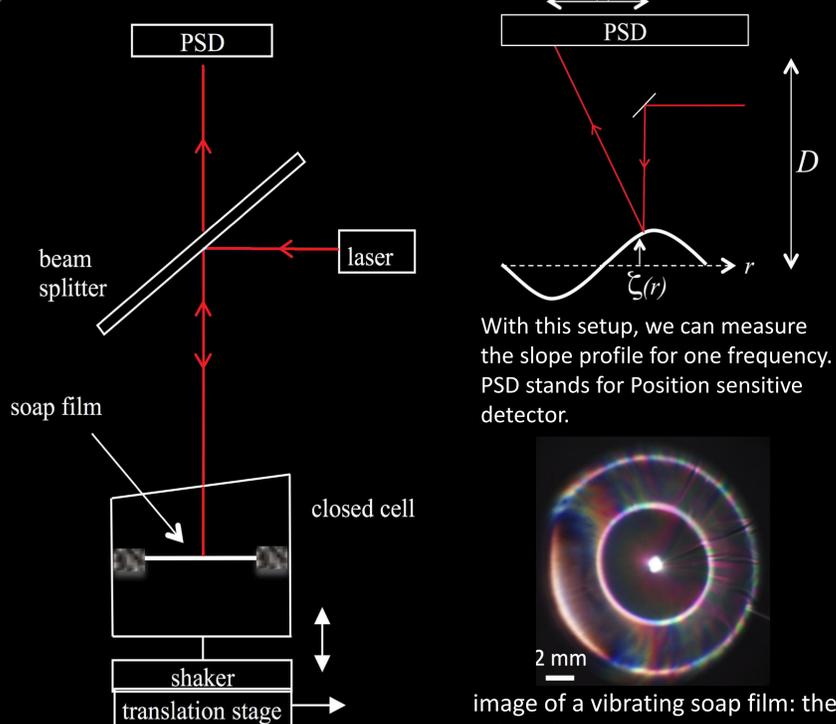


We investigate the complex dispersion relation of a transverse antisymmetric wave on a horizontal soap film. Experimentally, the complex wave number k at a fixed forcing frequency is determined by measuring the vibrating amplitude of the soap film: the wavelength (linked to the real part of k) is determined by the spatial variation of the amplitude; the decay length (linked to the imaginary part of k) is determined by analyzing the resonance curves of the vibrating wave as a function of the frequency. Theoretically, we compute the complex dispersion relation taking into account the physical properties of the bulk liquid and gas phase, and of the gas-liquid interfaces. The comparison between the computation (developed to the leading order in our experimental conditions) and the experimental results confirms that the phase velocity is fixed by the interplay between surface tension, and liquid and air inertia, as reported in previous studies. Moreover, we show that the attenuation of the transverse antisymmetric wave originates from the viscous dissipation in the gas phase surrounding the liquid film. This result is an important step to understand the propagation of an acoustic wave in a liquid foam, in a bottom-up approach.

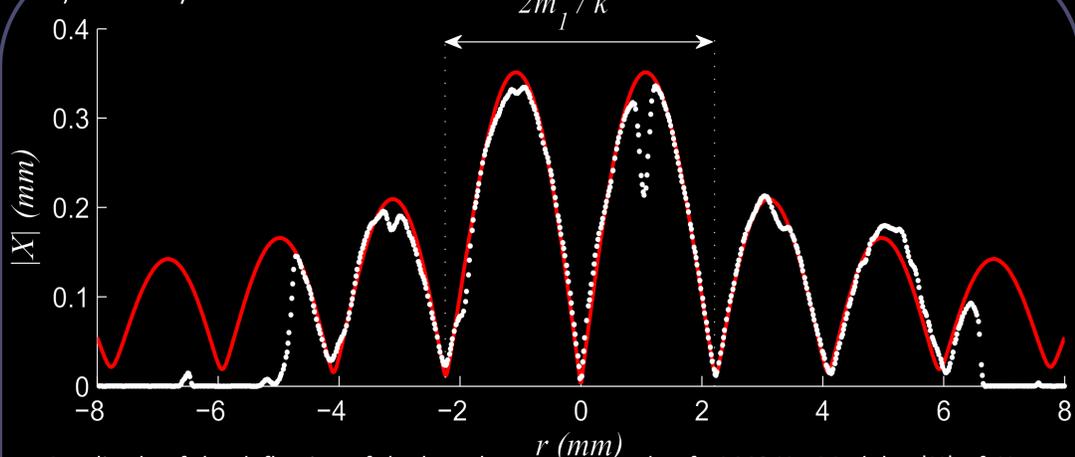
1) Experimental setup



The films are made with: water + 2.8 g/L TTAB + 10 %wt glycerol

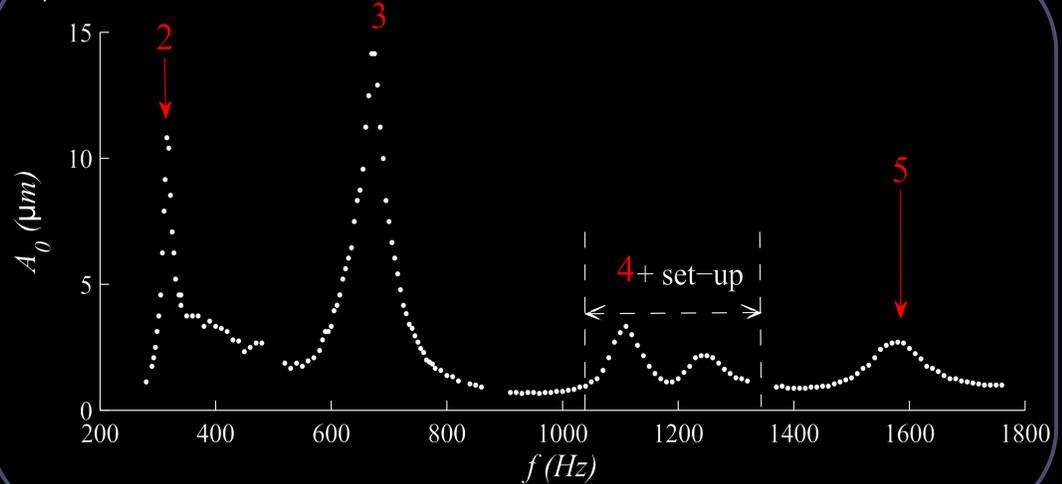
image of a vibrating soap film: the bright rings correspond to the antinodes of the standing vibrating wave

2) Data analysis



Amplitude of the deflection of the laser beam measured at $f = 1440$ Hz: Modulus $|X|$ of X as a function of the position r from the center of the film. The solid line is the best fit (expression $|X| = 2DkA_0J_1(kr)$, J_1 being the Bessel function of the first kind) of the data with $k = k' + ik''$. We measure from this fit the amplitude A_0 and real part of the wavenumber k' for one frequency.

3) measurements



Amplitude A_0 at the film center as a function of the frequency f .

4) Complex dispersion relation (long wavelength limit):

$$\left(\frac{\omega}{k'}\right)^2 \left(\rho h + \frac{\rho_a}{k} \frac{m_a}{m_a - k}\right) = \gamma + \frac{\eta\omega}{\eta\omega - i\varepsilon k^2 h} [\varepsilon(kh)^2 + E] \text{ with } E = \gamma \frac{2\eta}{k} (k + m_a)h + \frac{\rho_a \omega^2 h}{k(m_a - k)} \left(1 - \frac{\eta_a k^2 + m_a^2}{\eta k^2}\right) i\eta_a \omega h \left(2\frac{\eta_a}{\eta} - 3\right) + \frac{m_a}{k} \left(1 - 2\frac{\eta_a}{\eta}\right) \frac{m^2}{k^2} (k + m_a)h$$

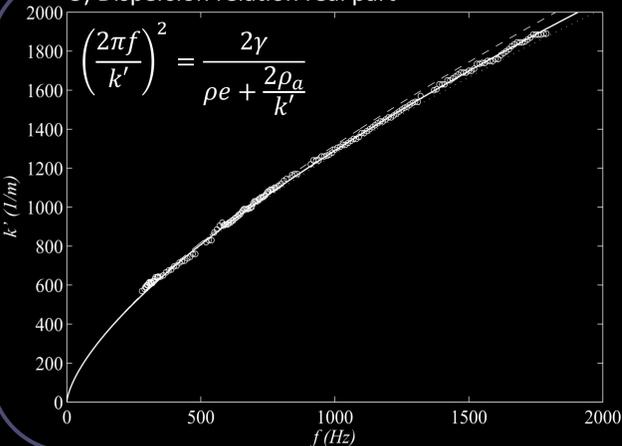
Where m_a is the air vibration mode and $e=2h$.

Air and liquid inertia and viscosity ($\rho_a, \eta_a, \rho, \eta$), interfacial viscoelasticity ε , and surface tension γ taken into account.

By simplifying to the leading order, we obtain two expressions:

$$\text{Real part: } \left(\frac{\omega}{k'}\right)^2 = \frac{2\gamma}{\rho e + \frac{2\rho_a}{k'}} \quad \text{Imaginary part: } \frac{-k''}{k'} \left(\rho e + \frac{3\rho_a}{k'}\right) = \rho_a \sqrt{\frac{\eta_a}{2\rho_a \omega}}$$

5) Dispersion relation real part

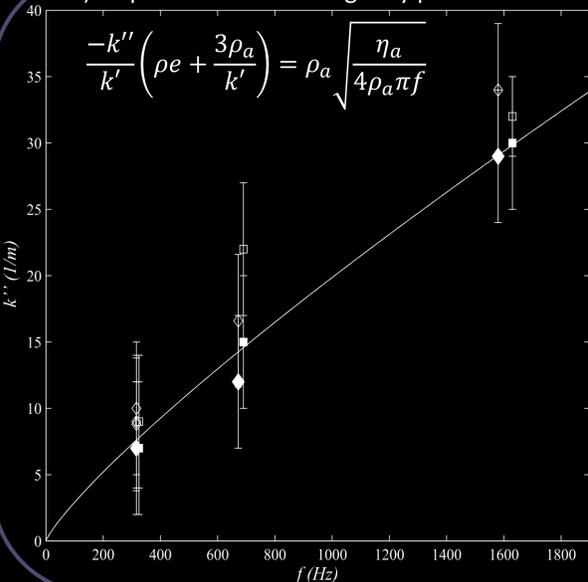


Solid line: fit using the equation in the inset, with the thickness e as a fitting parameter ($e = 0.08 \mu\text{m}$).

For comparison: The dotted line is the same expression with $e=0$, and the dashed line with $e=0.2 \mu\text{m}$.

Air mass is the main contribution to the film inertia.

7) Dispersion relation imaginary part

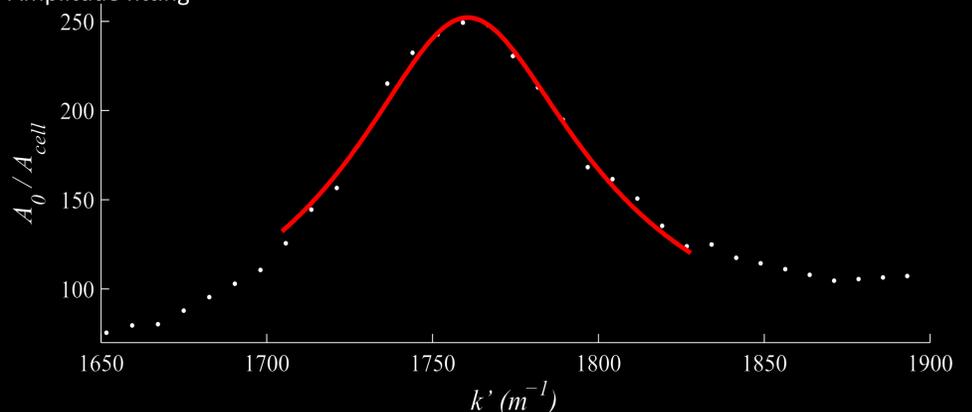


k'' as a function of the frequency f of the resonant modes. The experimental data are extracted from the fits of the amplitude. The theory is plotted from the equation in the inset.

Agreement between experimental data and theory is good.

This means that the main source of dissipation is viscous friction in the air around the film.

6) Amplitude fitting



Amplitude of the standing wave at $r = 0$, for the resonant mode $n=5$ (see above). The amplitude is rescaled by the amplitude of vibration of the experimental cell (A_{cell}). Solid line: fit of the data (expression $A_0/A_{cell} = \alpha / |J_0(R(k' + ik''))|$) with 3 fitting parameters (α, R and k'').

8) Conclusion and perspectives:

We have measured the attenuation of a transverse antisymmetric wave on a thin soap film. The amplitude and phase of the vibrating film were determined by measuring the deflection of a laser beam reflected by the film, at the forcing frequency.

We have computed the complex dispersion relation of the antisymmetric waves in the long wavelength limit. We have obtained analytical expressions for the real part and the imaginary part of the dispersion relation. The real part agrees with our measurements, with the film thickness e as a fitting parameter. Concerning the imaginary part, the very good agreement between the theory and the measurements evidences that, in our experimental conditions, the dominant source of attenuation is the dissipation by viscous friction in the air.

Further studies will investigate the dynamical coupling between the meniscus and the soap film, which is central to understand the acoustic propagation in liquid foams