ULTRASONIC PROPAGATION IN A RIGID POROUS MEDIUM WITH A FRACTIONAL SPATIAL DIMENSION

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I. Introduction

Understanding sound wave propagation in porous materials saturated by viscous fluid is important in various applications as architectural acoustics, geophysics, rock mechanics (Allard et al. 2009). In these media, wave analysis and understanding of the propagation of sound to the fluid transmitted by the passing wave. The wave creates local pressure gradients within the fluid phase, leading to the fluid flow. Because of the complicated structure of most porous materials, the induced fluid flow can take place on various length scales. The transport of porous media is characterized by a network of interconnected pores forming an extremely irregular geometry. A possible way of describing the complex structure of each media is to use the theory of fractal (Adlern 1997) sets without non integer dimension. A fractal is a quantity which displays self-similarity on all scales. In physics, behind this word, we understand object or phenomenon having no characteristic length or having structure at a hierarchy of scales which cannot be described by smooth functions. The object does not need to exhibit exactly the same structure at all scales but the same type of structures on all scales.

Modeling of acoustic propagation in non-fractional dimensional porous media, we use the notion of fractal dimensional (Feller 1977) to describe the visco-thermal exchange between fluid and structure. An original fractional propagation equation is obtained for the ultrasonic propagation in fractal porous medium with rigid structure.

II. EQUIVALENT FLUID MODEL

In the acoustics of porous media, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Boltzmann bi- (Biot 1956). In aerosaturated porous media, the vibrations of the structure can be neglected when the excitation is not very important and the waves can be considered to propagate only in the fluid. This case is described by a model of equivalent fluid which is a particular case of the Biot model. Express the Lagrangian density of the kinetic energy and $\gamma$ the potential energy of the porous medium. The expression of the kinetic energy is given by: $\gamma = \frac{1}{2} \rho \dot{x}^2$, where $x$ is the fluid displacement, and $\rho_f$ the fluid density. The expression of the potential energy is given by: $\gamma = \frac{1}{2} \gamma \frac{\varepsilon^2}{\varepsilon}$, where $\gamma$ is the compressibility modulus of the fluid. The Lagrangian density is then written:

$$ L = \frac{1}{2} \rho \dot{x}^2 - \gamma \frac{\varepsilon^2}{\varepsilon} $$

(1)

The principle of action for the Lagrangian system depends on the vector fields $\gamma$ and $\gamma$, and the action $\mathcal{A} (\gamma, \gamma)$ is given by the integral

$$ \mathcal{A} = \int \left[ \frac{1}{2} \rho \dot{x}^2 - \gamma \frac{\varepsilon^2}{\varepsilon} \right] dx $$

(2)

with integration between the initial instant $t_i$ and final instant $t_f$.

The action can be written in the Lagrange density by:

$$ L = \frac{1}{2} \rho \dot{x}^2 - \gamma \frac{\varepsilon^2}{\varepsilon} $$

where $D$ is the spatial dimension, $\partial_0$ is the boundary, and $\partial_0$ is the boundary for all coordinates. To take into account the variations and minimizing the action $S$, all equations of Euler-Lagrange are obtained:

$$ \frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = 0 $$

(5)

where $x$, and $\dot{x}$ are the Lagrange's equations of motion.

For propagation along the $x$-axis, we obtain the equation of wave propagation in a lossless medium:

$$ \frac{\partial^2 x}{\partial t^2} = c^2 \frac{\partial^2 x}{\partial x^2} $$

(6)

Within the framework of the acoustic propagation in a porous rigid structure, the density and compressibility of the fluid are considered by the fluid-structure interactions $\gamma = \gamma - \rho_0 \alpha (\gamma, x)$, where $\gamma_0$ is the compressibility modulus of the fluid. The Helmholtz equation in lossy porous medium with rigid structure:

$$ \frac{\partial^2 x}{\partial t^2} = c^2 \frac{\partial^2 x}{\partial x^2} - \frac{\partial}{\partial x} \left[ \frac{\gamma}{\gamma_0} \frac{\partial x}{\partial x} \right] $$

(7)

A prediction of the acoustic comportment of the porous material requires the determination of the dynamic tortuosity $\alpha (x)$ and dynamic compressibility $\gamma_0 (x)$. These functions depends to the physical characteristic of the fluid in the pore space of the medium and are independent of the dynamic characteristics of the structure.

III. FRACTIONAL MODEL

When the wave frequency is high, the skin depth is very narrow and the viscous effects are concentrated in a small volume near the fluid. In this case, the viscous effects in the fluid can be neglected and the fluid behaves almost like a perfect fluid (without viscosity). In the same way, the exchange between the fluid transfer between the air and the structure and it is a well-founded approximation to consider that the compression is adiabatic.

The high frequency approximation (Johnson et al. 1987) of the response factors $a (x)$ and $b (x)$ are:

$$ a (x) = e^{-\alpha x / c} $$

$$ b (x) = e^{-\alpha x / c} $$

(8)

(9)

where $\alpha$ is the fractional dimension of the domain. This approximation justifies the introduction of the Lagrangian derivatives multiplied by a constant fraction.

The parameter $\gamma = \alpha (x)$ introduces the notions of mass and material. In this case, we take $\gamma = 0$, when $x = 0$ and $\gamma = 1$ when $x = L$. However, we impose $\gamma = 0$, when $x = 0$ or $x = L$ and $\gamma = 1$ for $0 < x < L$. The right side of Eq. (5) becomes $0$, and $\gamma = 0$ for $x = 0$ or $x = L$.

IV. INTEGRATION IN SPACES WITHOUT INTEGER DIMENSION

Stiller (1977) developed a formalism for writing the Laplace operator in spaces having a fractional dimension. This dimension is defined from an integral calculus. Let us consider the integration of a radially symmetric function $f$ on a $D$-dimensional space:

$$ \int f (x) dx = \int f (r) r^{D-1} dr $$

(14)

where $(r, \theta, \phi)$ is the distance between points $x$ and $x_0$:

$$ r = \sqrt{\sum (x-x_0)^2} $$

(15)

D is an integer, $\theta$ and $\phi$ are on the unit sphere in Euclidean spaces. This justifies the generalization of fractional dimension to any value of $D$. Using this formalism, Stiller shows that the Laplace operator in a $D$-dimensional space is:

$$ \nabla^2 f (r) = \frac{D-1}{r^{D+1}} \frac{d}{dr} \left[ r^{D-1} \frac{d}{dr} f (r) \right] $$

(16)

For a non integer $D$-dimensional space, the Stiller's formalism leads to a Laplace operator for which the non integer dimension is located in only one direction. For example, in a space where only the dimension of the $x$ coordinate is integer, the Laplacian becomes:

$$ \nabla^2 f (x, y) = \frac{d^2}{dx^2} f (x, y) $$

(17)

V. ULTRASONIC PROPAGATION IN FRACTIONAL DIMENSIONAL SPACE

The Stiller's formalism of noninteger dimensional spaces has been generalized to $n$ orthogonal coordinates by C. Palmer and P.N. Stavrinou (2005). The authors derive the Euler Lagrange equations of a field theory in such spaces which follow from the stationarity property of the action integral with the respect to variations of the fields and their derivatives. So, if, the Lagrangian density by:

$$ S = \int d^d x \mathcal{L} $$

(18)

where $\mathcal{L} (\phi, \phi', \psi)$ is the Lagrangian density corresponding to a definite point of the space-time, the Euler-Lagrange equations are:

$$ \nabla^2 \phi (x, y) = \frac{D-1}{r^{D+1}} \frac{d}{dr} \left[ r^{D-1} \frac{d}{dr} \phi (x, y) \right] $$

(19)

V. Conclusion

An original fractional propagation equation is established for the ultrasonic propagation in non-fractional dimensional rigid porous media. The coefficients of this equations depends on the acoustic properties of the porous material and to the fractional dimension of the medium. By increasing the fractional dimension of the material, the amplitude of the wave decreases.


