Wave Propagation in Sandwich Panels with a Poroelastic Core

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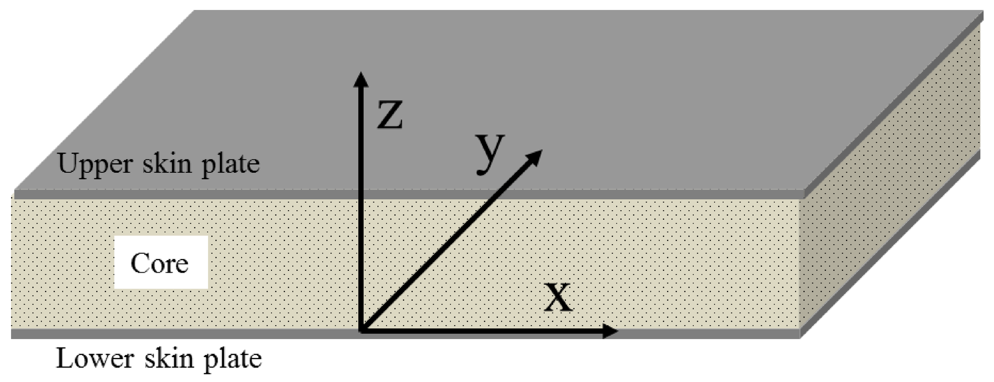
Motivations

Porous materials are of particular interest as the core of sandwich panels thanks to their acoustic absorbing property. An in-depth understanding of the panel's vibroacoustic behaviour will provide design guidance for such widely used structures in vehicles and buildings. This work studies the complex mechanism of wave propagation in sandwich panels with a poroelastic core, which is modelled with Biot's theory. The wave propagation characteristics may be used to predict sound transmission loss and estimate porous material properties.

Method

The semi-analytical finite element (SAFE) method as a powerful tool for wave propagation study is used to model the sandwich panel.



For porous medium which has constant property in the y direction, its field variables, based on the $\mathbf{u}-p$ weak form [1], are partially discretized using finite element ($\mathbf{N}_i$ where $i=u,v,w,p$) for the cross section and analytical functions ($\mathbf{v}$) for the y direction, given by

$$\begin{bmatrix} \mathbf{u}(x,y,z) \\ p(x,y,z) \end{bmatrix} = \begin{bmatrix} \mathbf{N}_u(x,z) \\ \mathbf{N}_v(x,z) \\ \mathbf{N}_w(x,z) \\ \mathbf{N}_p(x,z) \end{bmatrix} \mathbf{v}(y).$$

With an assumption that the analytical functions have a form as

$$\mathbf{v}(y) = \hat{\mathbf{v}} e^{i\kappa y},$$

the equations of motion yields a quadratic eigenvalue problem of wavenumber κ, given by

$$\left[\mathbf{A}_{00} + i\kappa(\mathbf{A}_{01} - \mathbf{A}_{10}) + \kappa^2 \mathbf{A}_{11}\right]\hat{\mathbf{v}} = 0,$$

where $\mathbf{A}_{00}$, $\mathbf{A}_{01}$, $\mathbf{A}_{10}$ and $\mathbf{A}_{11}$ are the generalized dynamic stiffness matrices.

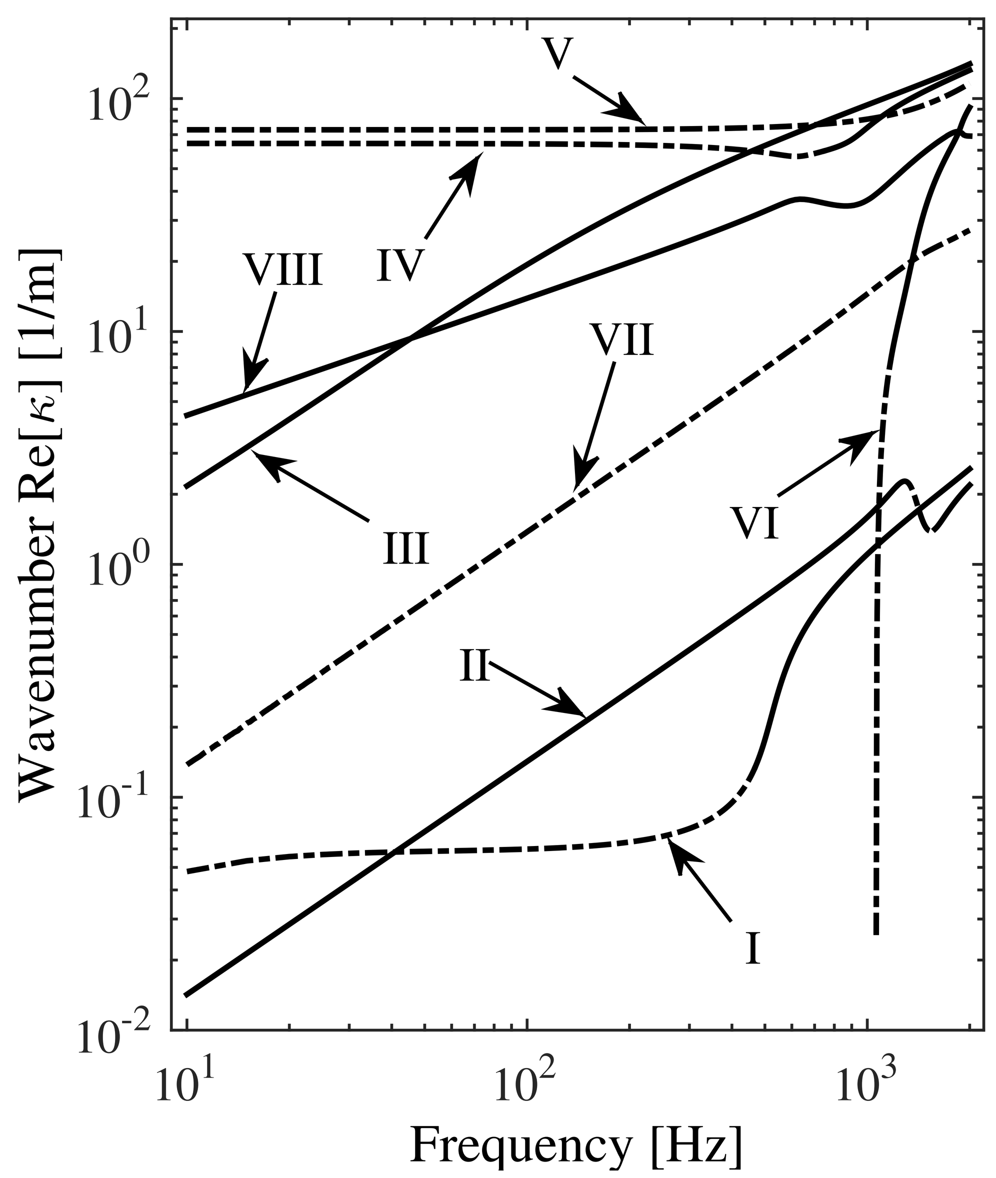
Model Parameters

The sandwich panel studied consists of two 0.5 mm thick aluminium skin plates and a 20 mm thick porous core, whose parameters are given in the table below.

Young's modulus *in-vacuo* (E), [Pa]	414×10^3
Density *in-vacuo* (ρ_1), [kg/m^3]	57
Poisson ratio (ν), [-]	0.24
Frame loss factor (η_c), [-]	0.19
Static flow resistivity (σ), [Ns/m^4]	55×10^3
Tortuosity (α_∞), [-]	1.05
Porosity (ϕ), [-]	0.95
Viscous characteristic length (Λ), [μm]	37
Thermal characteristic length (Λ'), [μm]	120

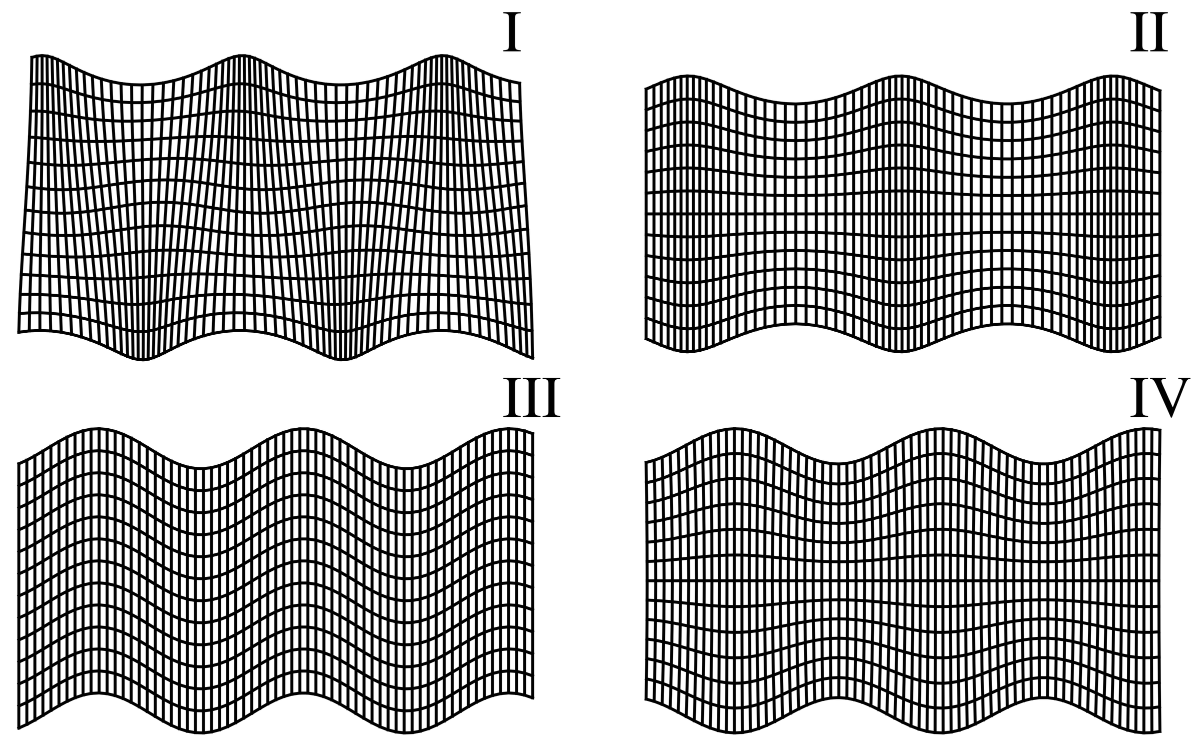
Dispersion Curves

The dispersion relation between wavenumber κ and frequency, by solving the quadratic eigenvalue problem, are plotted for 8 waves: I-VIII.



Propagating (solid lines) and evanescent (dashed dotted lines) waves are identified by observing variation of the Im[κ] against damping.

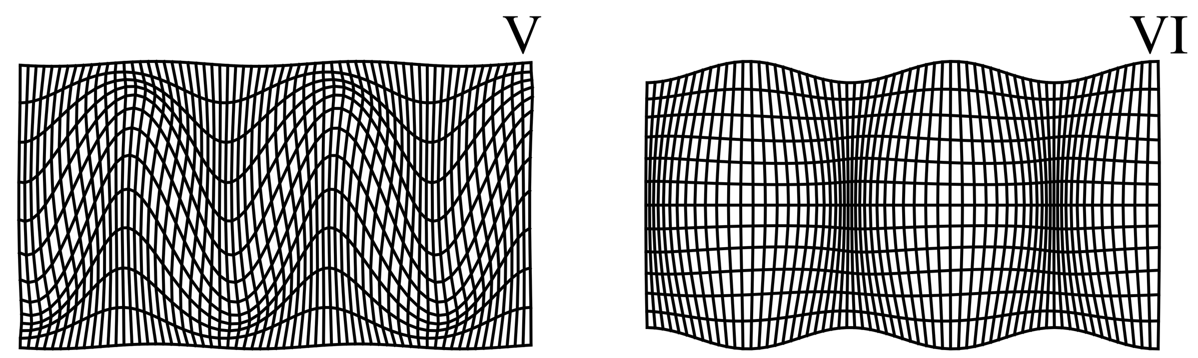
Skin Plates Dominated Waves



The bending and compressional waves existing in the two skin plates are antiphase and in-phase coupled by the core, leading to four waves.

- I Antiphase compressional waves coupled by the shear of the core.
- II In-phase compressional waves of the skin plates and the core.
- III In-phase bending waves of the skin plates.
- IV Antiphase bending of the skin plates coupled by the compression of the core.

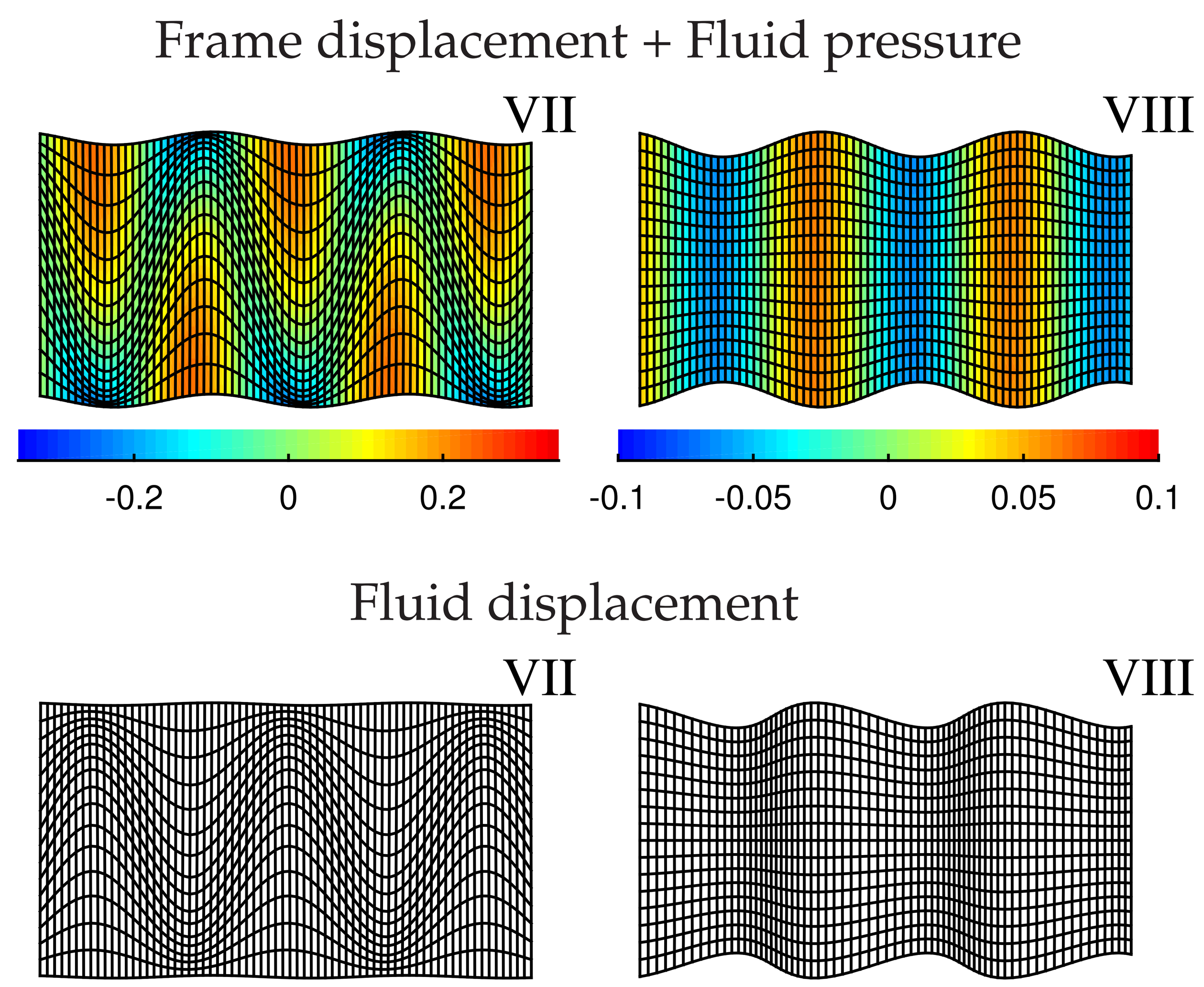
Core Frame Dominated Waves



The dilatational and rotational waves in the porous medium are coupled within the core.

- V Displacement v (in y) has one wavelength, and w (in z) has half a wavelength.
- VI Displacement v has half a wavelength.

Core Fluid Dominated Waves



The fluid-borne dilatational wave in the porous core plays an important role in the interaction with the skin plates.

- VII An oblique fluid-borne wave bouncing between the almost immobile skin plates. The fluid has high pressure at frame extension and low pressure at frame compression, which corresponds to an antiphase motion of the fluid and frame displacements. This antiphase motion introduces much viscous damping.
- VIII A plane wave with a "breathing mode" which pushes away the skin plates at high pressure and pulls the plates toward each other at low pressure.

Concluding Remarks

The propagation characteristics of 8 waves are investigated for a sandwich panel with poroelastic core, which is modelled using the semi-analytical finite element based on a $\mathbf{u}-p$ weak form. The 8 waves are studied within three groups, namely skin plates dominated waves, core frame dominated waves and core fluid dominated waves. More details such as energy distribution and simplified analytical models to calculate the dispersion relations are given in Ref. [2].

References

[1] N. Atalla, M. Hamdi and R. Panneton. *Enhanced weak integral formulation for the mixed ($\mathbf{u}-p$) poroelastic equations.* J. Acoust. Soc. Am. 109, 3065-3068 (2001).

[2] H. Liu, S. Finnveden, M. Barbagallo and I. Lopez Arteaga. *Wave propagation in sandwich panels with a poroelastic core.* J. Acoust. Soc. Am. 135, 2683-2693 (2014).