Characterization of porous materials having a rigid frame via transmitted waves at low frequencies.

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Abstract

An acoustic method is proposed for measuring the flow resistivity and the thickness of air-saturated porous materials. The conventional methods [1-3] for the measurement of the flow resistivity (or the viscous permeability) require the prior knowledge of the porosity. The method presented in this work is based on a temporal model of the direct problem in which a simplified expression (independent of frequency and porosity) of the transmission coefficient at the Darcy's regime (low frequency range) is established, this expression depends only on the flow resistivity and the thickness of a porous sample. The inverse problem is solved based on the least-square numerical method using experimental transmitted wave in time domain. Tests are performed using two samples of different thicknesses to same industrial plastic foam, thereby enabling the determination the thickness and flow resistivity of foam plastic. This method has the advantage of being simple, fast and efficient.

I. Model

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory [9]. In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid [4], which is a particular case of the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al*.[7] and the dynamic compressibility of the fluid included in the porous material $\beta(\omega)$ given by Allard [6], (ω is the pulsation frequency). In the frequency domain, these factors multiply the density of the fluid and its compressibility, respectively, and represent the deviation from the behavior of the fluid in free space as the frequency changes. In low frequency range the expressions of the responses factors $\alpha(\omega)$ and $\beta(\omega)$ when $\omega \longrightarrow 0$ are given by the relations [4]:

$$\alpha(\omega) = \frac{\sigma\phi}{j\omega\rho} \qquad\qquad \beta(\omega) = \gamma$$

III. Direct problem

The direct scattering problem is that of determining the scattered field as well as the internal field that arises when a known incident field impinges on the porous material with known physical properties. The reflected and transmitted fields are deduced from the internal field and the boundary conditions. The geometry of the problem is shown in Fig. 1.

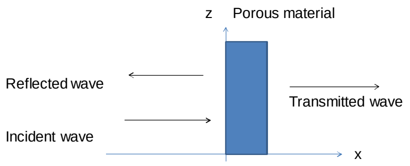

Fig.1 Geometry of the problem

The Darcy's regime [4,8] corresponds to the range of frequencies such that viscous skin thickness δ is much larger than the radius of the pores *r*, this is called the low-frequency range. In this domain the expressions of the transmission coefficient is given by:

$$T^{TBF} = \frac{1}{1+\frac{LC_2}{2C_1}}, \qquad \omega \to 0$$

with:

$$C_1 = \sqrt{\frac{\gamma\rho_f k_0 \phi}{\eta}} \qquad\qquad C_2 = \sqrt{\frac{\gamma\eta\phi}{K_a k_0}}$$

The incident $p^i(t)$ and transmitted $p^t(t)$ fields are related in time domain by the transmission scattering operator [1,3] T,

$$p^t(t) = \int_0^t \tilde{T}(\tau)\, p^i\left(t-\tau-\frac{(x-L)}{c_0}\right) d\tau. \qquad (6)$$

$\tilde{T}(t)$ is calculated by taking the inverse Fourier transform of the transmission coefficient of slab of porous material .

IV. Inverse problem

The inverse problem is to fond values for parameters, flow resistivity σ and thickness L, that minimizes the functions:

$$U(\sigma,L) = \sum_{i=1}^{i=N}\left(p^t_{exp}(x,t_i) - p^t(x,t_i)\right)^2$$

where $p^t_{exp}(\omega,t_i)$ is the determined transmitted signal and $p^t(\omega,t_i)$ is the transmitted wave predict from Eq. (6). The inverse problem is solved numerically by the least-square method.

Consider a simple of plastic foam, M, of two different thicknesses, the first M1 with a thickness of 10.1±0.1cm and the second M2 with thickness of 20.2±0.1cm, sample M was characterized using classical methods [5] given flow resistivity σ = 6500 ±500 $Nm^{-4}s$. Different frequency bandwidth have been investigated between (50 – 100)Hz. figure 2 show the experimental incident signal (solid line) generated by the loudspeaker and the experimental transmitted signal (dashed line) for the plastic foam samples M1 and M2.

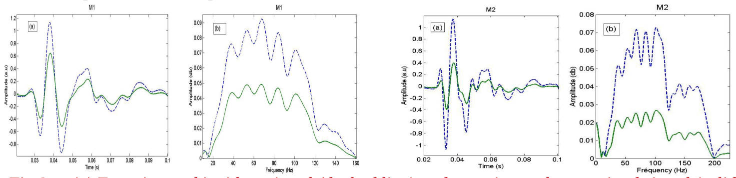

Fig.2. – (a) Experimental incident signal (dashed line) and experimental transmitted signal (solid line).(b) Spectra of the incident (dashed line) and experimental transmitted signal (solid line) for the plastic foam samples M1 (left) and M2 (right)

By solving the inverse problem for the flow resistivity and minimizing the cost function U, the obtained optimized values of the flow resistivity $\sigma1$ and $\sigma2$ are given by the table 1

Fréquency (Hz)	40-60	70-90	90-120	Average
$\sigma1$ (10^{+3} $Nm^{-4}s$)	6.40	6.51	7.00	6.63±0.30
$\sigma2$ (10^{+3} $Nm^{-4}s$)	6.51	7.00	6.75	6.75±0.25

Table1. Characteristics of samples M1 and M2 obtained by solving the inverse problem for the resistivity σ

We show the result of the inverse problem in Figs.3.

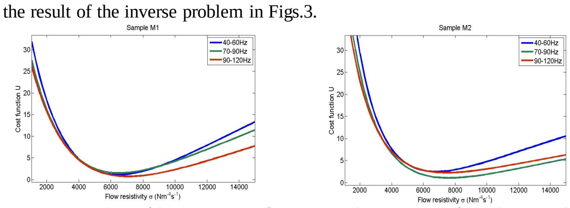

Figure 3 – Variation of the minimization function U with resistivity σ of samples M1 and M2

In the second case, we solve the inverse problem for measuring the thickness of the samples M1 and M2, assumed unknown, in the same frequency bandwidth of (40-100)Hz. The flow resistivity is fixed to σ = 6500 Nms^{-4} . By solving the inverse problem and minimizing the cost function U we obtain the following optimized values of the thickness of both samples M1 and M2 given by the table .2. In figs 4 we show the variation of the cost function U (L).

Fréquency (Hz)	40-70	60-80	70-100	Average
L1 (cm)	9.79	9.79	10.37	09.98±0.29
L2 (cm)	20.04	20.09	20.33	20.15±0.15

Table 2 – Characteristics of samples M1 and M2 obtained by solving the inverse problem for the Thickness L

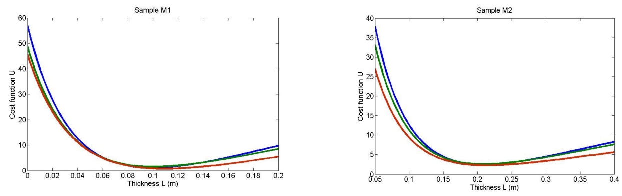

Figure 4 – Variation of the minimization function U with thickness L of samples M1 and M2

Using these optimized values; we compare the simulated transmitted signals and experimental signals. The results of the comparison are shown in figs.5. The correspondence between experiment and theory is good,

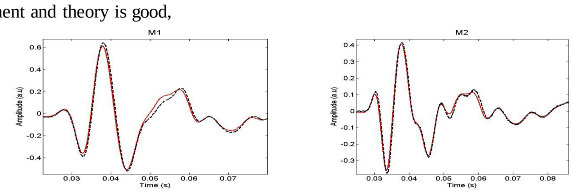

Figure 4. – (Color online) Comparison between the experimental transmitted signal (dashed line) and the simulated transmitted signals (solid line) using the reconstructed values of σ of samples M1 and M2

V. Conclusion

In this work, an inverse scattering estimate of flow resistivity and thickness was given by solving the inverse problem for waves transmitted by a slab of air-saturated porous material. The inverse problem is solved numerically by the least-square method. The reconstructed values of flow resistivity and thickness are close to those using classical method. The important result in this study is that is now possible, with the simplified expression of the transmitted coefficient, to measure the flow resistivity and thickness without knowing the porosity or any other parameters of the materials and just by using the experimental transmitted wave at low frequencies.

VI. References

[1] Z.E.A. Fellah, M.Fellah, F. G., Mitri. N.Sebaa, C.Depollier and W.Lauriks "*Measuring permeability of porous materials at low frequency range via acoustic transmitted waves*", Review of Scientific Instruments, 78 : 114, (2007).

[2] Z.E.A. Fellah, M.Fellah, F. G.Mitri, N.Sebaa and W.Lauriks, C.Depollier "*Transient acoustic wave propagation in air-saturated porous media at low frequencies*". J. Appl. Phys. 102, 084906, (2007)

[3] Z.E.A. Fellah, M.Fellah, N.Sebaa, W.Lauriks, C.Depollier "*Measuring flow resistivity of porous materials at low frequencies range via acoustic transmitted waves (L)*" J. Acoust. Soc. Am. 119(4):1926, (2006)

[4] Z.E.A. Fellah and C. Depollier, "Transient acoustic wave propagation in rigid porous media: a time-domain approach," J. Acoust. Soc. Am. **107**, 683–688 (2000).

[5] D. A. Bies and C. H. Hansen, "Flow resistance information for acoustical design," Appl. Acoust. 13, 357–391 (1980).

[6] J. F. Allard, Propagation of Sound in Porous Media Modelling. Sound Absorbing Materials (Elsevier, London, UK, 1993), pp. 1–284.

[7] D. L. Johnson, J. Koplik, and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media," J. Fluid Mech. **176**, 379–402 (1987).

[8] Z. E. A. Fellah, M. Fellah, and C. Depollier, Modelling and Measurement Methods for Acoustic Waves and for Acoustic Microdevices, edited by M. G. Beghi (InTech, Rijeka, Croatia, 2013), pp. 127–160.

[9] M. A. Biot, "The theory of propagation of elastic waves in fluid-saturated porous solid. I. Low frequency range," J. Acoust. Soc. Am. **28**, 168–178 (1956).