

Conventional ways to obtain the complex reflection coefficient r

$$r = \frac{p_r}{p_i}$$

The incident sound pressure p_i and the reflected one p_r are evaluated on the surface of the porous material (point 1)

$$r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}}$$

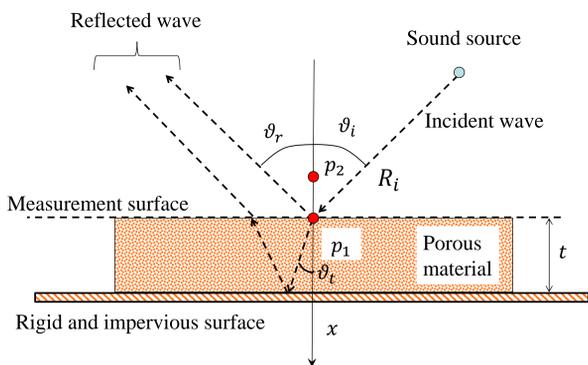
The reference impedance Z_o can be obtained considering the hypothesis of:

- Plane wave $Z_o = \rho_0 c_0$;
- Spherical wave $Z_o = \rho_0 c_0 \frac{jkR_i}{1+jkR_i}$.

The surface impedance Z_s is given:

- directly by the ratio $Z_s = \frac{p_t}{v_t} = \frac{p_r + p_i}{v_{i,x} - v_{r,x}} = \frac{p_1}{v_{i,x} - v_{r,x}}$;
- by the measurement of the normal velocity using Euler approximation:

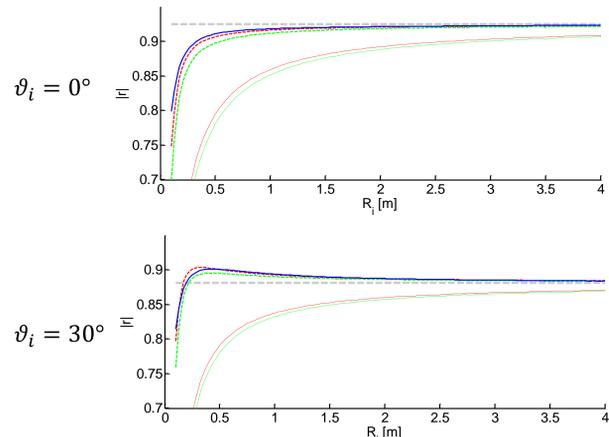
$$Z_s = \frac{p_t}{v_t} = \frac{p_1}{\frac{j(p_1 - p_2)}{\omega \rho_0 (x_1 - x_2)}}$$



Complex reflection coefficient r for end-less porous material

Ex. Fiber material $\sigma=2254$ [Pa s/m²]

$f=500$ Hz $t=0.03$ m



--- Theoretical reflection coefficient for plane wave

— $r = p_r/p_i$ — Reference value ?

$$r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}} \quad Z_s = \frac{p_1}{\frac{j(p_1 - p_2)}{\omega \rho_0 (x_1 - x_2)}} \quad Z_o = \rho_0 c_0 \frac{jkR_i}{1 + jkR_i}$$

$$r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}} \quad Z_s = \frac{p_1}{\frac{j(p_1 - p_2)}{\omega \rho_0 (x_1 - x_2)}} \quad Z_o = \rho_0 c_0$$

p_1, p_2, p_r, p_i are obtained by the theoretical Allard model¹ valid for an infinite porous sample.

With this model no assumption about the local reactive hypothesis must be taken into account. The input parameters of the material are the complex density and the complex wave number.

The complex density and the complex wave number can be obtained using a suitable model. In this case the simple model of Delany and Bazley² is used because it depends just on one parameter: the air-flow resistivity.

Since the edge effects are not considered having the material infinite surface dimensions, the only differences are due to:

- 1- the relation used to obtain r ;
- 2- the proper value of the reference impedance.

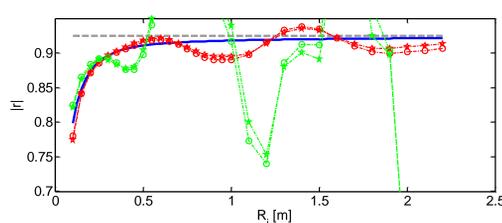
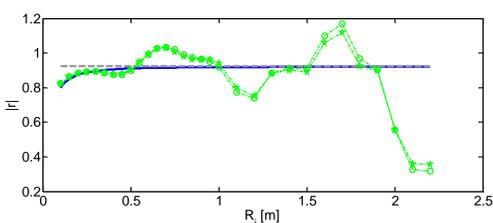
The main question arising by these results is related to what is the **true value** of the reflection coefficient.

1 - "The acoustic sound field above a porous layer and the estimation of the acoustic surface impedance from free-field measurements" Jean-François Allard, Walter Lauriks, and Christine Verhaegen, JASA, 91, pp. 3057 (1992).

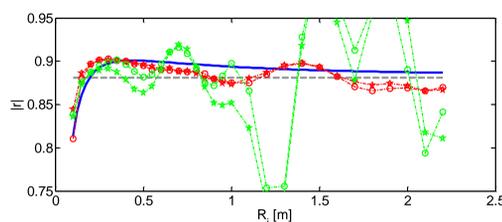
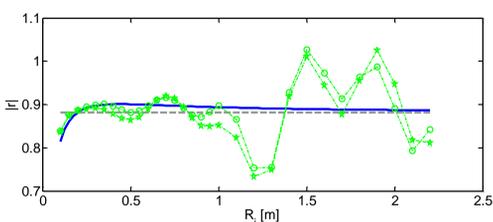
2- Delany M. E. and Bazley E. N., Acoustical properties of fibrous absorbent materials, Applied Acoustics 3, pp. 105, (1970).

Complex reflection coefficient r for a sample of finite dimensions

$\vartheta_i = 0^\circ$



$\vartheta_i = 30^\circ$



--- Theoretical reflection coefficient for plane wave

— $r = p_r/p_i$ — End-less surface – Allard model

—○— Point A $r = p_r/p_i$
—☆— Point B

$$r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}} \quad Z_s = \frac{p_t}{v_t} = \frac{p_r + p_i}{v_{i,x} - v_{r,x}}$$

Finite dimensions

Numerical simulations

$r = p_r/p_i$ where p_r and p_i are obtained by the theoretical Allard model and numerical simulations.

$Z_s = \frac{p_i + p_r}{v_{x,i} - v_{x,r}}$ where $p_r, p_i, v_{x,i}$ and $v_{x,r}$ are obtained by numerical simulations.

The difference between $r = p_r/p_i$ evaluated on the material surface by means of numerical simulations and with the Allard model become greater as the distance of the sound source from the material increases.

In numerical analysis, for a finite sample of material, the influence of edge effects is greater on r value evaluated

as $r = p_r/p_i$ than one evaluated as $r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}}$.

In numerical analysis, neither $r = \frac{Z_s - \frac{Z_o}{\cos \vartheta_i}}{Z_s + \frac{Z_o}{\cos \vartheta_i}}$ nor

$r = p_r/p_i$ approach the values of the end-less surface model.

Considerations

The complex reflection coefficient takes into account the amount of energy that is not absorbed by a sound absorbing system. A porous material backed by a rigid and impervious surface has been considered in this study. The common way to obtain this parameter is to consider the surface impedance and a reference impedance value as well. By considering $r = p_r/p_i$ as reference value, where the incident and reflected sound pressure are evaluated by the Allard model, it is possible to observe that the plane wave reference impedance is mostly inappropriate for small distance of the sound source from the material surface.

In real situations the dimensions of the material are finite. The simulations show that the reflection coefficient value depends strongly on the edge effects especially when the distance of the sound source from the material surface increases. This aspect is already known for local reactive porous material and, in this preliminary study, it is also highlighted for non-local reactive porous material and for spherical wave.

The results of these analysis are useful in statistical acoustics where, at given frequency, a porous material is characterized by a single parameter: the sound absorbing coefficient.