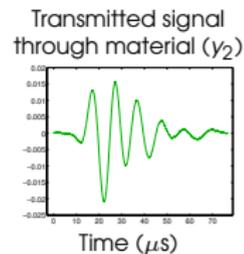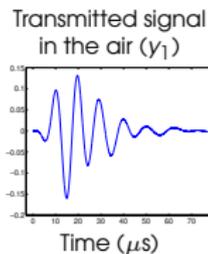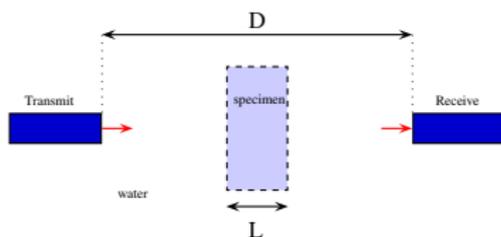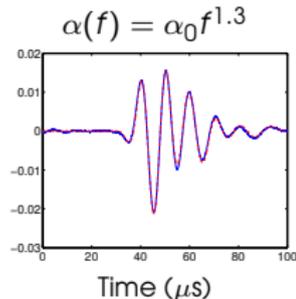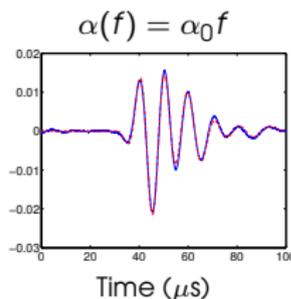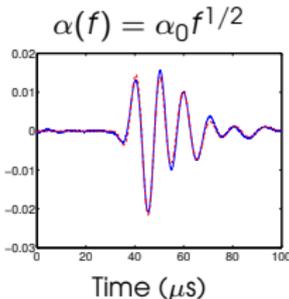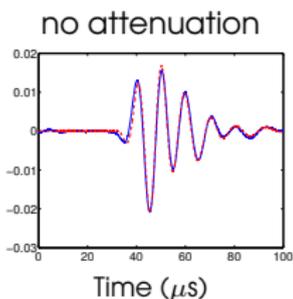
Conclusions and Perspectives

- Propagation model in the time domain ⇝ efficient inversion methods

- Experimental validation with polycarbonate, PMMA, Castor oil, metals

- More complicated in porous materials. . .
  - More experiments are needed !
  - However, fair prediction models in the time domain

Work to come : coupling deconvolution with existing characterization methods

$$\boldsymbol{y} = \mathbf{H}(\boldsymbol{\theta})\boldsymbol{x} \rightarrow \text{ joint estimation of } \boldsymbol{\theta} \text{ and } \boldsymbol{x}$$

$\rightarrow$ Estimation of porosity and tortuosity
  - oblique incidence + identification of the first reflexion (Fellah *et al.*, JASA, 2013)
  - thin material layer : separation of overlapping echoes

$\rightarrow$ Identification of $\alpha(f)$ ⇝ characteristic lengths (Leclaire *et al.*, J. Appl. Phys., 1996)

$\rightarrow$ Default detection / thickness measurement in material with unknown properties

Introduction
○○○○

Propagation model
○○○○○

Validation and deconvolution
○○○

Application to porous materials
○○○●

One particular perspective : biphasic media

- Propagation of acoustic waves through biphasic media : $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$
- Identification and separation of Biot waves by introducing appropriate models
$$\boldsymbol{y} = \mathbf{H}(\boldsymbol{\theta}_1)\boldsymbol{x}_1 + \mathbf{H}(\boldsymbol{\theta}_2)\boldsymbol{x}_2 + \mathbf{H}(\boldsymbol{\theta}_3)\boldsymbol{x}_3$$
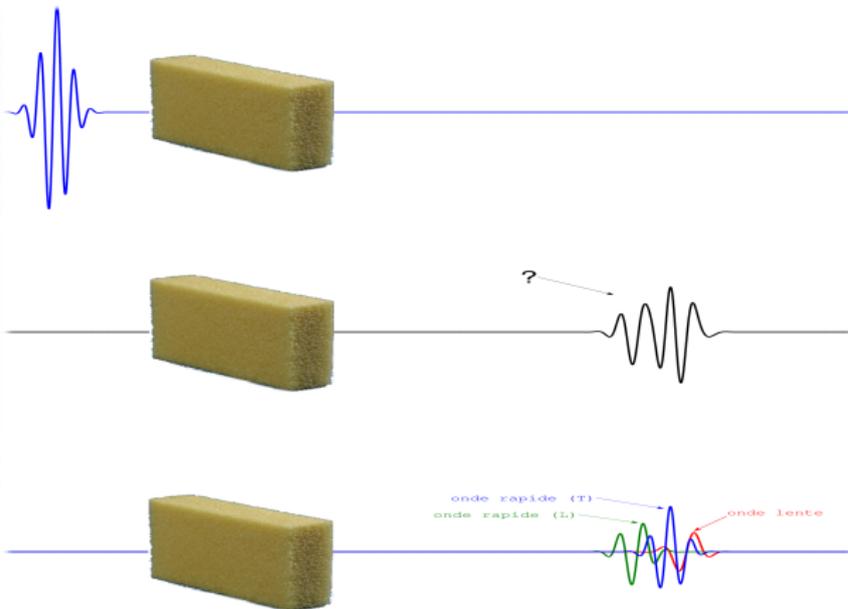
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