# *Sound Propagation in Rigid-Framed Porous Materials : Local Models*

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*Symposium on the Acoustics of Poro-Elastic Materials 19 December, Stockholm*

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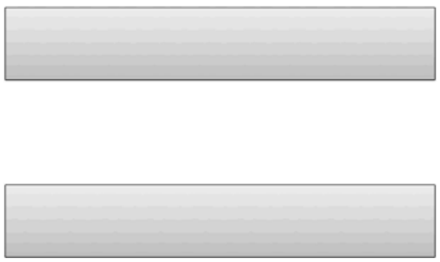
# LOCAL APPROACH
## Pore shapes

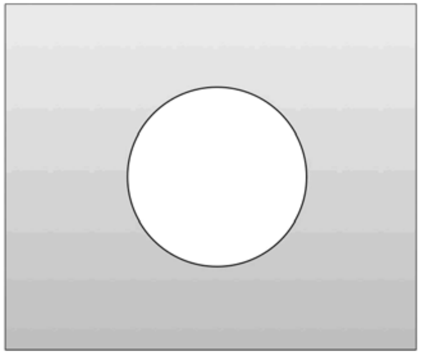
Slits



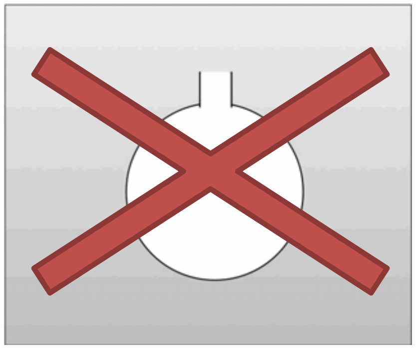
Cylinder

**More complex periodic geometries**

E.g. Circular/square inclusions



Helmholtz inclusions



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# LOCAL APPROACH

$$\rho(\omega) = \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial x}$$

$$\chi(\omega) = \frac{\partial P}{\partial t} = -\frac{\partial V}{\partial x}$$

Pore volume averages

$$k(\omega) = \omega\sqrt{\rho(\omega)\,\chi(\omega)}$$

$$Z(\omega) = \omega\sqrt{\rho(\omega)\,\chi^{-1}(\omega)}$$

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# FLUID VISCOTHERMAL EQUATIONS

*Momentum:*

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \eta_S \nabla^2 \mathbf{v} + \left( \eta_B + \frac{\eta_S}{3} \right) \nabla \left( \nabla \cdot \mathbf{v} \right)$$

*Mass:*

$$\frac{\partial b}{\partial t} + \nabla \cdot \mathbf{v} = 0$$

*Energy:*

$$\frac{\partial \tau}{\partial t} = \frac{\beta_0 T_0}{\rho_0 C_P} \frac{\partial p}{\partial t} + \frac{\kappa}{\rho_0 C_P} \partial^2 \tau$$

*Constitutive eq.:*

$$\gamma \chi_0 p = b + \beta_0 \tau$$

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## LOCAL APPROACH
## Action – Response Problems

$$-\frac{i\omega}{\nu}\mathbf{w} = -\nabla\Pi + \nabla^2\mathbf{w} + \mathbf{e}$$

$$\nabla\cdot\mathbf{w} = 0 \qquad \mathbf{w} = 0^*$$

$$\Big\} \quad \rho(\omega)$$

$$-\frac{i\omega \mathrm{Pr}}{\nu}\tau = \nabla^2\tau + 1$$

$$\tau = 0^*$$

$$\Big\} \quad \chi(\omega)$$

* At pore boundaries

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# Analytical Models

*Johnson:*

$$\alpha_J(\omega)=\alpha_\infty\left(1+\frac{8}{MS_T^2}\sqrt{1+\frac{M^2}{16}S_T^2}\right)$$

*Pride-Lafarge:*

$$\alpha_P(\omega)=\alpha_\infty\left(1+\frac{8}{MS_T^2}\left(1-q+q\sqrt{1+\frac{M^2}{16q^2}S_T^2}\right)\right)$$

$$M(\phi,\alpha_\infty,k_0,\Lambda) \qquad S_T(\Lambda) \qquad q(\phi,\alpha_0,\alpha_\infty,k_0,\Lambda)$$

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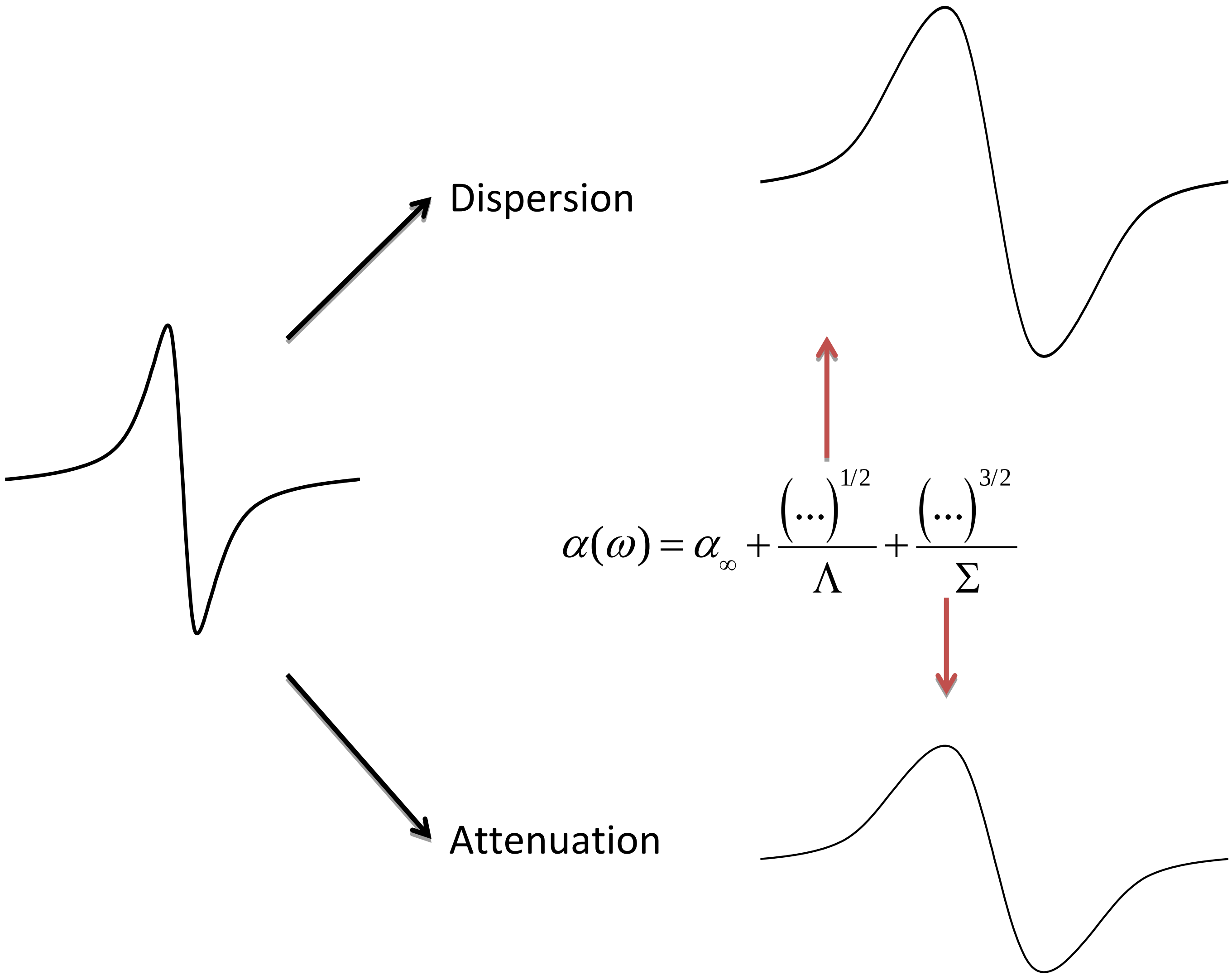
# Effect of the leading high-frequency terms




$$\alpha(\omega) = \alpha_{\infty} + \frac{(\ldots)^{1/2}}{\Lambda} + \frac{(\ldots)^{3/2}}{\Sigma}$$

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1 – Determination of $k_0, \alpha_\infty, \alpha_0, \alpha_1, \Lambda$

2 – Determination of $k'_0, \alpha'_0, \alpha'_1$

3 – Dynamic compressibility $\chi(\omega)$

4 – Dynamic density $\rho(\omega)$

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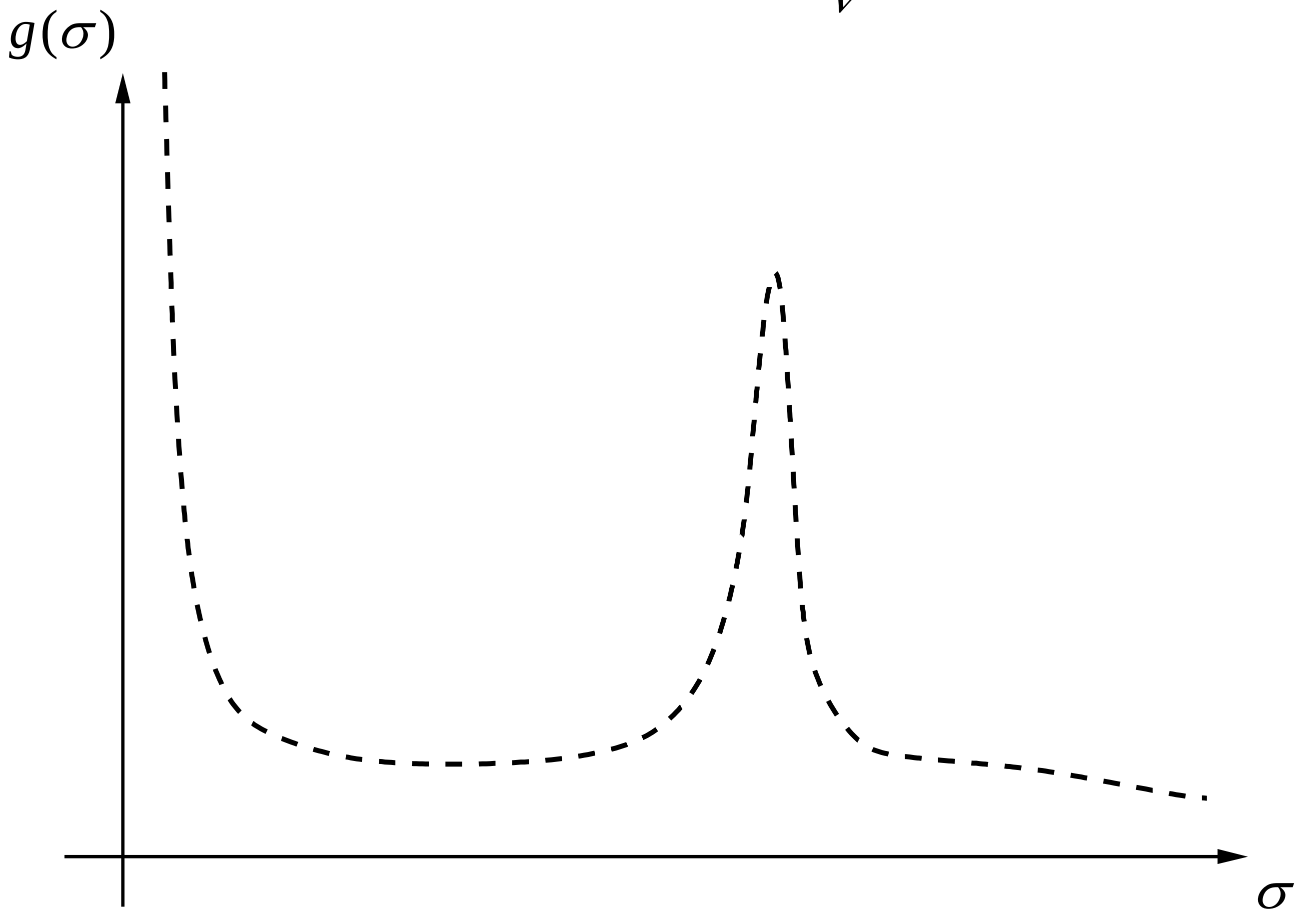
$$\frac{k(\omega)}{\phi} = \int_0^\infty \frac{\sigma g(\sigma)\, d\sigma}{1 - \frac{i\omega}{\nu}\sigma}$$



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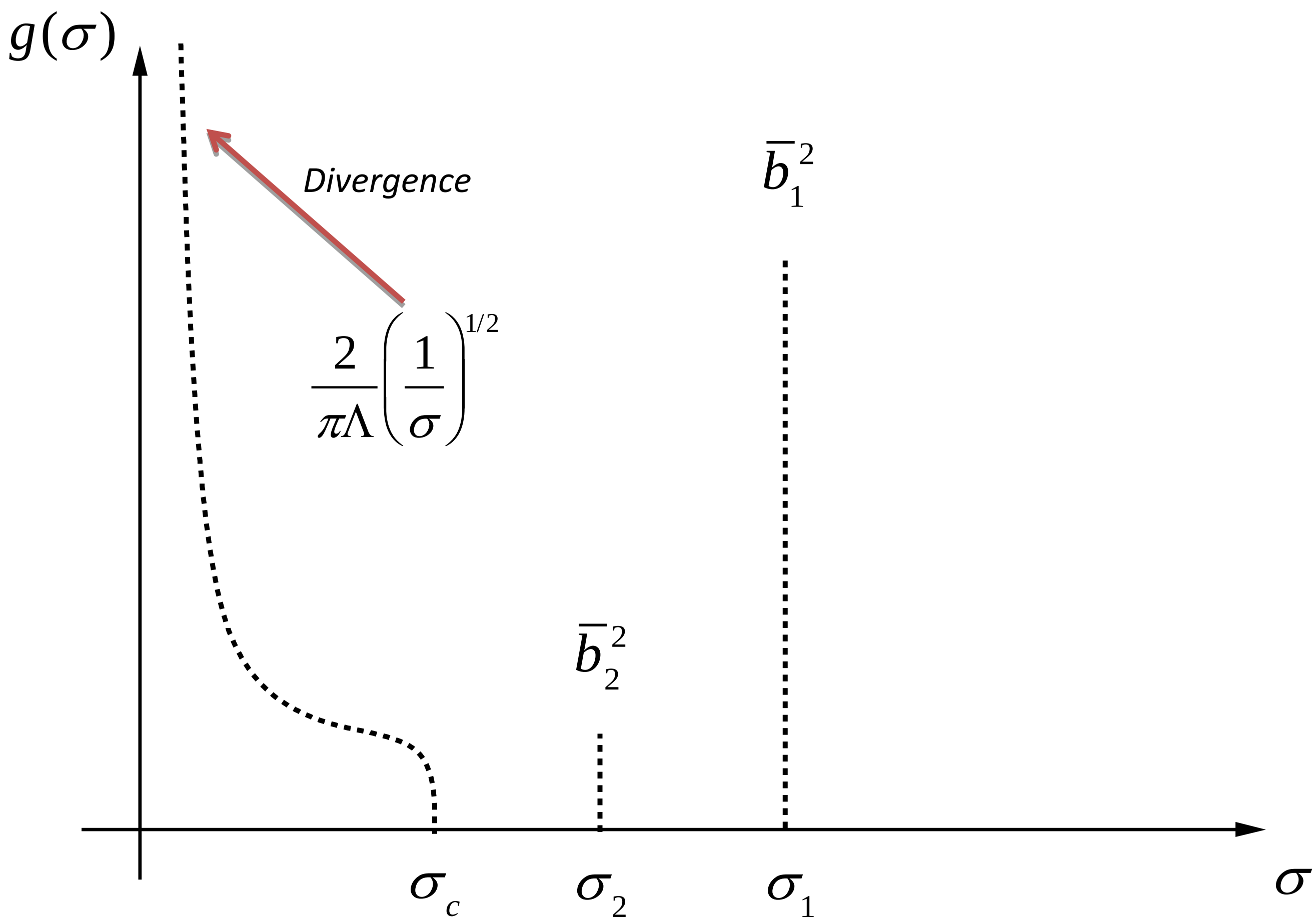


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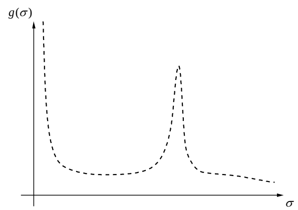
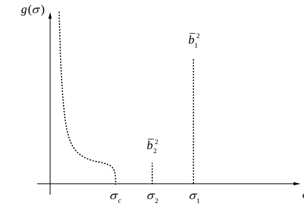
# LOW FREQUENCY LIMIT




$$\alpha(\omega) = \frac{\nu\phi}{-i\omega k_0} + \alpha_0 + \frac{i\omega k_0}{\nu\phi}\alpha_1^2$$

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$$\int_0^\infty \sigma g(\sigma)\, d\sigma = \frac{k_0}{\phi}$$

$$\frac{4}{3\pi\Lambda\alpha_\infty}\sigma^{1/2} + b_2^2\sigma_2 + b_1^2\sigma_1 = \frac{k_0}{\phi}$$

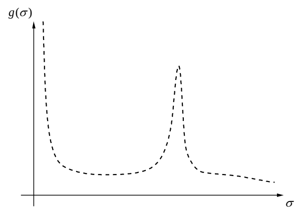
$$\int_0^\infty \sigma^2 g(\sigma)\, d\sigma = \alpha_0\left(\frac{k_0}{\phi}\right)^2$$

$$\frac{4}{5\pi\Lambda\alpha_\infty}\sigma^{5/2} + b_2^2\sigma_2^2 + b_1^2\sigma_1^2 = \alpha_0\left(\frac{k_0}{\phi}\right)^2$$

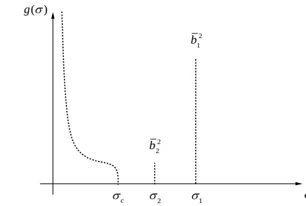
$$\int_0^\infty \sigma^3 g(\sigma)\, d\sigma = \left(\alpha_0^2 + \alpha_1^2\right)\left(\frac{k_0}{\phi}\right)^3$$

$$\frac{4}{7\pi\Lambda\alpha_\infty}\sigma^{7/2} + b_2^2\sigma_2^3 + b_1^2\sigma_1^3 = \left(\alpha_0^2 + \alpha_1^2\right)\left(\frac{k_0}{\phi}\right)^3$$

# HIGH FREQUENCY LIMIT




$$\alpha(\omega) = \alpha_\infty + \frac{2\alpha_\infty}{\Lambda}\left(\frac{\nu}{-i\omega}\right)^{-1/2} + \frac{3\alpha_\infty}{\Sigma}\frac{\nu}{-i\omega}$$



$$\int_0^\infty g(\sigma)\,d\sigma = \frac{1}{\alpha_\infty}$$

$$\frac{4}{\pi\Lambda\alpha_\infty}\sigma^{1/2} + b_2^2 + b_1^2 = \frac{1}{\alpha_\infty}$$

$$g(\sigma) \underset{\sigma\to 0}{\approx} \frac{2}{\pi\Lambda\alpha_\infty}\frac{1}{\sigma^{1/2}}$$

$$\frac{b_1^2}{\sigma_1} + \frac{b_2^2}{\sigma_2} - \frac{4}{\pi\Lambda\alpha_\infty} = \frac{3}{\alpha_\infty\Sigma} - \frac{4}{\alpha_\infty\Lambda}$$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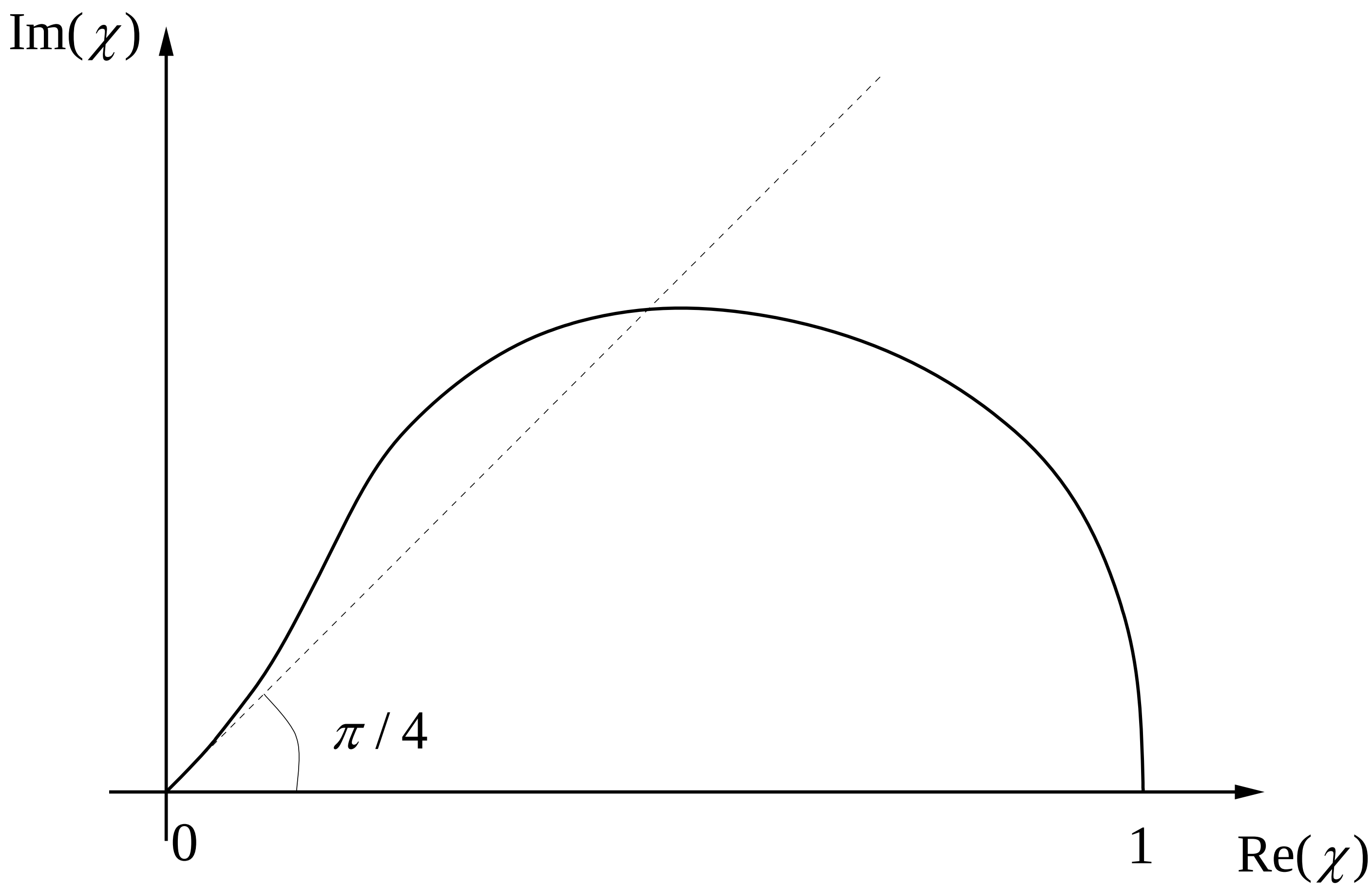
# ARGAND PLOT

$$\frac{1}{\alpha(\omega)} = \frac{1}{\alpha_\infty}\left(1-\chi(\omega)\right)$$

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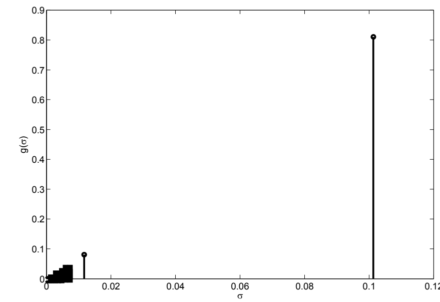
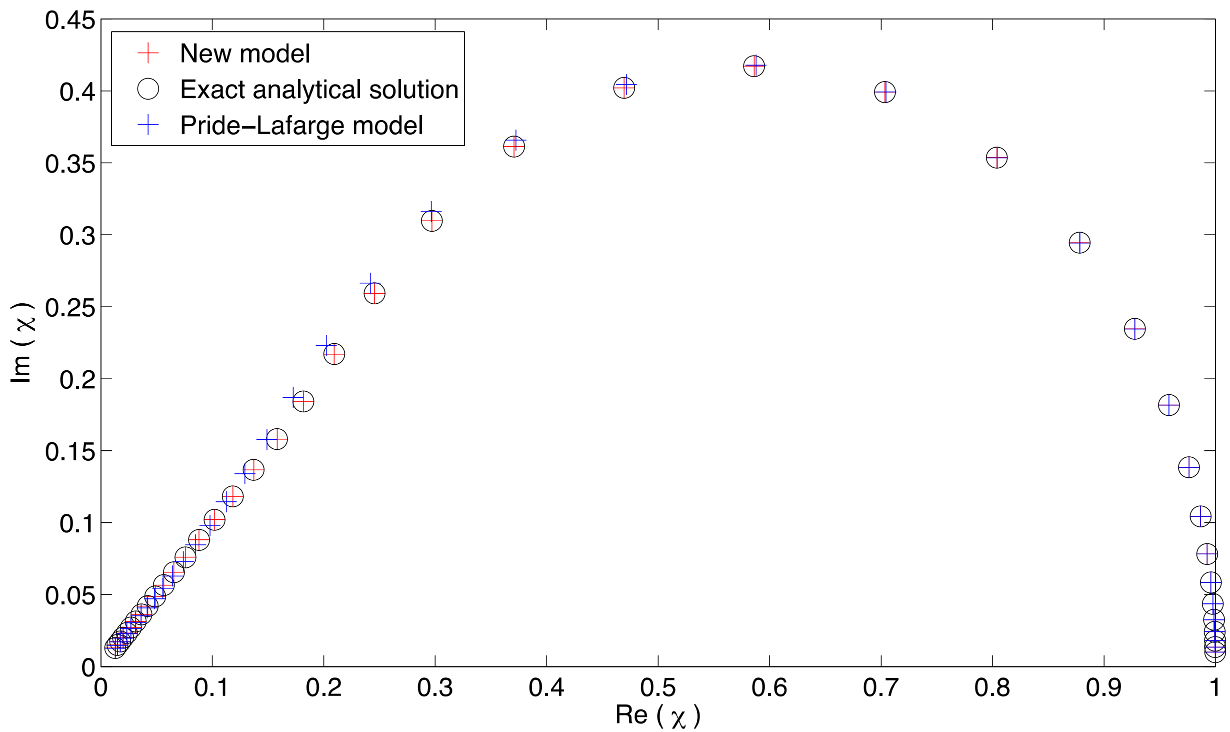
# SLITS

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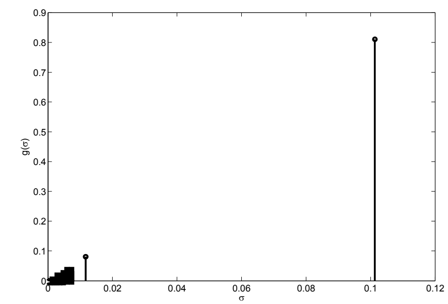
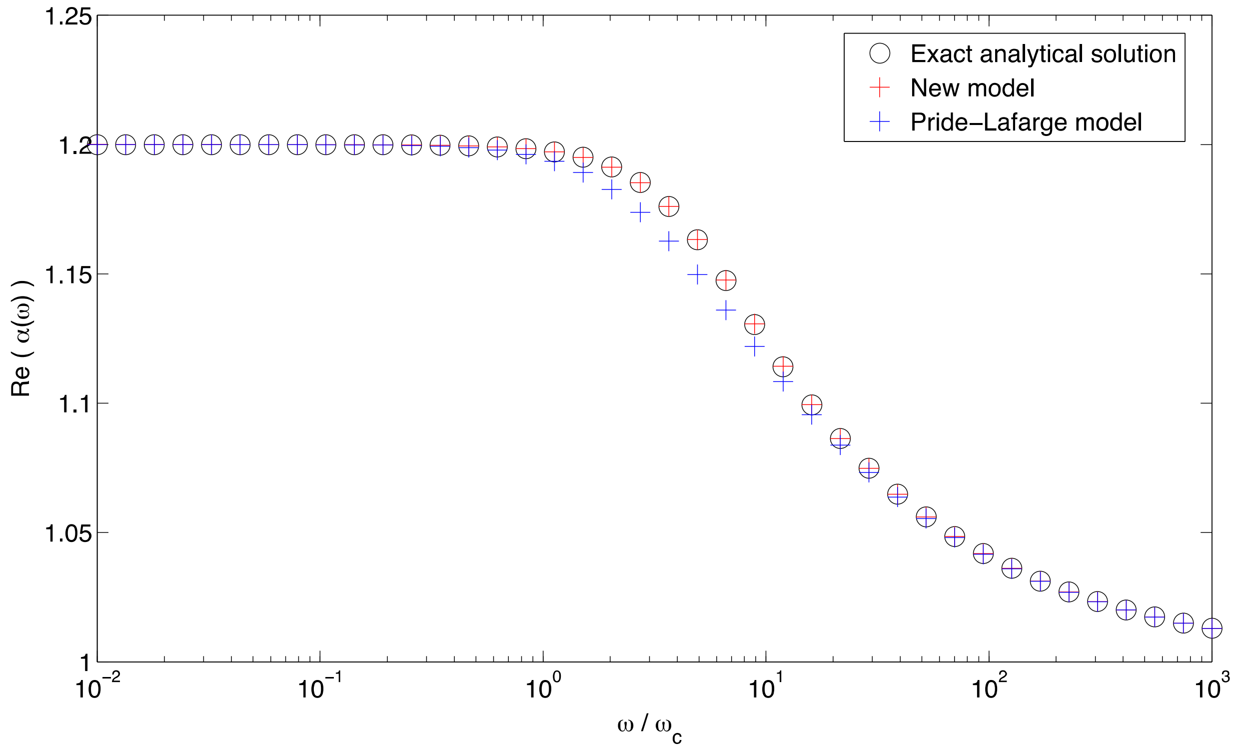
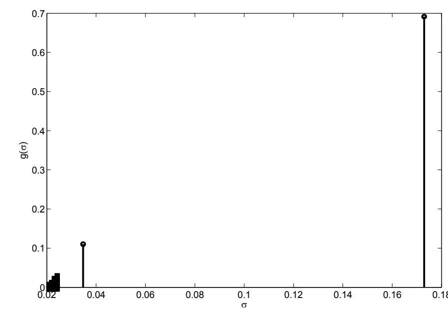
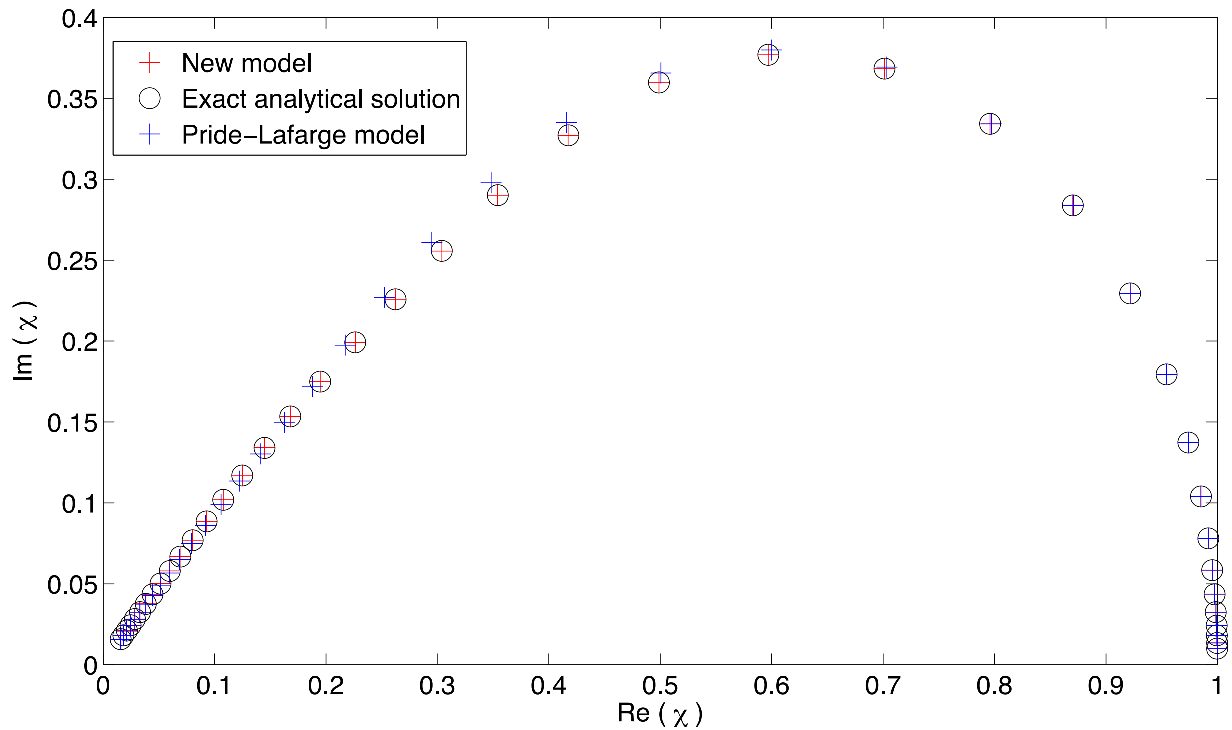
# SLITS



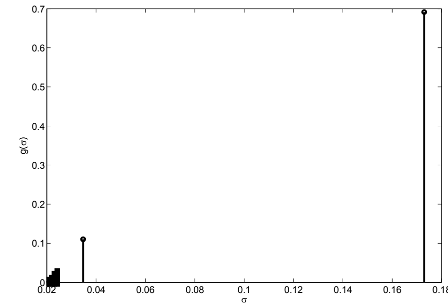



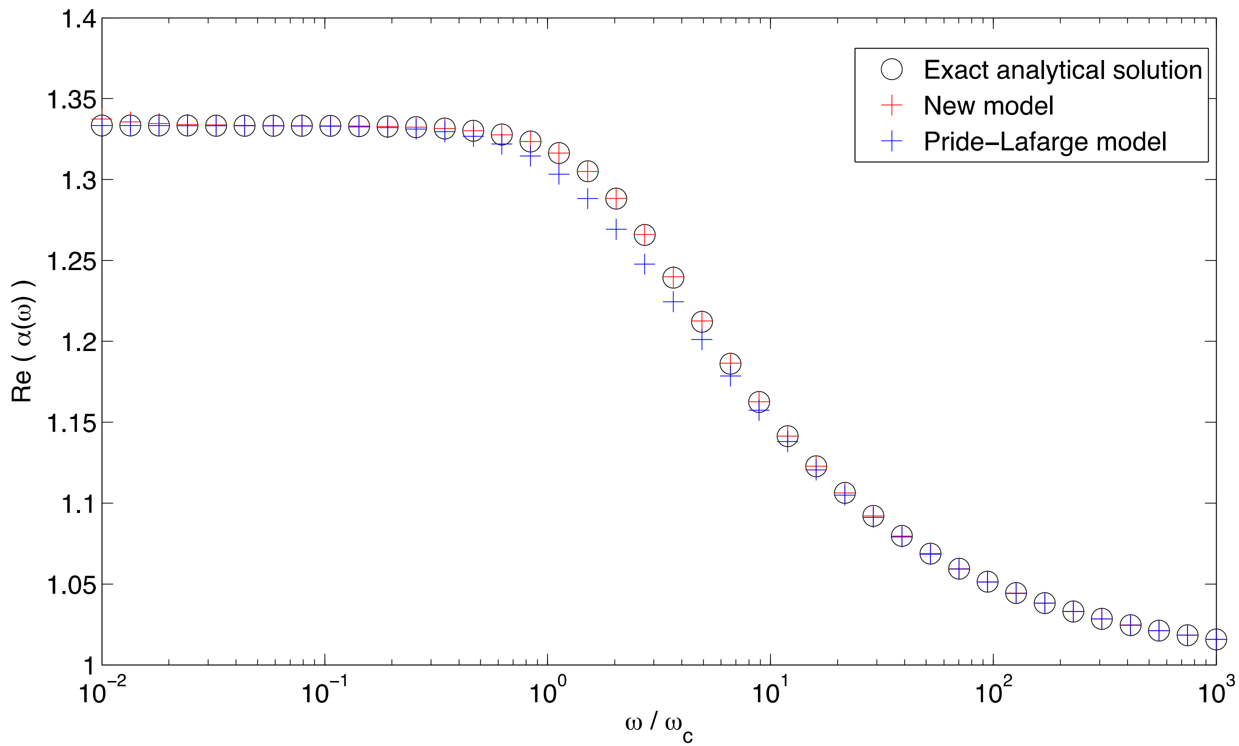
# CIRCULAR TUBES



# CIRCULAR TUBES

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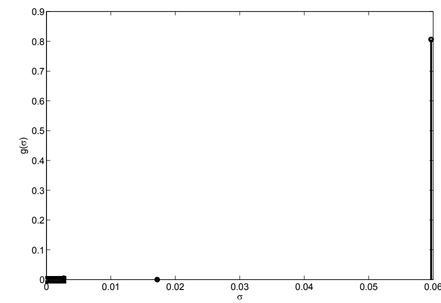
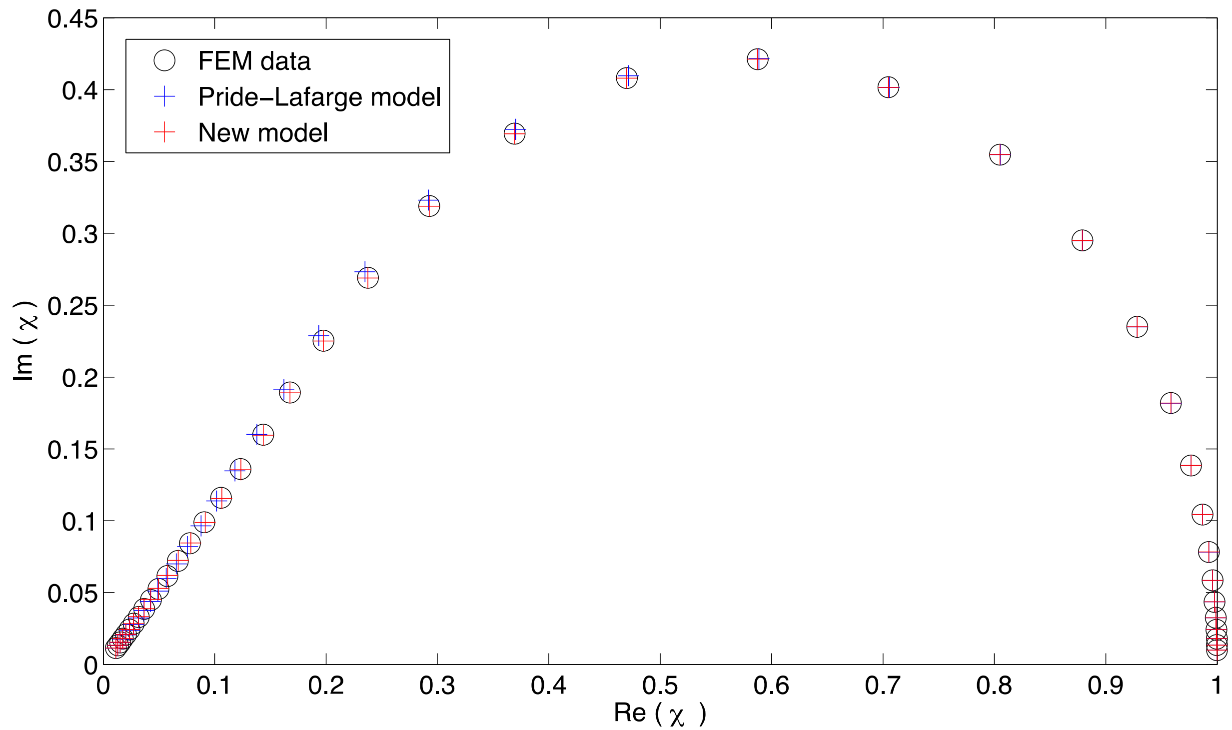
# UNIT CELL WITH CIRCULAR INCLUSION

$\phi = 0.90$



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# UNIT CELL WITH CIRCULAR INCLUSION

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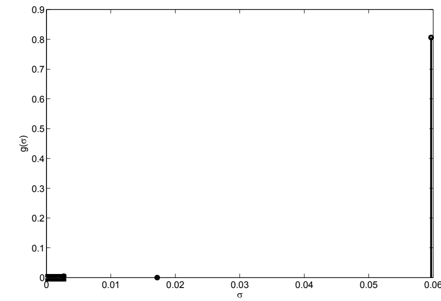


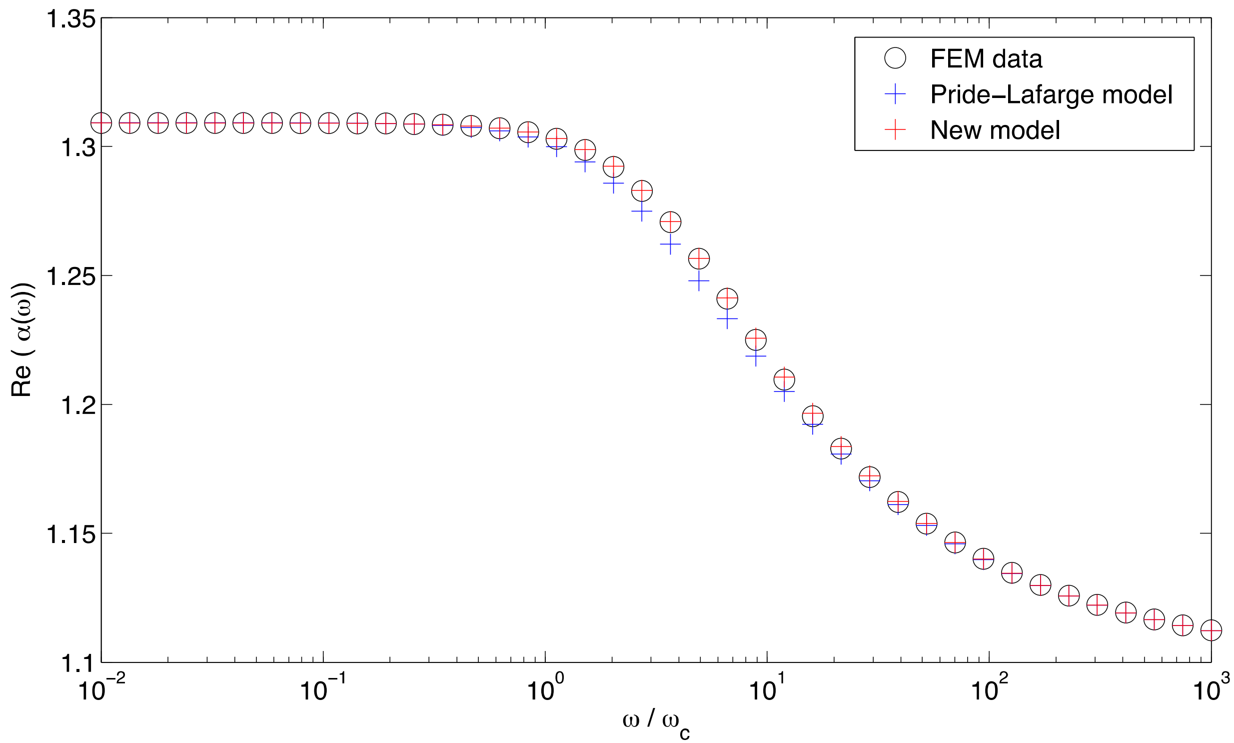

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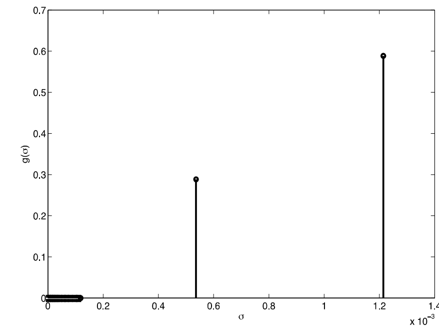
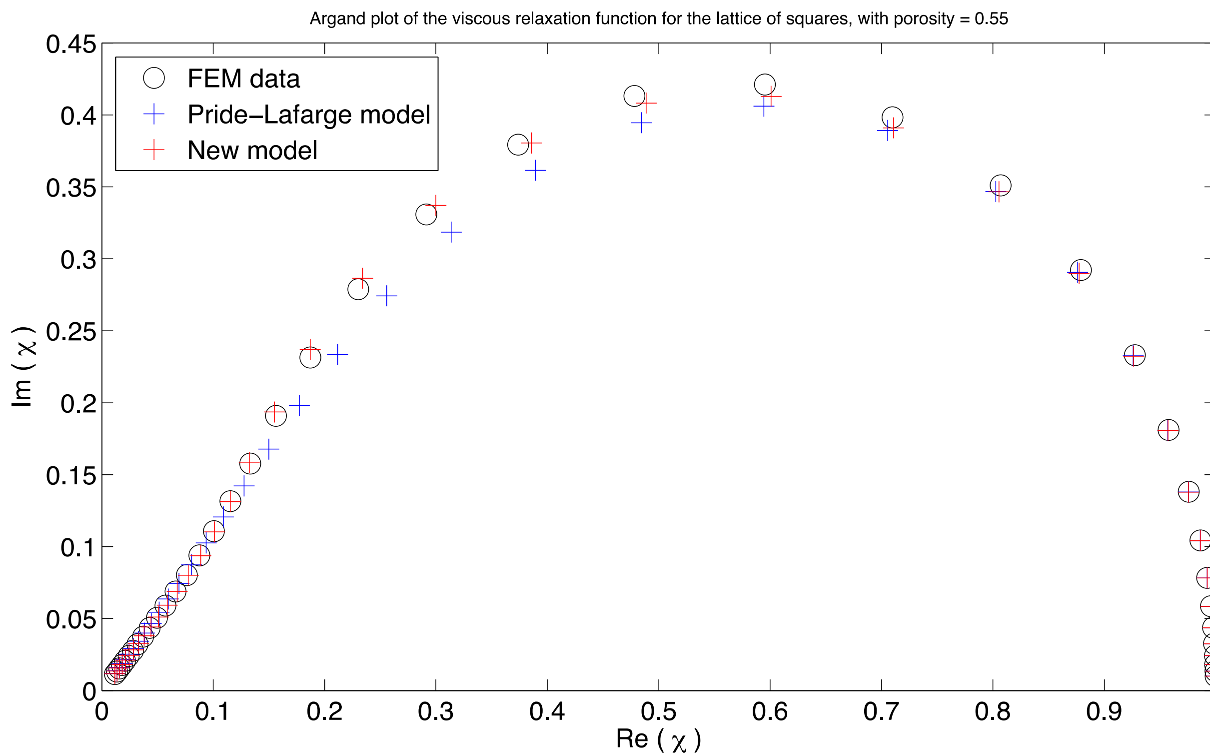
# UNIT CELL WITH SQUARE INCLUSION

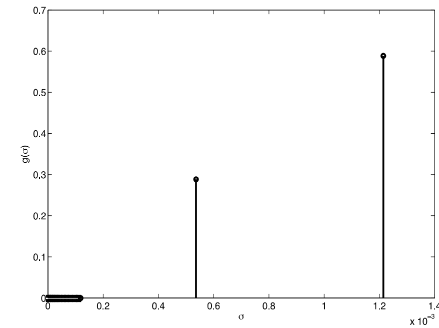
$\phi = 0.55$

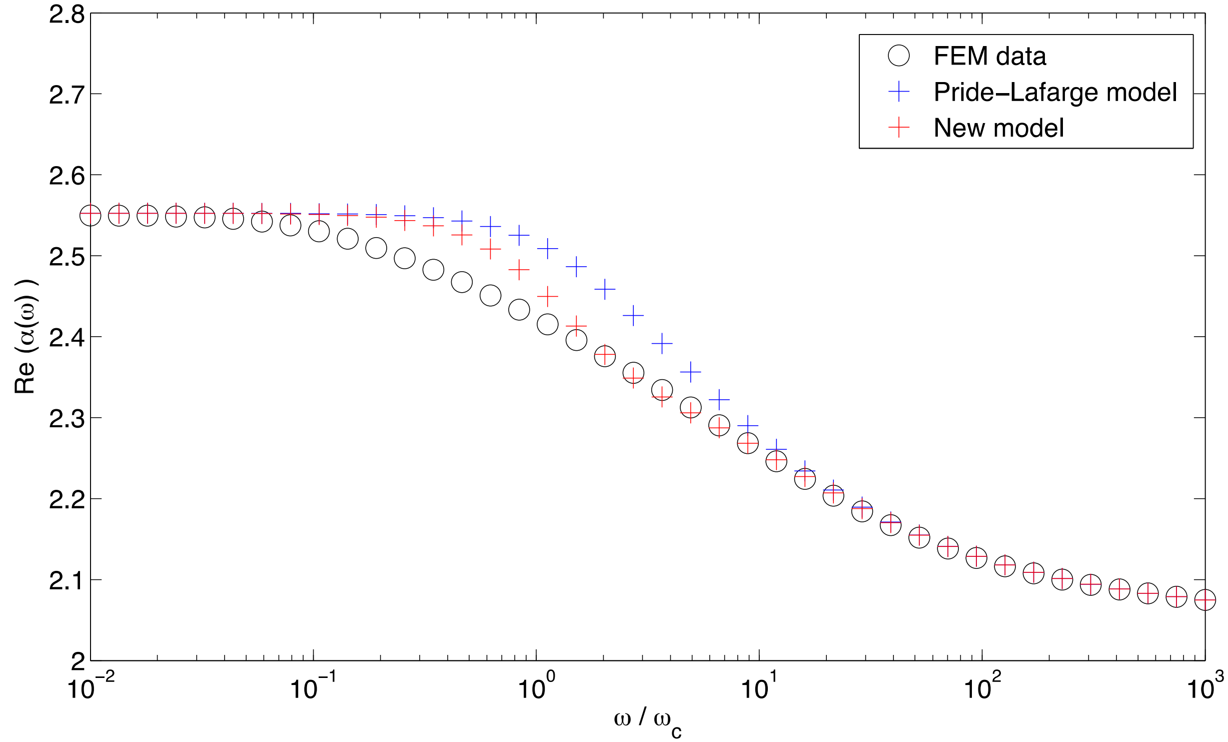


# UNIT CELL WITH SQUARE INCLUSION

$\phi = 0.55$

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# CONCLUSIONS

1 – Test of a new Local model

(unfinished … we still have to improve the precision of numerical simulations)

2 – “Better” accuracy for simple geometries

(but two more parameters than Pride-Lafarge)

3 – Probably limited use in complex geometries

*   *   *

In sufficiently simple geometries, could be get all information from intermediate frequency data only ?



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# FUTURE LINES

1 – Improve model roughness

2 – Check it with “bizarre” geometries

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# *MANY THANKS FOR YOUR ATTENTION !!*

# *Sound Propagation in Rigid-Framed Porous Materials : Local Models*

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# EXTRA SLIDES
# From simulations to model parameters

1 – FEM simulations to extract parameters

Freefem++

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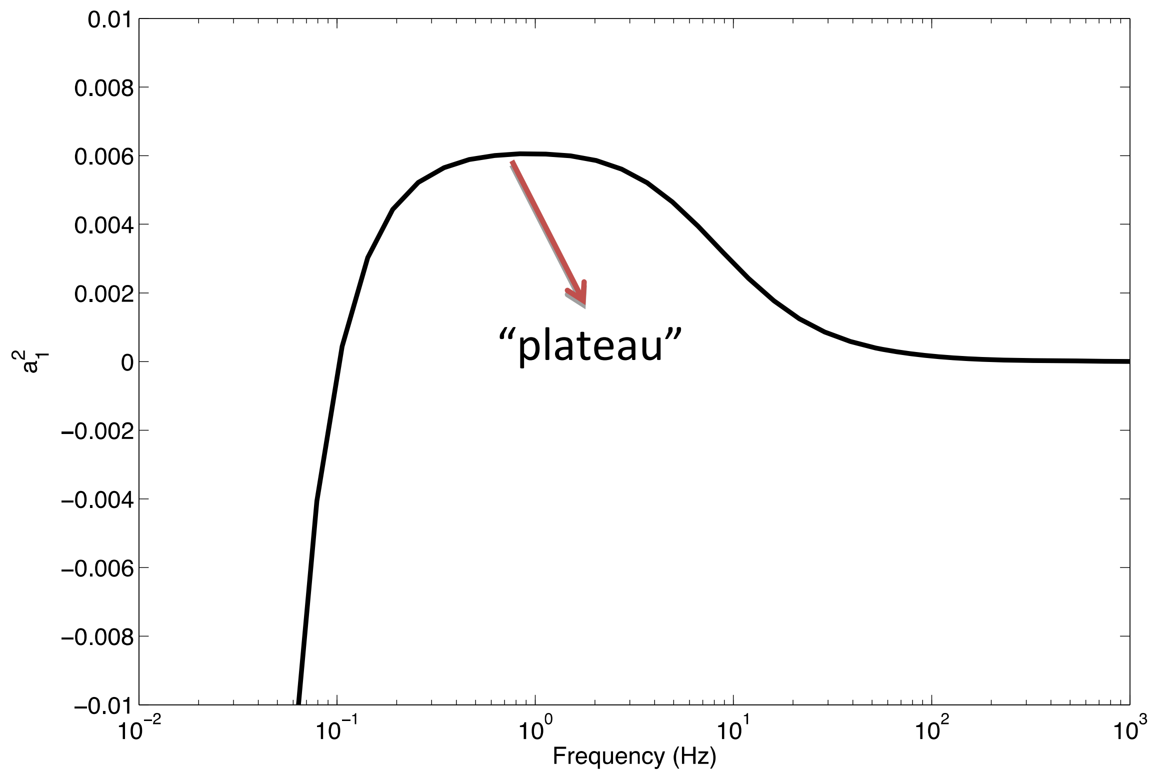
# EXTRA SLIDES
# From simulations to model parameters

2 – We need to know $\alpha_1^2 = \lim_{\omega \to 0} \frac{\mathrm{Im}\,\alpha(\omega) - \frac{\alpha_\infty}{\tilde{\omega}}}{\frac{\tilde{\omega}}{\alpha_\infty}}$



# EXTRA SLIDES
# From simulations to model parameters

3 – We need to know $\frac{\Sigma}{L^2} = \lim_{\omega\to\infty} \frac{\frac{3\alpha_\infty k_0 \alpha_\infty}{-i\tilde{\omega}\phi L^2}}{\alpha(\omega) - \alpha_\infty - \frac{2\alpha_\infty L}{\Lambda}\left(\frac{k_0\alpha_\infty}{-i\tilde{\omega}\phi L^2}\right)^{1/2}}$

4 – Compute model parameters

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